Chapter 5
Preparing Students for Decision-Making in the 21st Century – Statistics and Probability in The Australian Curriculum: Mathematics

Jane Watson
Denise Neal

The place of statistics in *The Australian Curriculum: Mathematics* has been controversial from several perspectives. These perspectives are considered throughout the chapter. Discussion includes a comparison of the content of the Statistics and Probability strand with the model of a statistical investigation leading to beginning inference. Relevant research that underlies the current or alternative content is presented. An important focus of the chapter is the relevance of context to the study of statistics and probability, along with the need for numeracy across the entire curriculum and the contribution of statistical literacy to this goal. Interviews with classroom teachers are used to document implementation of Statistics and Probability in the classroom and to gain suggestions on changes to the curriculum and professional learning required to assist further implementation. Work samples illustrate some of the implementation that has taken place. Synthesizing these aspects results in recommendations for further research in classrooms with teachers and students in order to obtain outcomes that will assist students in the 21st century to make meaningful decisions in relation to data and risk.

The Mathematics Curriculum

Following the lead in the United States (National Council of Teachers of Mathematics (NCTM), 1989), Chance and Data became one of the components of *A National Statement on Mathematics for Australian Schools* in 1991 (Australian Education Council). This *National Statement*, although not creating a "national curriculum" for the country, influenced most state curriculum documents for nearly 20 years. When finally *The Australian Curriculum: Mathematics* first appeared in 2010, Chance and Data had transformed into Statistics and Probability. This change was itself somewhat contentious due to the fear of intimidating primary teachers with the more sophisticated terms. The terminology move, however, reflected the perceived

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need to link concepts from preschool to Year 12, where the latter two terms were more appropriate.

In 1991, there was virtually no research available on the development of school students' understanding of concepts associated with statistics and very little related to probability. Curriculum decisions were hence made based on advice from tertiary statisticians, with very much of a top-down approach reflecting the needs of introductory units at university. In the intervening years, much research was carried out both in Australia and overseas that could have informed the writing of the new curriculum from a bottom-up developmental perspective. Some of this research was considered, whereas other aspects were ignored. Also apparently given slight attention were other nations' recent curricula (e.g., NCTM, 2006; Franklin et al., 2007; and Ministry of Education, 2007, 2009). What is presented in the new curriculum is, hence, a mixture of good suggestions and questionable propositions, constrained by pressure from the other two strands of Mathematics, Number and Algebra and Measurement and Geometry. Given that some mathematicians continue to hold the view that statistics should not be in the mathematics curriculum at all (e.g., Dean, 2010), it is important to work consciously to build on what is there to create a meaningful component of the curriculum that can lead to the critical statistical literacy required by students as they become citizens of Australia in the 21st century.

Although it is possible to argue that aspects of theoretical probability are part of mathematics based, for example, on calculating permutations and combinations to produce probabilities, statistics is arguably different in its non-deterministic nature, which demands special consideration. "Doing statistics" is about carrying out investigations that have more than one step and that encompass the stages shown in Figure 1.

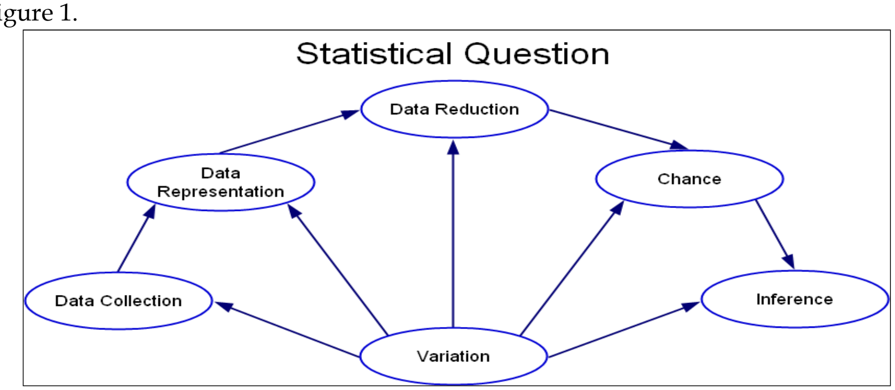


Figure 1. Model for a statistical investigation (Watson, 2009).

Understanding statistics is not just about calculating a mean or drawing a graph; it is about considering all parts of the model in Figure 1, drawing a conclusion to an investigation of a question based on evidence from data in a context, acknowledging variation and ultimately the uncertainty of the conclusion. To do all of this in every year, with ever-increasing complexity of data sets, context and available techniques, would be desirable but space in the curriculum puts constraints on what can be done. It is, hence, necessary to consider the components of the model in Figure 1 and explore the curriculum to find them.

A Statistical Question is the starting point of any investigation and immediately raises the issue of context and cross-curriculum links. Because there is no statistics without a problem from another field (Rao, 1975), immediately this part of the mathematics curriculum lends itself to satisfying the "numeracy across the curriculum" requirements of the National Curriculum Board (NCB, 2009). This link to other subjects may be one of the negatives that pure mathematicians see in a curriculum that they would like to see focused more specifically on mathematical theory and techniques. The Statistics and Probability component of *The Australian Curriculum: Mathematics* (Australian Curriculum, Assessment and Reporting Authority [ACARA], 2011c) begins with asking questions in the Foundation Year, with more complex scenarios being introduced over the years. Although the word "context" is not explicitly used until Year 5, the questioning suggested before then includes contexts such as hair colour, birdlife in the playground, popular breakfast cereals, and examples in the media. By Year 7 the descriptor, "identify and investigate issues involving continuous or large count data collected from primary and secondary sources" (p. 37), implies that context is involved and the elaborations provide examples. Further in Year 7 the elaboration for mean, median, and mode includes "linking them to real life." Working with context meaningfully in relation to fundamental statistical concepts, however, may involve both connection and disconnection at various stages of an investigation, as illustrated by the intervention of Pfannkuch (2011) in a Year 10 classroom.

The underpinning concept for the entire field of statistics is that of Variation (Moore, 1990) and although the term is not defined in the glossary, it is explicitly mentioned in Year 3 in relation to Chance. Why it is first linked to Chance and not to Data Representation and Interpretation is a mystery, but at least it is there and teachers can discuss variation across all phases of an investigation, where they begin with data from random generators, such as dice, or from other sources in real-world contexts. In Year 4, however, variability is one of the criteria used for evaluating the effectiveness of different data displays. Later in Year 8, a descriptor asks students to "explore the variation of means and proportions in representative data" (p. 40). This statement is important but it is unfortunate there are no elaborations to help

teachers interpret it. The type of elaborations that are appropriate are found in Watson (2005) for students from age 6 to 15 and in Shaughnessy (2006) especially related to studying graphs to characterise distributions. Hopefully by high school, thinking about variation will accompany every investigation.

