Misperceptions in Mathematics and Music: An Intra- and Inter-Disciplinary Investigation Employing Applied Metacognition via Computer-Aided Learning

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This presentation deals with the common but largely unnoticed and undocumented experience of misperceptions (as opposed to misconceptions) in the contexts of mathematics and music. While learners may possess an accurate conceptual understanding of specific tasks, misperceptions may hinder successful problem solving. Using the study's computer software, this presentation will demonstrate misperceptions existing in these subjects, show how misperceptions confound learning problems diagnosis, how they sabotage the process of conceptualisation in learning, and how synergy between the disciplines might enhance learning.

This interdisciplinary ongoing project aims to investigate the common but largely unnoticed and undocumented experience of *misperception* as identified in the learning contexts of music and mathematics. While learners may possess an accurate conceptual understanding of a specific task, their misperceptions may hinder their successful problem-solving. Everyday examples of misperceptions exist: proof readers misperceive typographical errors (by not seeing them) and people opening fridges may unsuccessfully search for what is right in front of them (by misperceiving what they see). In no way do proof readers and fridge openers have misconceptions about what they are looking for. While *misconceptions* are widely reported in the literature (Perso, 1999; Treagust, Harrison & Venville, 1996; Metioui, Brassard, Levasseur & Lavoie, 1996; Saxena, 1992; Brown, 1992; Linder & Erickson, 1989), the effects of misperception on learning, although having potential catastrophic educational consequences, are being largely neglected. This could be because this phenomenon might not be widely understood and/or because learning problems are misperceived to be the result of student misconceptions.

Rationale

In music education, a critical interaction between student misperceptions and misconceptions has been identified and documented in a limited way (Lamb, 1984). The confounding effects of misperception on musical learning (as distinct from misconception) were first detected when incorporating metacognitive strategies into aural training (Leong,1998). Subsequently, this issue has become a very significant challenge as the authors developed teaching materials including software in an ongoing CUTSD project (2000-2001). The misperception of a musical example can interfere with the conceptualisation of what is heard. To cite two extreme cases: a student may listen to a musical example and be able to sing it perfectly in tune and with impeccable rhythm but make mistakes while notating it. These mistakes cannot be due to misperception (because the student reproduced it accurately); rather, they are likely to be consequences of misconceptions regarding the symbolic representation of what was just heard. When translating from sound to symbol (and vice-versa), various levels of cognition are involved; a crucial component involves the conceptualising of musical patterns. In order to accurately notate a musical example, one must correctly perceive it, and properly conceptualise the musical patterns in what was perceived, thereby translating faultlessly the sound to symbols. In a second case, a student who sings completely different notes to the musical example just heard might correctly notate what was sung, thereby producing a 'wrong' answer. In the latter case, there are no misconceptions but the misperceptions sabotage the learning process and create the illusion that the student is incompetent. This misdiagnosis is a tragedy. The phenomenon of misperception interfering with learning is probably widespread in all disciplines and causing much distress to students and teachers who are completely unaware of it.

Hence the purpose of this intra-disciplinary study to determine the nature and extent of misperception within each learning area. Recently, there has been an international interest in having an inter-disciplinary approach adopted in education. Venville et al. (1999) report this phenomenon as follows: "Internationally, the past decade has seen several influential documents advocating integrating science, mathematics, technology and other content areas" (p. 2). By integrating mathematics, music and information technology, this study supports such an approach by seeking the synergistic benefits of cross-curricular studies and attempts to identify the misperceptions and misconceptions that would impede learning in a multi-disciplinary environment.

The main purpose of this study is to identify *misperceptions* in mathematics education, based on our understanding of misperception in music education; and in so doing, integrate our findings with the extensive research in *misconceptions* in mathematics and science education. This research is of importance because its applications will impact upon the teaching of mathematics (and science) at many levels.

It is anticipated that applications of this research will lead to: (1) improved student understanding and retention of new knowledge over an extended period of time, (2) more efficient and effective teaching practices in which the re-teaching of concepts will be unnecessary, (3) improved teacher understanding of student learning difficulties, (4) new techniques for diagnosis and remediation of student learning problems, and (5) a better understanding of how to implement an inter-disciplinary integration of subjects in schools.

Research Questions

The four main research questions are:

1. To what extent do misperceptions exist in mathematics and music for the general student population?
2. How do misperceptions confound the diagnosis of student learning problems?
3. How do misperceptions sabotage the process of conceptualisation in student learning?
4. How might the synergy between the disciplines enhance student learning?

