The Value of Play to Enhance Mathematical Learning in the Middle Years of Schooling[1]

Rosemaree Caswell
Griffith University
<rosemaree.caswell@pacific.net.au>

The value of play has been well established in the early years of schooling, however in the years that follow, a transmission approach where the learner is 'drilled' in mathematical concepts and processes often dominates the curriculum (Von Glasersfeld, 1992a). This paper identifies the need for rethinking the approach to mathematics learning in this phase of development by recognizing the need for pedagogy where concrete materials, sensory-motor experiences and metalanguage are employed through play activities to support learners in the transition to abstract mathematical processing. Both cognitive and affective factors are identified as essential elements of a play-based approach to optimising mathematical understandings in the middle years and beyond.

Mathematics teachers and researchers working with children in the early childhood years have long recognized and advocated for the need to include play and concrete materials into the curriculum. In contrast to this constructivist foundation, mathematical learning in the middle phase of schooling that follows tends to reflect a behaviourist paradigm where the early years' methods of learning and inquiry are replaced by drill and practice with an emphasis on abstract mathematical thinking and processing. The traditional drill and practice paradigm often employed in mathematical learning in the middle years of schooling, has long been thought to reinforce computational skills and mathematical processes, yet it is also claimed to produce rote memorization that does not translate into meaningful understanding or long term retention of mathematical knowledge. von Glasersfeld (1992a) criticizes this mindset, claiming:

> for 50 years in this century we have suffered the… domination of mindless behaviourism. The behaviourists succeeded in eliminating the distinction between training (for performance) and *teaching* that aims at the generation of understanding. (von Glasersfeld, 1992a, p. 4)

Some decades ago, Dienes (1963) argued that the aims of mathematics education were unclear. He puts forward the view that mathematics should engage students in enculturation and "appreciation of mathematics as a beautiful structure" (Dienes, 1963, p.155). English (2002) notes that the aims of mathematics education in the 20th century were largely associated with computational skills, whereas the 21st century presents an entirely different goal for mathematics education: engaging students effectively in mathematical modelling, visualizing, problem solving and problem posing. To do this requires an epistemology grounded in the constructivist approach with open-ended inquiry (Bauersfeld, 1992; von Glasersfeld, 1992a). Mathematical games offer an attractive alternative to standard teaching methodologies and engage learners in constructivist learning contexts. (Kanes, 1991). The challenge addressed by this paper is to investigate the value of engaging students in play activities beyond the early years of schooling, to enhance their mathematical understanding and engage them more effectively in working mathematically.

The study focused on two main goals: to describe activities that constitute 'playful learning' in the middle years and to analyse and explain the elements of play that enhance

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student engagement in learning and contribute to deep conceptual development. The focus of this research is to understand from a student's perspective the value of play activities in enhancing mathematical understanding.

Theoretical Background

Essential to validating a play-based approach to mathematical learning beyond the early foundation years is an epistemology that supports the view that knowledge needs to be constructed not just in the early concrete developmental stages but continuously through to abstract mathematical thinking and reasoning. Learning theory is not therefore defined by age or developmental cognition – but applies when mathematicians of any age engage in constructing meaningful mathematical conceptualisations. The learning theory that underpins play-based learning is the constructivist approach as outlined in the works of Piaget (1965) and von Glasersfeld (1992a). The theory of constructivism has many interpretations and has been acknowledged as a learning theory for decades, yet in practice, constructivism is rarely observed in mathematical classrooms in the middle phase of schooling. Play-based learning though can be argued to more closely reflect social constructivism as advocated by Bauersfeld (1992), in which meaning is constructed through discourse and interaction. Play-based learning also strongly mirrors the 'Zone of Proximal Development' as outlined by Vygotsky (1962). In this theory of learning, Vygotsky maintains that the optimal zone of learning is established when there is a supportive, interactive environment where the learner is challenged to stretch beyond their current level of knowledge with the support of peers and teachers. In a play context, the learner can engage with mathematical concepts and construct their own meaningful pathways when working with mathematical activities. Bastick (1993) further states that children need a learning climate which recognizes both affective and cognitive factors. Too often in the middle phase of schooling, mathematics classrooms are a serious study in concentration – there is little communication and even less 'fun'. Yet studies into learning theory advocate for recognition of affective factors that enhance learning (Steffe & Wiegel, 1994; Marshall, 1989). Open-ended, play-based approaches to learning mathematics are argued to effectively address key elements of each of these learning theories.