The first component of an investigation (Figure 1) is Data Collection and from the Foundation Year students are "collecting information." In Year 1, they "gather responses" and in Year 2, "gather data." By Year 3, students "conduct chance experiments … plan methods of data collection and recording … and collect data …" In Year 4, survey questions are introduced as a data collection method. Year 5 contains the first explicit mention of collecting "categorical or numerical data". Although the elaboration of this descriptor in Year 5 describes a sampling situation, the actual term "sample" is not introduced until an elaboration in Year 6. The associated descriptor asks students to "interpret secondary data presented in digital media and elsewhere" (p. 33), an important inclusion, although the potential for using such data is clear in earlier years where context could lead to exploring data sets from outside of the classroom. As well at Year 6 the elaboration of this descriptor introduces the critical thinking necessary to question aspects of data collection and possible biases. Large data sets are mentioned in Year 7 and the implications of collecting representative data are made explicit in Year 8. In Year 9, the descriptor tying surveys from digital media to estimated population means and medians appears misplaced under Chance rather than Data Representation and Interpretation. It is not until this level that collecting data involving "at least one numerical and at least one categorical variable" (p. 43), including secondary sources, is mentioned. Research has shown (e.g., Watson & Donne, 2009) that students as young as Year 5, and certainly in Year 7, can handle these two types of variable, for example using class data and data from the Australian Bureau of Statistics website Census@School (www.abs.gov.au).

Data Representation is the second stage in statistical investigation (Figure 1) and is mentioned at every year level in the curriculum, becoming more complex with increasing years. There are many types of data representation and it was difficult for the writers to select ones that should be explicitly mentioned in the curriculum and at what year. Because technology is allowing much more variety in the types of graphs available in the media and hence in other subject areas (e.g., Wall & Benson, 2009), what the curriculum needs to encourage is flexibility in thinking about and interpreting graphical forms. The acknowledgement of the use of digital technology to create graphs from Year 3 is encouraging and the technology should be employed at every year level from there. Some of the specific suggestions of graph types are controversial, because they appear to underestimate the ability of students to handle the task. Explicit mention of graph types in the descriptors includes picture graphs in Year 2, picture graphs and simple column graphs in Year 3, the same in Year 4 with many-to-one representation, column graphs and dot plots in Year 5, side-by-side column graphs for two categorical variables in Year 6, stem-and-leaf plots in Year 7, back-to-back stem-and-leaf plots and histograms in Year 9, and box plots in Year 10. The most concerning omission is the scatterplot, which is not mentioned until an elaboration at Year 10. Research has shown that students can create (with technology) and interpret scatterplots as young as Year 6 (Watson & Donne, 2009; Fitzallen & Watson, 2010). In dealing with two numerical variables, a scatterplot is a natural representation.

Data Reduction is the next stage in an Investigation, usually associated at the school level with measures of central tendency (i.e., average) and spread (i.e., variation). As noted earlier, variation is considered generally throughout the years. The calculation and interpretation of the mean, median and mode, to measure central tendency, are not introduced until Year 7. The idea of "most" (or mode) is accessible to children as soon as they can count and compare numbers and should be noted much earlier. Working with the middle value of an ordered data set, such as the heights of students lined up at the front of the classroom, is also accessible to much younger children and builds intuitions about centre and representativeness. The measures are applied in exploring samples and populations in Year 8. This is an important point because although measures of average have been a focus of research for many years (e.g., Mokros & Russell, 1995), it has also been shown that often students do not employ the measures naturally when completing open-ended tasks, for example comparing two groups (e.g., Watson & Moritz, 1999). The range, as a measure of spread, is first mentioned in Year 7, outliers are noted in Year 8, and centre and spread are used to compare data displays in Year 9. In Year 10, the box plot provides a visual representation of both the centre and the "middle 50% of the data," as well as the spread. The importance of box plots is seen in the research of Pfannkuch (2006) exploring a teacher's struggles with comparing box plots.

Chance is separated from Data Handling and Interpretation in the curriculum and there is very little effort to link the two, which is unfortunate. There should be an intuitive recognition that chance and probability are involved in determining the confidence with which decisions are made in statistics. Much time is spent in the early years talking intuitively about the likelihood of events happening. After initial work with language, which progresses very little up to Year 4, the treatment of chance is very mathematical. Having introduced the frequency approach to probability in Year 6, sample spaces appear in Year 7. Explicit mention of contrasting the two approaches would be useful, as would acknowledgement that often chances are determined in a totally subjective manner. From the introduction of two-way tables in Year 8 through to Year 10, the descriptors are not explicit

enough in making clear to teachers and students what is meant. It appears that in trying to avoid language such as "conjunction" and "conditional," a vagueness associated with Venn diagrams and two-way table makes it difficult to understand the explicit intentions of the curriculum. Further elaborations or worked examples for problems exemplifying the descriptors are needed. Over the years there has been considerable research on students' understanding of probability (e.g., Jones, 2006) and it is unfortunate that outcomes were not used to inform the curriculum. Given the statistical tools that students have acquired by Year 10 they cannot determine specific numerical probabilities associated with the likelihood of a decision, for example about whether two populations are "different." Using box plots, however, and criteria suggested by Wild, Pfannkuch, Regan and Horton (2011), they could make informal likelihood statements about the populations represented by samples of different sizes.

Inference, the last stage in a statistical investigation, is acknowledged to a limited extent in the curriculum with the Interpretation part of Data Representation and Interpretation. The word interpret is used frequently from Year 2 to Year 10. Its use in later years should imply a sense of "beginning inference" (Watson, 2006) or "informal inference" (Makar & Rubin, 2009), where *evidence* is used to make a *generalisation* beyond the data with an acknowledgement of *uncertainty*. Although evidence is implicit in referring to data, it needs to be made explicit, as does uncertainty throughout and generalisation in later years. From the beginning, data in a context provide the evidence that students can interpret, for example, "the most popular fruit in our class is the banana." To create awareness of uncertainty, teachers can ask, "are we certain that banana will be the most popular fruit tomorrow?" To create awareness of generalisation beyond the classroom, teachers can ask, "will banana be the most popular fruit for our whole school?" or "would banana be the most popular fruit in Australia?" This sample-population relationship is alluded to in elaborations in Years 10 and 10A, but it is not obvious that there are opportunities for appropriate intuitions to be developed across the years of schooling. Over 30 years ago, Kissane (1978) illustrated the feasibility of addressing "intuitive inference" in the school curriculum. Recent research (e.g., Makar, Bakker, & Ben-Zvi, 2011) suggests many avenues for developing the reasoning behind informal statistical inference that can be introduced across the years before Year 10. Recognising the importance of conflict and context provides a contrast to much of the other content within the mathematics curriculum. Another aspect of inference is prediction (e.g., Watson, 2007) and although there is a promising start in Foundation to Year 2 in making "predictions about chance events" (p. 5), this is only mentioned once again, in Year 6, and the usage does not encompass data and the opportunity to predict with uncertainty in many other contexts.