While some researchers from music psychology have examined perceptual problems in decontextualised and over-simplified scenarios (Louhivuori, 1999), this study aims to grapple with the 'messiness' (including 'unpredictability' and 'irrationality') of real life that the use of metacognitive software can help unravel the intertwining of misconceptions and misperceptions in the learning process of individual students. We have identified two techniques to deal with this: (1) by analysing what students sing immediately after hearing a musical example, student misperceptions can be identified; (2) by playing back the notated version, students can be confronted with their misperceptions and misconceptions simultaneously. Corresponding techniques in mathematics and other disciplines might be: (1) listening to students 'think-aloud' while attempting to solve a problem (Gorgy, 1998; Gorsky & Finegold, 1994), and (2) simulating the consequences of the students' incorrect answers (Gorsky & Finegold, 1994). While these techniques are already successfully being applied in both maths and science education (Confrey, 1990; Driver, 1995; Driver & Erickson, 1983; Duit, 1993; and Pfundt & Duit, 1994), their applications are not being directed specifically to diagnose or treat misperceptions. An extensive literature search has identified only two examples of misperceptions in mathematics found in students either with dyslexia or cerebral palsy (Malmer, 2000; Bottge, 1999; Shaw, Durden & Baker, 1998). Although otherwise intelligent, these students exhibited extreme forms of misperceptions; similar misperceptions may be experienced by the general student population to a lesser degree.

The music software Leong and Lamb (2001) have developed has helped reveal some of the misperceptions and misconceptions of music students in rhythmic and melodic dictation, both non-trivial real-world tasks. It has enabled the diagnosis and remediation of these problems in an initial exploratory manner (Leong, 1998). Despite the limited scope of previous studies, the mystery of misperceptions needs to be further unravelled in music and mathematics, and we believe that the diagnostic and remedial techniques developed in music can be beneficial to other disciplines such as physics.

The area identified for this particular study is the mathematics of spatial visualisation where misperceptions and misconceptions are known to occur but have not been adequately investigated.

Approach

Guided by the research questions, there are four phases in this intra- as well as inter-disciplinary project. Each of the disciplines (mathematics and music) are being studied simultaneously and separately during the first three phases, with the authors/researchers meeting regularly to share observations and problems, and to discuss lessons gained in the respective disciplines.

Method

This study employs a mixed-method approach to address the four research questions stated earlier. It supports the recommendations by the Curriculum Council of Western Australia to give students "frequent opportunities to see the connections between different areas of knowledge and endeavour" (Curriculum Council, 1998, p. 27). It also incorporates the "qualitative, visualisable models" which Brown (1992) recommends to be developed to provide "mechanistic explanations for phenomena" (p.17).

Participants

As there is some indication that problems with student learning originate in the middle school years (Cormack, 1996; Hargreaves, Earl & Ryan, 1996; Speering & Rennie, 1996), participants of this study involve one class each of students from Years 8, 9, 10 and 11 as well as their teachers. Two schools per discipline (totalling six) with networked computer laboratories are involved. A sample of convenience was employed since there is no evidence to suggest that any particular group of the general student population suffers from learning problems caused by misperception.

Phase 1

Step 1: Develop instruments. Tests were developed for identifying students' misperceptions. Two types of tests (instruments) were created: pencil and paper and computer-based. The computer enables snap-shots with time stamps to be taken of student problem-solving processes that facilitate accurate diagnosis and analysis. It also correlates the thinking-aloud of students with the problem-solving actions of students. The software enables the immediate construction and presentation of the consequences of students' misperceptions and/or misconceptions. Moreover, the non-intrusiveness of the software in tracking student 'think-alouds' is another benefit of applying information technology to this study.

The following provides some examples of how the tests are used to identify misperceptions in the two disciplines. In *mathematics*, students are shown a picture and given a transformation to apply to it. They are asked to transform it in their mind, and then draw the result of the transformation requested. Any discrepancy could be due to faulty visualisation or faulty drawing skills. In *music*, students will be asked to sing what they have just heard. Any discrepancy might be caused by a perceptual (hearing) problem or a vocal reproduction problem. The former is the more common in our experience. Students will also be asked to verbalise ("think aloud") while they solve the given tasks (the verbalisation will be recorded). A computer-aided approach enables students to be presented with the reality of their descriptions and be confronted by any discrepancies between their descriptions and what the computer presented. The initial exploration of student misperceptions and misconceptions in mathematics and music is being carried out in the Physics Studio at Curtin University.