Within the literature on mathematical play, a clear definition of 'play' is difficult to find and most references relate to early years of schooling. There is little reference to play in the middle phase. Dienes (1963) presents one of the few diagrams outlining the elements of mathematical play in a comprehensive study completed nearly forty years ago, yet still very relevant when defining play. He outlines three stages – exploratory, representational and rule-bound play. Dienes' findings indicate that the stages of play are reflective of students' levels of understanding in mathematics and that students enhance their understanding of mathematical concepts through increased awareness of their engagement in these stages of play. The most interesting aspect of Dienes' study was this element of meta-awareness and engagement with play-based activities.

The second part of this study specifically sought to identify characteristics of a play-based approach that enhance learning beyond the early years of schooling and validate the need for play-based learning through to engagement with abstract mathematical concepts. Most prominent in research on play activities in mathematics is the element of interactive learning, involving social discourse and domain specific communication. Burnett (1993) identifies students' engagement in discourse as a feature of instructional games and play activities, in which both metalanguage and metacognition are facilitated through a supportive social setting. Onslow (1990) supports the value of play, but argues against ‘play-only’ learning, advocating instead for play that is supported by metacognitive discussions. He claims the impact of this learning supports improved retention. In order to communicate mathematical understandings, Cobb (1994) argues that learning mathematics requires effective engagement with mathematical metalanguage. Gawned (1990) produced a model of mathematical language development that recognizes the gradual transition from students own language to domain specific applications which can be argued to accurately reflect the mathematical discourse that occurs in play-based learning.

As well as engaging students in a mathematical discourse, play activities also support students learning through engagement with sensory motor experiences (von Glasersfeld, 1992b). Salomon and Perkins (1998) also argue for materials that provide students with mathematical ‘tools’. They claim that manipulating materials engages students in the conceptual development of mathematics. (‘Mathematical tools’ are most often associated with sensory motor stimuli, but should also include cognitive representations and manipulations). Sowell (1989) further supports this argument claiming that mathematical achievement is increased with the long term use of concrete materials, thus supporting arguments for play-based learning beyond the early years of schooling. The perception of play activities as pre-abstract is in fact a misrepresentation of the application of sensory-motor stimuli and cuing using visual and kinaesthetic representation.

Play activities also have the characteristic of engaging students in the learning context. Marshall (1989) points out that engagement is essential for effective learning to occur. Oldfield (1991) and Ernest (1986) emphasize motivation as a factor in learning mathematics and argue that games and play activity enhance learning through engaging students in an environment of fun whilst working mathematically. They add that games provide a learning context in which students are supported to confidently engage in mathematical dialogue and metacognition without fear of failure. Motivation, enjoyment and confidence to engage with mathematical concepts have been recognized as factors that impact on students’ learning (Steffe & Wiegel, 1994; Marshall, 1989).

Methodology

As the intent of the research was to document and analyse students’ reflections on the value of play, a retroductive approach as outlined by Blaikie (2000, p.108-114), was adopted. In this approach the researcher seeks to study the participants in their normal surroundings, to identify characteristics or patterns of behaviour. The research therefore took the form of a case study of a single primary level class. The classroom where the research study was conducted had the benefit of providing a natural setting where students had already been working together as a unit for six months prior to the study and were engaged in a constructivist approach to learning mathematics through ‘play-based learning’. By this it is meant that students were familiar with the interactive nature of play and the opportunities provided to explore their own pathways to mathematical reasoning and problem solving. Students were also engaged actively, not only in discovery learning, but in reflecting on their learning, thus making it possible to collect student reflections on the value of the activities in which they engaged to support their mathematical understandings. The class consisted of 27 students ranging from 9 years to 12 years and demonstrating varying mathematical competencies. The ratio of boys: girls was 15:12.