Besides the Content strands of *The Australian Curriculum: Mathematics*, there are Proficiency strands, which describe the "actions in which students can engage when learning and using the content" (p. 2). The four proficiencies – understanding, fluency, problem solving, and reasoning – are all exemplified at various points within the Statistics and Probability component of the curriculum. The "how" and "why" of understanding are seen across the stages of an investigation when data are collected, graphs drawn, and statistics calculated. Fluency is seen when students recall methods for graphing and calculating statistics and carry them out accurately and efficiently. Students demonstrate problem solving when they design investigations to answer questions, plan strategies, and check outcomes; the verbs interpret, model, and investigate are particularly relevant to Statistics and Probability. The logical thought associated with reasoning includes evaluating, explaining, inferring, justifying and generalising, all of which are involved in a complete statistical investigation. Given the contextual nature of statistical investigations the ability to transfer learning across the curriculum illustrates all four proficiencies, but particularly reasoning.

The Wider Curriculum

Turning to the other three areas of the curriculum released at the same time as Mathematics, it is of interest to explore reference to data or statistics that reflects the NCB (2009) requirement for "numeracy across the curriculum," as well as providing meaningful contexts. *The Australian Curriculum: History* (ACARA, 2011b) provides a promising start with the statement, "Students need to organise and interpret historical events and developments and this may require analyses of data to make meaning of the past, for example to understand cause and effect, and continuity and change. This requires skills in numeracy such as the ability to represent and interpret quantitative data" (p. 7).

Although not explicitly stated, the gathering of information on families past and present and communities, as well as localities up to and including Year 2 would lend itself to tallying or creating pictographs as suggested for the same years in Data Representation and Interpretation. From Year 3 the descriptor under Historical Skills, "Locate relevant information from sources provided" (p. 20), evolves into "Identify and locate relevant sources, using ICT and other methods" in Year 8 (p. 40), continuing to Year 10. Posing questions begins in Year 2 with Years 3 and 4 suggesting a "range of questions" and Year 5 moving to "identify questions to inform an historical inquiry" and "identify a range of relevant sources" (p. 24). These reflect elements seen in Figure 1 as part of a statistical investigation. An elaboration in Year 6 suggests "retrieving census data to construct arguments for and against migration" (p. 27). In Year 9, an elaboration asks students to "[graph]

historical data to identify past trends and to draw conclusions about their significance" (p. 46). The Mathematics curriculum provides ample graphical forms for this enterprise. Further, in History in Year 10 students should, "[use] data from immigration records and [process] it (sic.) using ICT to identify historical trends over time" (p. 52) and "[combine] historical data from a range of sources to identify and explain the impact of World War II" (p. 52). Historical sources such as a data base on the First Fleet created by the University of Wollongong (firstfleet.uow.edu.au/index.html) can be used across year levels and have links to data handling skills in the statistics part of *The Australian Curriculum: Mathematics* (e.g., Watson, Beswick, Brown, Callingham, Muir, & Wright, 2011).

For Science, the links to statistics through "numeracy across the curriculum" are very strong, particularly in the Science Inquiry Skills strand where the five sub-strands of Questioning and predicting, Planning and conducting, Processing and analysing data and information, Evaluating, and Communicating fit very well with the model in Figure 1. Under the heading, Planning and conducting, for example, students are "Making decisions regarding how to investigate or solve a problem and carrying out an investigation, including the collection of data" (ACARA, 2011d, p. 4). Expanded under Numeracy (p. 10), these skills include "practical measurement and the collection, representation and interpretation of data from investigations … [considering] issues of uncertainty and reliability … collecting, analysing, and representing quantitative data in graphical forms, … identifying trends and patterns from numerical data and graphs … using statistical analysis of data and linear mathematical relationships to calculate and predict values." The intention is that graphical representations will be used across the years as they are developed in parallel within the mathematics curriculum, including scatterplots, linear graphs, and gradients (p. 14). It is salient that "using simple column graphs (bar graphs) with guidance from the teacher" is suggested in Year 2 (p. 22), whereas it does not explicitly occur in Mathematics until Year 3. In Years 5 and 6, "compare data with predictions and use as evidence in developing explanations" (p. 31, 34) reflects better the expectations of beginning inference than what is found in Mathematics, and describing "simple cause-and-effect relationships as shown by trends in quantitative data" (p. 34), does not appear in the Mathematics curriculum. Calculating means and ranges in Science appears in Year 9 (p. 45), a year or two after Mathematics. In Science, skills are repeated in the descriptors at later year levels with the elaborations intended to illustrate the more complex contexts for their consideration. The contexts suggested are those related to the understanding of the content sub-strands of the biological, chemical, earth and space, and physical sciences, and they vary widely. English (2009), for example suggests data modelling in relation to a class project exploring pollution in a local creek. Examples of explicit links between data handling and activities involving measurement of data, such as the number of drops of water that will fit on a five-cent coin or the percentage of weeds in a garden (Brown, Watson, & Wright, 2011), provide many opportunities for cross-curriculum classroom engagement.