We have identified the software that can help us diagnose misperceptions and misconceptions. Although these programs have been used successfully in mathematics, music and physics education (Lamb & Beirne, 1986; Mee, 1988), they were designed without an awareness of student misperceptions and misconceptions. The software is being modified and customised for the specific purposes of this study. The developer and owner of Mathemagic and MusicLand (described in detail below) is one of the authors/researchers and is modifying his software continually.

Steps 2-5. The remaining steps in Phase 1 were as follows.

2. Identify participating schools and obtain permission for research access.
3. Pilot the paper instrument.
4. Initial meetings with teachers.
5. Findings of pilot (paper instrument) to be applied to features in the software instrument.

Phase Two

Interviews with teachers to ascertain the diagnostic techniques employed by them in their teaching. What teachers attribute the causes of learning problems to be will also be a focus here. There are six steps:

1. Processing of interview data using the NUDIST software (Richards & Richards, 1994).

2. Students’ construction of concepts will be studied using the paper instrument.
3. Analysis of data from the paper instrument.
4. Findings of paper instrument applied to features in the software instrument.
5. Piloting of the software developed in Phase One followed by its refinement.
6. Dissemination of findings from Phase I.

Phase Three

Students’ construction of concepts will be studied using the software test.

Analysis of data from the software instrument using the ‘analytic induction’ technique (Goetz & LeCompte, 1981): “This strategy involves scanning the data for categories of phenomena and for relationships among such categories, developing working typologies and hypotheses upon examination of initial cases, then modifying and refining them on the basis of subsequent cases ... Thus, the discovery of relationships, that is, hypothesis generation, begins with the analysis of initial observations, undergoes continuous refinement throughout the data collection and analysis process, and continuously feeds into the process of category coding. As events are constantly compared with previous events, new typological dimensions, as well as new relationships, may be discovered” (cited in Gorsky & Finegold, 1994, p. 82).

There are two steps in this phase:

1. Meeting with teachers for validation and feedback purposes.
2. Identification of students suffering from problems with misperceptions (for Phase 4).

Phase Four

Selection of students suffering from problems with misperceptions. Using the software developed, the effects of an interdisciplinary approach on student learning will be studied over a semester. This phase also includes a final meeting with teachers¾with “crystallised” examples presented for feedback¾and the analysis of the data, including verbal protocols, computer-tracked problem-solving files of student work.

The combination of computers with sound synthesis technology permits us to work with real maths and real sounds. This is in contrast to most other subjects where the computer can only simulate reality. Virtual reality is a pale substitute for the real thing. Furthermore, virtual reality is perceived by many students as ‘fictitious’, thus it is not always believable to them (Yeo, Loss, Treagust & Zadnick 1999; Gorsky & Finegold, 1994). Because music, via technology, provides the explicit reality corresponding to mathematical and physical models, students can directly perceive the validity of the models which they manipulate on the computer screen and thereby make the essential connections to effect authentic learning. Two software products which possess such functionality have been identified: Mathemagic (Lamb, 1995) and MusicLand (Mee, 1988; Lamb, 1981, 1982). By providing an interactive environment that is instantaneously responsive, this software can be modified to help us identify misperceptions and misconceptions of individual learners.

Briefly, spatial visualisation is addressed in MusicLand by allowing students to perform spatial (mathematical) operations, viz. reflection, enlargement and translation (sliding), applied to graphical musical notation to produce musical results. Students can then hear the corresponding musical transformations, i.e., retrograde and/or inversion, augmentation/diminution, and transposition, respectively. Thus, students are able to see and hear their spatial transformations, thereby reinforcing these concepts in a multi-modal and multi-disciplinary manner.

The Mathemagic software allows students to explore the spatial transformations of reflection, enlargement, translation and rotation. These transformations can be applied individually or in combination, and students are asked to solve problems by constructing a sequence of transformations and visualising the results mentally.

The MusicLand software demonstrates how musical instruments produce sounds that are combination of sine waves. Students can manipulate a histogram to alter the timbre of a sound by changing the relative strengths of the sine waves (overtones). As the mathematical model is manipulated, students instantaneously hear the corresponding changes in the timbre. Students can also edit the shape of the sound waves and hear the consequent effects instantaneously. Concepts that students can explore include the superimposition of sound waves the addition of sine waves, sum- and difference-frequencies, the attack-decay-sustain-release of a sound (amplitude-time graph in maths), pitch (frequency), duration, loudness (amplitude), consonance and dissonance and psycho-acoustics.