Data were collected over a ten week period. The normal practice in the classroom was to engage in a particular area of mathematics for a weekly cycle. During this cycle, a range of activities would be provided to stimulate student learning and support their individual understanding of the mathematical concepts. The mathematical tasks varied and included real-life maths using shopping catalogues, simulations, games – including card games and number challenges, problem solving and drill and practice. The activities were not always meant to be play-based, so that students could compare activities and define which ones they considered to be 'playing with mathematics' and which activities were simply practice activities. Throughout the weekly cycle of activities, students experienced a range of interactions - working individually, in pairs, small groups or as whole class. Communication was a focus of the classroom and weighted towards student contributions, so that students were consistently being asked to communicate their understanding of mathematical knowledge, problem-posing and problem solving as well as feelings and attitudes. Throughout the weekly cycle of activities, students were asked the questions – "Was this a play activity? Why/ Why not?" At the end of each weekly cycle of activities, students were engaged in class conferences which summarised the weekly cycle and they were asked to identify common elements of the activities that constituted 'play'. They also described student demonstrations of understanding that had been observed and made comments or written reflections on how well the activities supported their learning.

The following sources of qualitative data were collected: anecdotes of classroom conversations during feedback sessions, audio-recordings of weekly conferences, student written reflections in journals/surveys, and conceptual models created by the class. A survey was compiled at the end of the ten weeks using anecdotal comments recorded during the classroom feedback sessions and weekly conferences. A selection of comments students had made about play and the value of play were used to form a questionnaire which each student was asked to consider and rate (using the Likert scale) according to how much each statement truly reflected their own opinions. The completed survey was used to provide an overall picture of the importance placed by the students on different factors in the study. The class model to define "what is play?" was gradually constructed over eight weeks until nothing further was being added. Photographs of the concept diagram were taken each week to compare additions to the model. After eight weeks the model was removed for two weeks, after which time, the class was asked to collaboratively reconstruct the model. This was also photographed and used to analyse student thinking about play.

The teacher's role in this research project was identified as a factor that could impact on the study as the teacher was also the researcher. For this reason, questioning about "Was this play? Why/ Why/ not?" was very standard and little or no direction was provided to influence student thinking. The teacher's role was defined as proactive in engaging students in communication (Cobb, 1994) as the classroom practice focussed heavily on open communication. Throughout the weekly activities, the teacher adopted the role of facilitator while students engaged with the activities. After each activity the teacher would initiate discussion about the activity and the characteristics of 'play'. The teacher's role became a fine balance between open-ended questioning and semi-structured interviewing, particularly during weekly conferences and feedback sessions. This effectively maintained students' awareness of factors that impacted on their learning, but allowed students to respond naturally.

Findings

The first part of the research collected information to identify characteristics of 'Play Activities' in order to define 'Play' in mathematics in the transition from concrete to abstract. Each week the students contributed ideas to a class concept map describing 'play activities'. In the first few weeks of the study, students began by referring to 'games' rather than 'play' (see Figure 1).

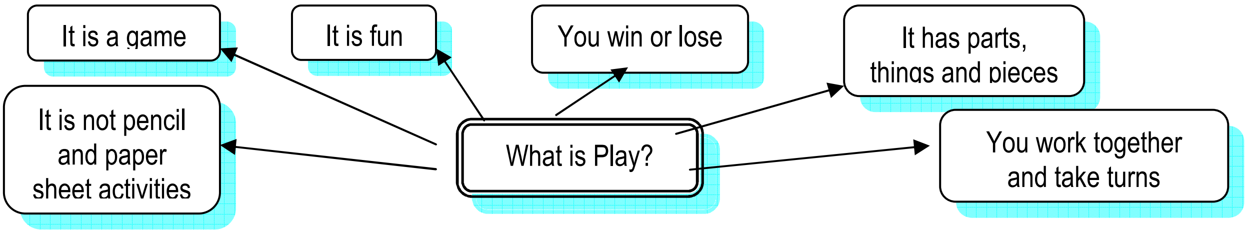

Figure 1. Elements of play activities – Week 1.

They identified first the objects of play - such as dice, cards, etc. and recognised the sensory-motor elements as addressed by von Glasersfeld (1992b). As playdough was frequently used to stimulate mathematical thinking and conceptual development, students began changing the term from 'games' to 'play activities'. Students also recognized that 'play activities' were different because social discourse and interaction were encouraged to support learning. Their comments began to reflect an understanding of the 'Zone of Proximal Development' (Vygotsky, 1962), expounding the benefits of working together, discussing ideas and acknowledging alternative ways to approach a task.