Context and critical thinking are the major links between *The Australian Curriculum: English* (ACARA, 2011a) and *The Australian Curriculum: Mathematics* (ACARA, 2011c). “Numeracy can be addressed in English learning contexts across all year levels. Students select and apply … graphical [and] statistical … concepts and skills to real-world situations when they comprehend information from a range of sources and offer their ideas. When responding to or creating texts that present issues or arguments based on data, students identify, analyse and synthesise numerical information and discuss the credibility of sources and methodology” (ACARA, 2011a, pp. 10-11). Under the Literacy strand of English, the sub-strands of “Texts in context” and “Interpreting, analysing, and evaluating” provide the avenues for intersections with statistical thinking. Although there is no direct mention of numeracy- or statistics-related usage, an elaboration at Year 5 suggests “bringing subject and technical vocabulary and concept knowledge to new reading tasks” and “using research skills including … gathering and organising information … and summarising information from several sources” (p. 45). The introduction of media texts in Year 6 provides opportunity for synergies with statistical goals, also introduced in Mathematics in Year 6 (e.g., Watson, 2004). Cause-and-effect is mentioned in a descriptor at Year 7 (p. 55), following Science in Year 6 but there is no mention in Mathematics. At Year 8, the descriptor “Apply increasing knowledge of vocabulary, text structures and language features to understand the content of texts” (p. 60) could well apply to language from the Statistics and Probability strand of Mathematics. Similarly, “Interpret, analyse and evaluate how different perspectives of issue, event, situation, individuals or groups are constructed to serve specific purposes in texts” (p. 65) in Year 9 could well apply to investigations of media claims based on questionable statistics. Despite these general statements the lack of any specific mention of a context involving numeracy (or statistical literacy) is somewhat disappointing given the initial introductory statement on numeracy.

A final comment from across the curriculum arises in the preliminary *Shape* statement for Geography (ACARA, 2010, p. 8) in relation to Numeracy: “Students … count and measure, calculate statistics and interpret them, and construct and interpret graphs. In the senior secondary years, interpreting the results of statistical analyses requires an understanding of mathematical concepts such as probability.” Whether this final requirement will be met from the mathematics curriculum in Years 11 and 12 is uncertain.

There are three Cross-curriculum Priorities in the Australian Curriculum, all of which, given the previous discussion, lend themselves to data analysis. At the introductory level, “Aboriginal and Torres Strait Islander histories and cultures” provide a link between History and Statistics. “Asia and Australia’s engagement with Asia” can combine Geography, History and Statistics. “Sustainability” is likely to begin with Science and Statistics but also link to Geography. In all cases studying data bases can add depth and rigour to classroom investigations.

The overall General Capabilities within the Australian Curriculum are Literacy, Numeracy, Competence in ICT, Critical and creative thinking, Ethical behaviour, Personal and social competence, and Intercultural understanding. The concepts of variation and expectation, along with acknowledged uncertainty, the foundation of the Statistics and Probability curriculum, are readily enlarged to underpin the complexities of the general capabilities, particularly the final three. The first four are encompassed in the earlier discussion in this chapter, with “statistical literacy” listed as one of the six mathematical skills expected to be developed by students related to the Numeracy competency. In terms of ICT it is somewhat disappointing to note that although digital technology is mentioned periodically in descriptors in the Statistics and Probability section of the mathematics curriculum, statistical software does not rate a mention in the generic list of digital technologies, which includes “spreadsheets, dynamic geometry software, and computer algebra software [to] engage students and promote understanding of key concepts” (ACARA, 2011c, p. 9). Further the Achievement Standards at the end of each year level do not mention achievement related to technology in any form across the entire F-10 curriculum.

Reaction to the Curriculum

In the first stages of the rollout of *The Australian Curriculum: Mathematics*, teacher feedback was sought from 17 staff of a primary school in Tasmania and 5 secondary teachers who were leaders in their schools or consultants. When asked the areas in which they had confidence the majority of primary teachers were happy with basic chance/probability or the language associated with it (as covered in the early years of the curriculum), with the basic notions of data collection, and with graphing. A few teachers mentioned tables and picture graphs, as well as interpreting data. The picture reflects confidence in the basic skills of chance and data as reflected in previous curriculum documents but not the subtle or higher level expectations as students move in to middle school (e.g., Department of Education, 2007). For secondary teachers who were qualified in mathematics, teachers had confidence in all content areas. For less qualified teachers, usually in rural schools, despite knowing the “basics,” there was concern about struggling with the curriculum for students at specific year levels. Without the separation of lower, middle, and higher expectations for different students within a year, some teachers expressed lack of confidence with respect to both teaching and assessment, a difficulty not experienced with the standards in the Tasmanian curriculum. The "new" language was also seen as an issue for some teachers in the middle school.

When asked to name the areas where the primary teachers felt they needed more support, fifteen different aspects of the curriculum, with specific content, were noted. These included chance phrases, where a teacher desired "certainty" about the interpretation of words like "maybe" and "possible"; the linking of probability to fractions, decimals, and percents; comparing experimental and expected frequencies across (random) trials; interpreting the descriptors under Chance in the curriculum document; formal ways of displaying data in the early years; working with secondary data (from sources outside the classroom); different ways to collect data; recording data in various ways; relating data displays to the sample size; use of the media; an overall understanding of the terminology; and the need for cross-curriculum links. More generally the most frequent request by primary teachers was for access to and help with digital technology to represent data, including access to data bases. For the secondary teachers, professional learning needs across schools and teachers were varied but included the following: access to resources to move teachers away from text books; exposure to "good" large data sets; introduction and practise on software; data representation and interpretation in the senior years; designing tasks for specific proficiencies; finding and developing rich tasks; understanding correlation and its relationship to r-squared; and developing chance activities beyond rolling dice.

When asked for their concerns about the curriculum, teachers were generally happy with the Statistics and Probability component, although they felt there had been a "narrowing down" from the existing Tasmanian curriculum; for example, there was not the same emphasis on variation and expectation as in the Tasmanian document. They felt there was a need for more elaborations for the content descriptors, more like what is promised in Scootle <www.scootle.edu.au>, and directly linked to content. Concern about defining content for specific year levels was expressed by several teachers but others said they only viewed the content as a "progression," quoting one section of the curriculum acknowledging that teachers first should "identify current levels of learning and achievement and then select the most appropriate content (possibly from across several year levels)" (ACARA, 2011c, p. 12).

Specific improvements to the curriculum suggested by the secondary teachers included moving box plots and stem-and-leaf plots to earlier years; making clear the distinction between discrete and continuous variables in Year 7; taking a practical

approach to probability in Year 7 linking it to statistics via collecting data from chance activities to use for statistical investigations; improving the treatment of scatterplots (much earlier than Year 10) and incorporating technology to include lines of best fit; and including cumulative frequency, and a contrast of percentage and percentile, as precursors to studying box plots. Concern was also expressed that the expectation for time allocation for *The Australian Curriculum: Mathematics* is 5 hours/week in Primary and 4 hours/week in Secondary. It was claimed that schools that value mathematics may currently allocate only 3 hours/week to the subject and many allocate much less; this is not closely monitored, at least in state schools. With this issue in mind the curriculum was seen as "aspirational" with Year 6 outcomes perhaps being realistic at the end of Year 7.