Another important aspect of the study is the interaction between the disciplines of mathematics and music. For example, to what extent does a misperception of sound give rise to a misconception in maths? While we employ metaphors, analogies and mathematical models to describe the phenomenon of sound, mathematics provides idealised abstractions in the form of sine functions and logarithms. Music provides the reality and physical experiences corresponding to mathematical abstractions. For example, to play a musical scale on a violin is really about positioning the fingers sequentially in a logarithmic relationship to each other on the fingerboard. There are definite educational benefits in engaging student learning through an inter-disciplinary approach, and some evidence of this is beginning to appear. According to Linder & Erickson (1989), an understanding of the phenomenon of sound provides “an ideal foundation for some of the ideas that a student would encounter later in quantum mechanics” (p. 499).

The conference presentation will focus on the study’s software. Brief details follow.

Description of Software

Mathemagic

This software provides opportunities for problem solving using the following operations:

- Translation (sliding) up or down by one unit; left or right by one unit.
- Rotation through a quarter turn (90 degrees) clockwise or anticlockwise.
- Dilatation (scaling) by a factor of 2 or 1/2.
- Reflection in the x- or y-axis, or in the line $y = x$, or in the line $y = -x$.

A typical problem is presented as follows: the original picture on the screen (in black), is shown together with the end result (in blue) of several transformations. The challenge for the student is to find a combination of operations that turns the (original) black picture into the (transformed) blue one. The solution may be entered in one of two ways: (1) each of the student’s moves may be shown when it is selected, or (2) the moves may only be viewed *after* all the moves have been chosen, thus requiring the student to visualize the entire sequence of transformations.

MusicLand

Of the four interconnected learning programs (Music Sketchpad, Sound Factory, Timbre Painting and Music Blocks) within MusicLand, three are directly relevant to this study and will be demonstrated during the presentation.

Music Sketchpad. When the user draws a curve (on the computer screen), it is interpreted as a Pitch-Time graph. The curve "crystallizes" into a graphical notation (as used by some contemporary composers) and the computer plays the corresponding music. A portion of the notation may be selected by the user, and transformed as follows: (a) Translation (sliding) up or down, left or right, (b) Enlargement (or reduction) in the X direction, or in the Y direction, (c) Reflection in a horizontal line, or in a vertical line. The user may listen to the transformed portion of the music, and also "paste" copies (which may be further transformed) onto the Pitch-Time graph (i.e. onto the graphical notation). The original portion and all the pasted transformed versions are displayed in the same colour. Each time another portion of the music is selected for transforming, a new colour is used. In this way, the user sees a colour-coded thematic analysis of the evolving composition.

Sound Factory. Here, the user may examine the properties of the sound of a particular instrument. Each overtone is shown as a bar on a histogram, with the corresponding sine wave displayed beside it (the length of each bar indicates the amplitude of that overtone). One period of the sound wave (which is the sum of all the component sine waves) is also displayed. An Amplitude-Time graph shows the amplitude envelope (attack-decay-sustain-release) of each note, simplified as straight-line segments. Normally, the computer produces a steady sound corresponding to the histogram and its associated waveform, at a pitch chosen by the user. The user may edit the sound *in real time* by adjusting the bars on the histogram (thereby changing the relative strength of the overtones) or by redrawing part (or all) of the waveform. The timbre of the sound changes instantaneously as the parameters are adjusted. When the user edits the amplitude envelope, the sound is heard in pulses; the attack, decay, sustain, and release of each pulse are controlled by the graph. Each change to the graph is heard in the subsequent pulses.

Timbre Painting. The graphical notation for the music composed in Music Sketchpad can be imported to Timbre Painting where it is displayed in a neutral grey colour. Here, five coloured paint pots representing five different instrumental sounds (timbres) are shown on the screen. The user dips a brush in a paint pot, whereupon an instrumental sound is heard, and the brush is then used to colour some (or all) of the musical notes. When the music is played back, the coloured notes are heard with their corresponding instrumental sounds.

MusicLand has been enjoyed by 3-year old children (using Music Sketchpad and Timbre Painting) at MIT's "Project Headlight" in Boston. The entire program has been used by professional composer Denis Smalley to produce the composition "Vortex" which won an international composition award and has been released on CD. MusicLand is a permanent exhibit at the Ontario Science Centre and is used in schools around the world.

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