> We had to work together in teams and talk about what we were learning.

The most consistent element identified by students was the affective factor of fun, as identified by Ernest (1986). Students frequently referred to the enjoyment of the task and began associating enjoyment with engagement.

> Everyone at our table was doing the activity because it was a game and though it was work it was also fun. It wasn't just a sheet of sums.

By the third week of the study, the concept of 'play' was rapidly expanding to include elements of metacognition. Students were constantly engaged in discussions about how they were working mathematically to solve the task and began referring to the need to think about strategies in order to be successful at the task. The element of strategies associated with play meant not just knowing how to solve something mathematically but also finding more effective ways to work mathematically. The diversity of thinking required in a play activity was also identified. Students began referring to the need to 'make links' between different maths in their heads, to mathematical terms and to make links between mathematical activities and other fields of knowledge. Students also began to link working mathematically and play to real-life contexts as advocated by Ellerton and Clarkson (1996).

> I think it is more fun and more challenging to use real maths.
> It is 'play' because it uses real maths not just a sheet of sums to practice.

By this point in the study the concept diagram defining 'play' began to remain constant and after the eighth week it was removed from view. In the tenth week, students were asked to reflect again on aspects of play and recreate the concept diagram. The most interesting result was that the students' perception of play had changed from an initial focus on objects and games to an understanding of play as a mathematical activity that presented a cognitive challenge.

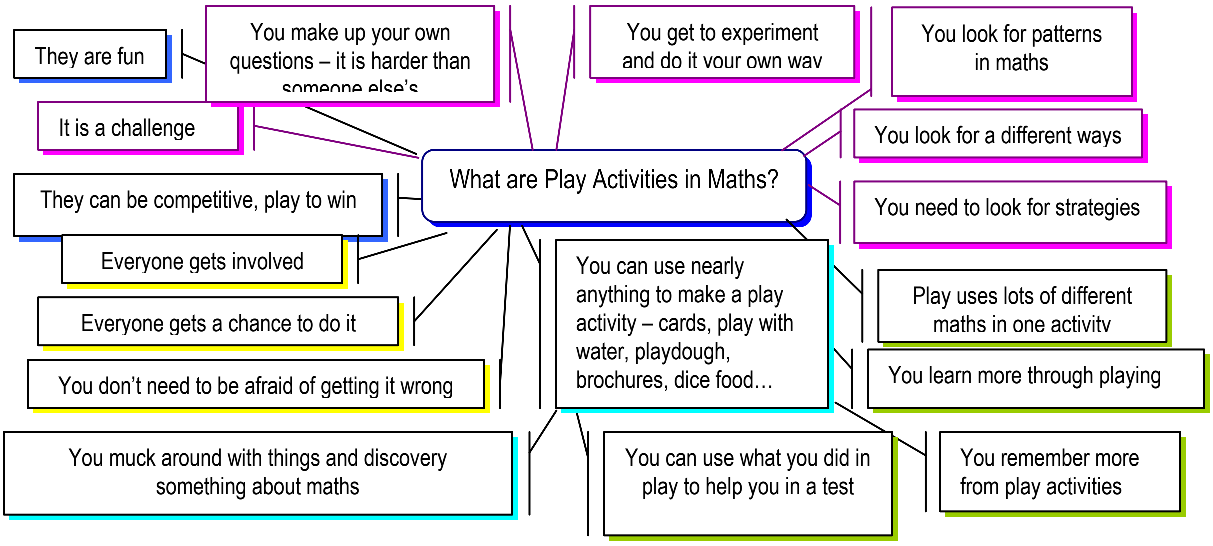

Figure 2. Elements of play activities – Week 10.

The Value of Play

Where students had previously realised play involved metacognition, now their comments indicated a growing awareness of working mathematically, the need to think divergently and apply mathematical knowledge. They associated play with discovery learning and construction of knowledge (von Glasersfeld, 1992a).