When teachers were asked what contexts they were aware of or could suggest that would link to Statistics and Probability from across the curriculum, teachers mentioned Science, with some specifically highlighting the weather and, Literacy in relation to genre, writing, vocabulary, claims made, and favourite books. Geography, History and Physical Education were noted, as was using data collected from the class, for example transport to school, birthdays and family data. One teacher said it was important to use the media in relation to advertising found there using data to persuade. Teachers as a group were hence aware of potential contexts but from their previous responses it appears they lack well developed resources and technology to use them in their classrooms.

Work Samples

Some of the teachers who were interviewed provided work samples from units of work completed by students in their schools. The first example is from a Year 1/2 class where the students recorded the suburbs in which they lived. After class discussion, one of the representations created was a stacked dot plot using cubes of different colours. A photo was taken of the plot and used to create a sheet for the students to record their stories about the data they had collected. The range of responses was large and shows the potential for such an activity at the year level. Four examples of student work are shown in Figure 2.

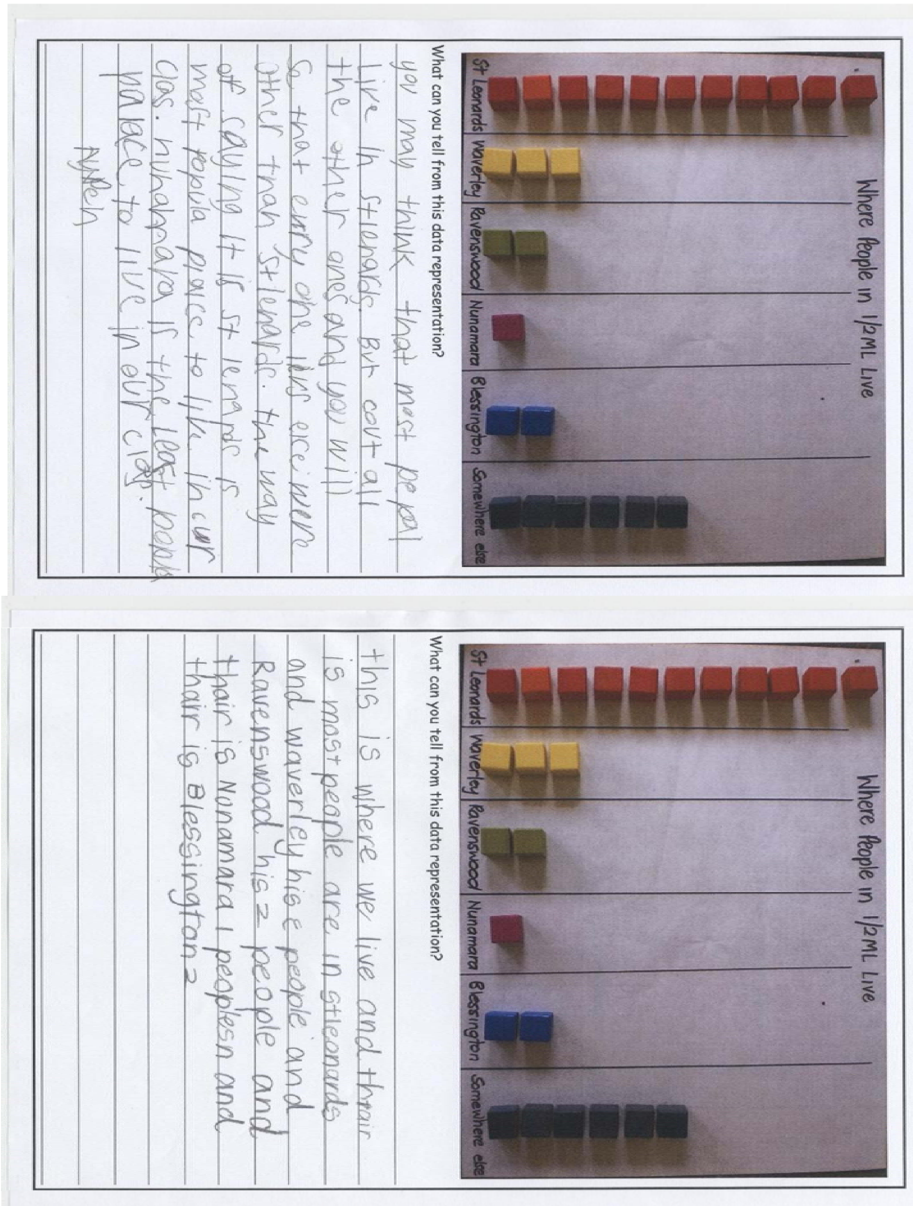


Figure 2. Year 1/2 student work.