> It is a challenge – much harder than just doing the same thing over and over; you have to think more deeply and use lots of different maths in the one activity.
> You need to look for different ways to do it. There is more than one way to do maths. You get to experiment and do it your own way

As well as being aware of the value of games to promote divergent thinking, students also connected play activities to deeper understanding and improved retention of conceptual knowledge. They explained that the play situation enabled them to transfer their knowledge and skills to other mathematical contexts.

> With play, you have fun and enjoy it, so you remember it more.
> With games, you learn a lot more and learn more quickly, but you remember it better.

Students associated play with positive attitudes and awareness of affect – in particular enjoyment and confidence as stated. They all agreed the play activities made mathematical learning fun which in turn was associated with participation and acquisition.

> I find I enjoy it more, which helps me to understand more and makes me more confident.

Students felt confident to engage with the tasks because there was no sense of failure, only discovery and shared learning. They began to associate the supportive network with enhanced learning – reflecting an understanding of Vygotsky's Zone of Proximal Development (1962).

> You don't need to be afraid of getting it wrong. We help each other and learn from each other.

As well as recognizing the value of communicating and working together, students also began to identify the open-ended nature of play activities that allowed them to progress to more abstract levels of working and understanding mathematically. Their comments reflected the 'Levels of Knowing' developed by Carpenter and Lehrer (1999), as they discussed the need to think more deeply, draw on a range of mathematical knowledge.

> We are all on different levels, but everyone can do maths at their level. Because we work multi-age, the maths we do is often higher – there are no limits.

Students referred to 'making links' between different mathematical knowledge, and other fields of knowledge. The diversity of thinking required in a play activity was identified and valued as a factor that contributed and enhanced the learning context.

> It is fun, but it is a challenge –It is much harder than just doing a sheet, makes you think a lot more and use much more maths at once.

Conclusions

The first objective of the study was to identify and describe the elements of 'play' in mathematical learning in the middle years of schooling. The results of the study indicated that the students believe 'play activities' are firstly activities that engage students in using their mathematical knowledge to solve open-ended challenges. They also indicated that play activities engage the learner; they are challenging and diverse, not repetitive. Both 'cognitive conflict' and 'cognitive challenge' were identified through the study as features of play activities that enhance mathematical understanding and application of mathematical knowledge. 'Cognitive challenge' exists where students are engaged in effectively linking mathematical knowledge to open-ended problems within multi-disciplinary contexts.

The study also identified that play activities can be structured around a central mathematical concept, but are not limited to one concept and in fact are more effective as a learning tool if they have a real world focus through which students are encouraged to find links between mathematical concepts and across fields of knowledge. Play activities required students to draw on prior knowledge and share understandings in order to create the basis of on-going discovery and experimentation. The study also revealed that play involves asking questions and posing problems with multiple pathways to finding solutions. Play does not establish a formal mathematical process to be 'practiced' – instead it should be seen as an opportunity to find a pattern or process.

The second aspect of the study was to analyse the value of play in enhancing students' mathematical learning in the transition from concrete to abstract. Throughout the study, students reflected on the mathematical activities and commented on the effectiveness of the activity in supporting their learning. The responses were overwhelmingly in favour of play activities as an effective learning context. Play activities allow a continuum from concrete to abstract that engages all students at their level of understanding. Bastick (1993) and Dienes (1963) argue that development is cyclic rather than linear and as such play activities provide a context in which students can engage at different levels of competency, consolidate their understandings and extend their knowledge.

The students in the study noted that interactive play created a supportive environment in which there was no failure. The satisfaction of meeting a mathematical challenge and sharing strategies with peers was a very effective and confidence boosting experience for all students in the study. The value of play is also clearly established not only in the interactions between 'players' but also in the communication that is required within the play activity. Because play activities are interactive, students were engaged in using mathematical language, communicating mathematical reasoning and explaining their processing to others. These actions of communicating mathematically in a meaningful context enhanced the value of the activity for the learners and from the responses of the participants in the study also enhanced their confidence to engage with mathematics.

Play activities then can be argued to present the platform for engaging students and educators in a new era of learning mathematics in the middle years of schooling. Instead of relying on transmission to develop mathematical competency, a play-based approach encourages students to use their mathematical knowledge, practice working mathematically and communicate their understandings with the aim of mathematics as the engagement of students in "making mathematical knowledge one's own" (Carpenter & Lehrer, 1999).

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