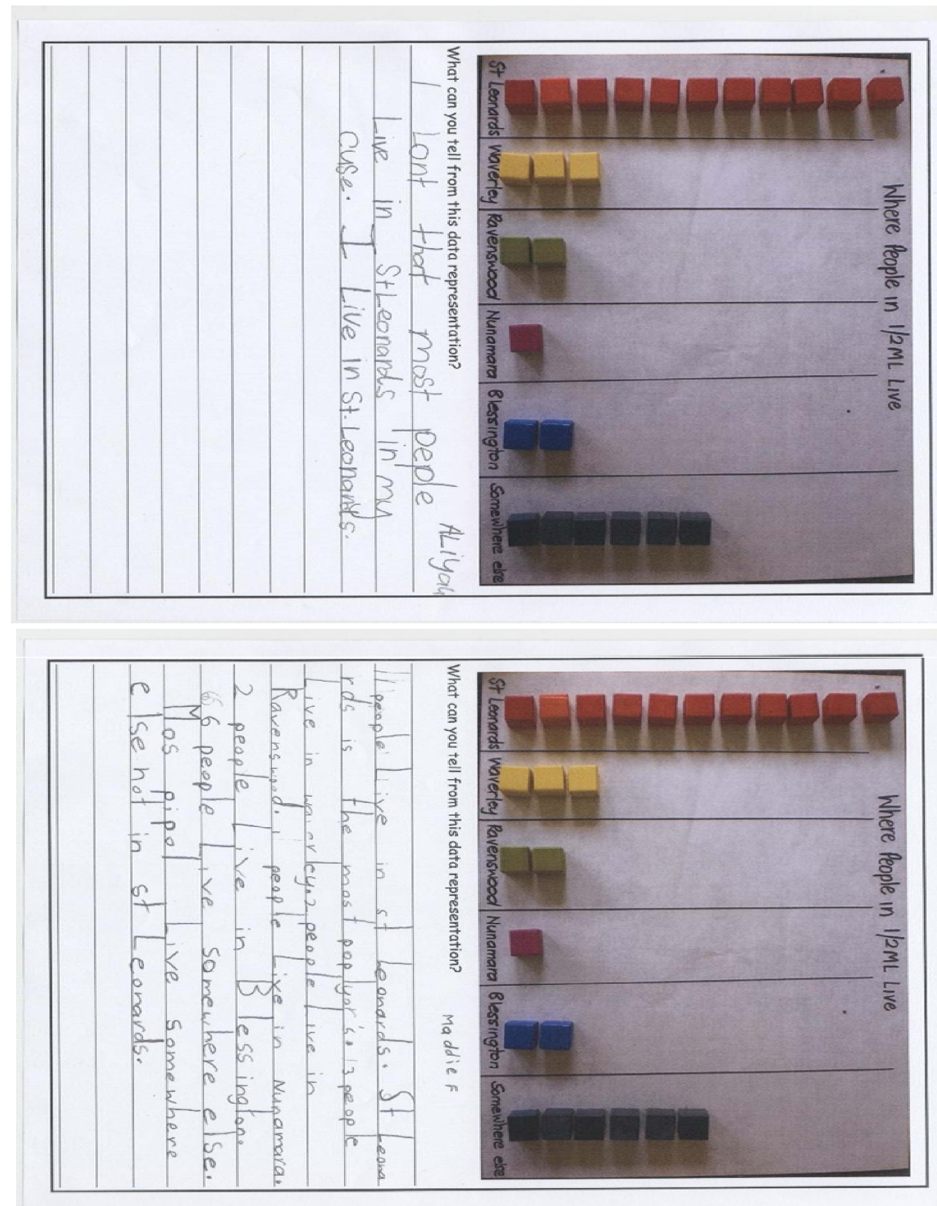


Figure 2. Year 1/2 student work (continued).

What is particularly interesting in the responses in Figure 2 is the reflection in the discussion about the "most popular" suburb and the total number of children who live in other suburbs. It appears clear that at least several students had picked up this distinction and could describe it. These two sets, "St Leonards" and "other suburbs" are disjoint and complementary with respect to the entire class. Although "Identify complementary events" is included as a descriptor in Year 8 for Chance, following the descriptor "Identify everyday events where one cannot happen if the other happens" in Year 4 (ACARA, 2011c, pp. 26, 40), it is clear that the foundation for understanding such concepts can be laid much earlier when children are discussing data related to themselves and their environment.

Figure 3 contains the "poster" prepared by a Year 4/5 class based on an investigation of the length of their first names, an investigation suggested by the Australian Association of Mathematics Teachers several years ago for National Mathematics Week. It illustrates the nature of a statistical investigation that is possible in the middle years, including much more than creating a graph, and satisfies the content descriptors for Data Representation and Interpretation in Year 5.

At the high school level one of the teachers explored data collected from the Australian War Memorial database to work with his mathematics class on data investigations before Anzac Day (P. Tabart, personal communication, 19/4/2011). This activity involved the use of the software *TinkerPlots* (Konold & Miller, 2005) with classes in Years 8 and 10. The data set was very motivating in an all-boys school and the reaction of the class was very positive. Figure 4 shows the type of data collected for 45 months of World War I that were held in data cards in *TinkerPlots*, some of the questions that the students considered, and three of the plots that were created for or by the class. The outcomes were consistent with expectations across Years 8 to 10, including working with representative data, investigating an international issue, using displays to investigate effects, using authentic data to construct scatterplots and draw conclusions, and use mean, median and range to interpret numerical data sets (ACARA, 2011c, pp. 40, 43, 46). This data set and the work in a mathematics classroom would provide an excellent adjunct to a unit in History at the same year level. Similar data sets exist or can be created for data from the Titanic, Australian explorers, or early Australian convicts (e.g., Watson et al., 2011).

Figure 3. Year 4/5 student work.

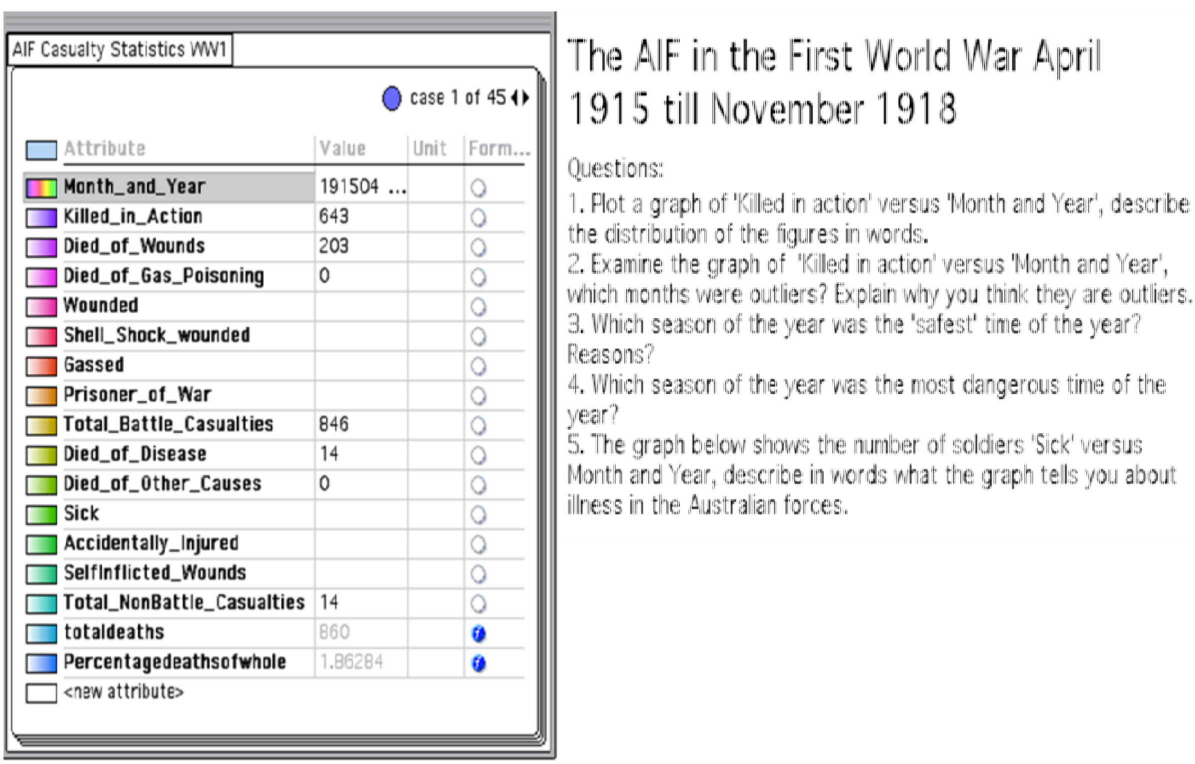


Figure 4. High school activities linked to Anzac Day.

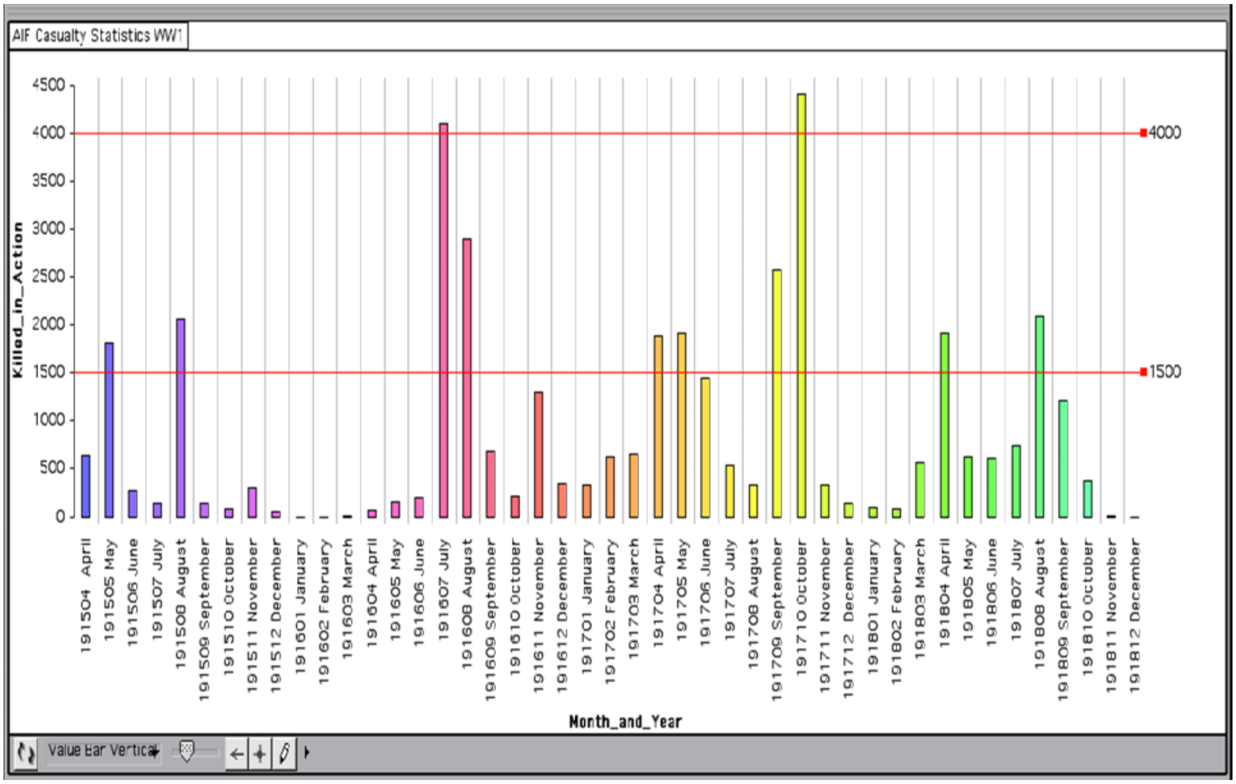
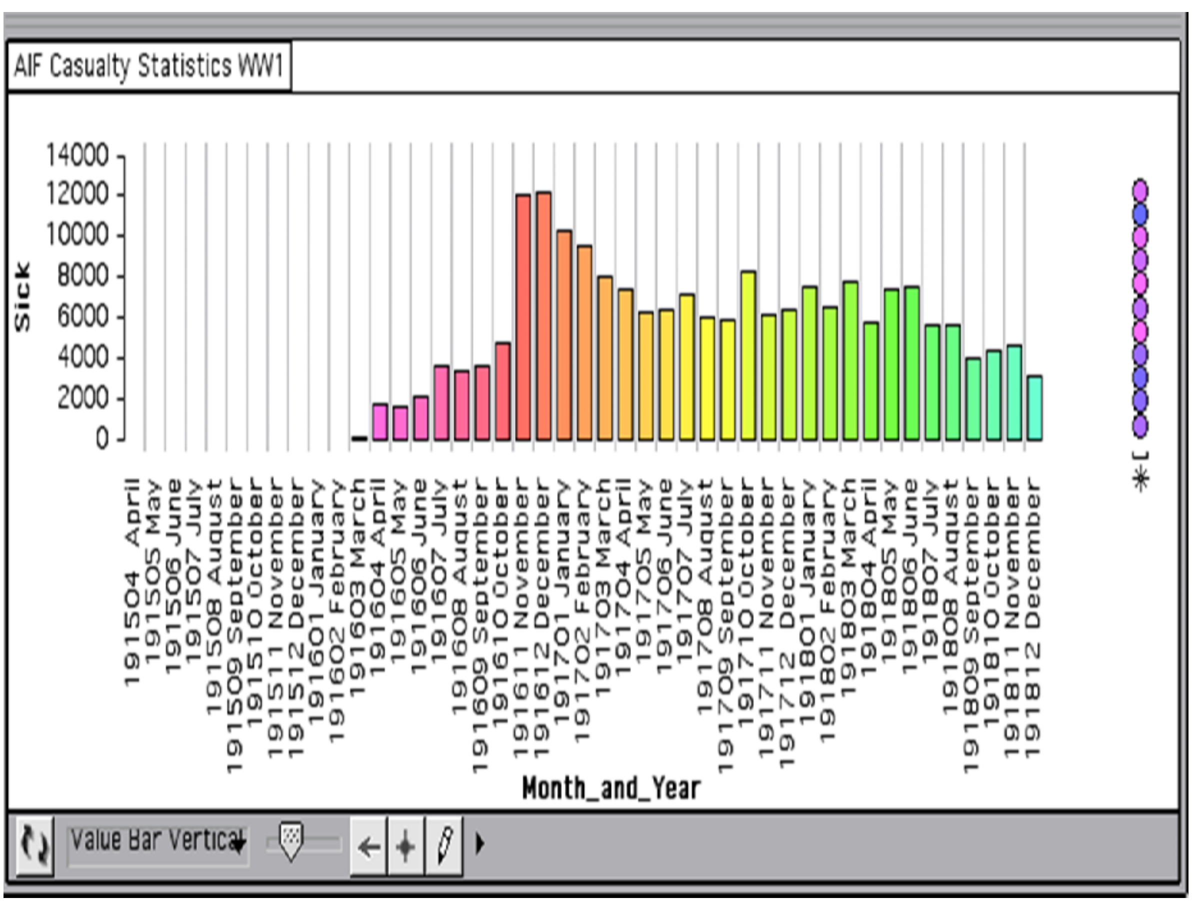


Figure 4. High school activities linked to Anzac Day (continued).



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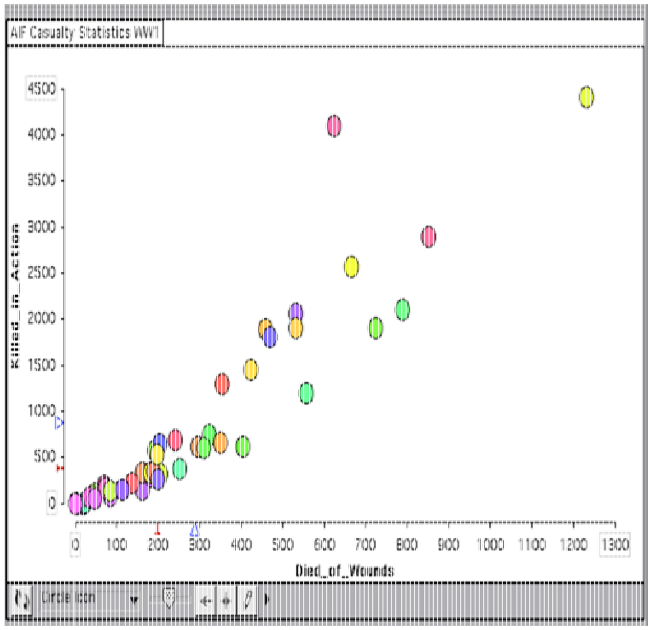

Problems to answer

1. Create a plot of Killed in Action. Use this to determine the following:
a) The mean number of soldiers killed in action per month.
b) The median number of soldiers killed in action per month.

2. Create a plot of Gassed against Month and Year. What conjectures can you make by looking at this graph? Give reasons.

3. Create two plots; one of self inflicted wounds versus month and year and the second plot of totaldeaths versus month and year. Examine both graphs side by side. What conjectures can you make in relation to the occurrance of self-inflicted wounds? Give reasons.

Figure 4. High school activities linked to Anzac Day (continued).

Implications and Further Research

As was seen at the beginning of this chapter, the components of a statistical inquiry are covered across the years of the K-10 mathematics curriculum. It is unfortunate that an effort is not made to reinforce their integrated relationship with each other at all levels of schooling. Teachers who are aware of the connections, however, can discuss and speculate with students each year, as was illustrated by the Year 1/2 class that provided the work samples in Figure 2. This is where professional learning can be of assistance, taking the elements of content with which teachers are familiar at each level and combining them to give a big picture of what statistics is about when considering a problem based in an authentic context. For primary teachers this can happen readily as most teachers cover the entire curriculum in their classrooms. For secondary teachers there is an urgent need to provide combined professional learning for teachers from differing curriculum areas, including mathematics. Teachers from other areas may need to increase their own skills in data handling and informal inference, likely supplemented by use of software, whereas mathematics teachers may need to increase their understanding of how project work and investigations are carried out in meaningful contexts in other areas. School time-tabling may be an issue for implementation of joint lessons but it was sorted out in one of the schools where a teacher was interviewed for this chapter. The three cross-curriculum priorities mentioned earlier related to Aboriginal and Torres Strait Islanders, Asia, and Sustainability all offer great potential for linking mathematics, through statistics, to other subject areas such as History, Geography, and Science.

Although the Content strands have been the focus of much of the discussion about the *Australian Curriculum: Mathematics* (ACARA, 2011c), the Proficiency strands provide the mechanism for implementation of the curriculum. The proficiencies are likely to be the foundation of meaningful professional learning for teachers, assisting with the development of pedagogies to aid understanding, fluency, problem solving, and reasoning. Teachers need to be aware of these four aspects of their own thinking before creating learning experiences for their students. As specifically stated in the curriculum, Problem solving for example is illustrated by "interpreting sets of data collected through chance experiments" (Year 7, p. 34), "interpreting data using two-way tables" (Year 8, p. 38), and "collecting data from secondary sources to investigate an issue" (Year 9, p. 41). Significantly, reasoning includes "inferring from the results of experiments" (Year 6, p. 30), "making inferences about data" (Year 8, p. 38), and "evaluating media reports and using statistical knowledge to draw conclusions" (Year 9, p. 41). The reference to "inferring," to "inferences," and to "drawing conclusions" in the Proficiencies, when the word inference does not appear in the Content descriptors or elaborations,

points to the need for professional learning for teachers based around the development of informal inference across the schools years (Makar, Bakker, & Ben-Zvi, 2011; Makar & Rubin, 2009) in order to enhance the meaning of "Interpretation" in the substrand heading Data Representation and Interpretation.

There is much research to be carried out in relation to the implementation of the new curriculum. Some of the questions to be answered include the following. What are the links between students' development of proportional reasoning and their understanding of box plots? How does the use of interactive software facilitate development of problem solving and reasoning for statistical investigations? What is the best way to transition between probability seen as trials with random devices and probability seen as the measure of uncertainty for statistical investigations? How can stem-and-leaf plots be used to reinforce student understanding of place value in the lower primary years, as well as providing early intuitions about the shape of data distributions? How can statistics contribute to the understanding of estimation and approximation in the Measurement and Geometry component of the curriculum by displaying and measuring variation (e.g., Watson & Wright, 2008)? Following the research of Watson (2005), how can variation and expectation be woven seamlessly throughout the curriculum to build the foundation and intuitions needed to understand formal inference in later encounters with statistics? As part of cross-curriculum work, how can students' appreciation of perceived risk as a function of hazard and outrage (Sandman, 1993) be built upon the Probability curriculum and transformed into social contexts to assist students in making safe life decisions?

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Authors

Jane Watson, University of Tasmania.
Email: Jane.Watson@utas.edu.au

Denise Neal, Department of Education, Tasmania.