Issues in Assessing the Impact of CAS on Mathematics Examinations

Peter Flynn
University of Melbourne
<p.flynn@edfac.unimelb.edu.au>

Barry McCrae
Australian Council for Educational Research
<mccrae@acer.edu.au>

Different classification schemes are applied to investigate the impact of CAS availability on the 2000 VCE Mathematical Methods examinations. The degree of CAS impact on individual questions is also investigated with specific examples of CAS sensitive and CAS resistant questions illustrated and assessment issues with different brands of CAS calculators discussed. For both examinations, CAS either did not impact or impacted slightly upon a similar combined proportion of the total marks (approximately 60%).

The University of Melbourne's Computer Algebra Systems in Schools—Curriculum, Assessment and Teaching (CAS-CAT) project aims to investigate the changes that regular access to CAS calculators may have on senior secondary mathematics. The study is funded from 2000–2002 by the Australian Research Grant Strategic Partnerships with Industry Scheme. The industry partners are the Victorian Curriculum and Assessment Authority (formerly Board of Studies) and three calculator suppliers and manufacturers: Hewlett-Packard, Shriro (Casio) and Texas Instruments. Further details can be found at the project web site http://www.edfac.unimelb.edu.au/DSME/CAS-CAT/, and in a series of papers (Stacey, McCrae, Chick, Asp, & Leigh-Lancaster, 2000; Stacey, Asp, & McCrae, 2000; Stacey, Ball, Asp, McCrae & Leigh-Lancaster, 2000) that address preliminary issues.

The CAS-CAT project will culminate with the trial in 2002 of a Victorian Certificate of Education (VCE) Mathematical Methods 3/4 (CAS) subject, as an alternative to the current Mathematical Methods 3/4 subject (Board of Studies, 1999). Mathematical Methods 3/4 (CAS) will assume that students have access to an approved CAS calculator throughout the year and in the examinations. Mathematical Methods 3/4 is essentially a functions and calculus based subject, with some study of probability distributions (about 20% of the content), and is intended to provide an appropriate background for further study in, for example, science, economics or medicine.

Classification Schemes for Examination Questions

The CAS calculators supplied to students participating in the CAS-CAT project are the Casio FX 2.0, the HP 40G or the TI-89. Each of these calculators possesses graphics capabilities, *but* (non-CAS) graphics calculators have been allowed in VCE mathematics examinations since 1997. Accordingly, schemes originally devised to assess the impact of graphics calculator availability on examination questions (Jones, 1995; Jones & McCrae, 1996), can be adapted to assess CAS by defining impact to mean that *a CAS user would have an advantage over a graphics calculator user*. By this we mean that a CAS user would have access to a *more efficient* solution strategy than a graphics calculator user and not just that CAS may broaden the number of solution methods possible. In particular, the Jones and McCrae classifications become:

- *CNI*: The availability of CAS would have no impact on the question.

- *CIU*: The availability of CAS would have an impact, but the question could remain unchanged.
- *CIO*: The availability of CAS would have an impact and the question would need to be omitted in its current form.

Schemes have been devised specifically to classify the impact of CAS on a question (Kokol-Voljc, 2000; MacAogáin, 2000). Kokol-Voljc suggests that each question should be analysed according to the extent it tests basic abilities such as concept knowledge, modelling of real world situations and reflecting about mathematical content, and the extent to which it requires algorithmic and calculation skills. Unfortunately, we were not able to understand Kokol-Voljc's descriptions of some her classifications sufficiently well (see McCrae & Flynn, 2001) to use it with confidence in the final analysis reported here.

Kutzler, cited in Kokol-Voljc (2000), proposes classifying questions according to the role that CAS plays in answering them. His two-way classification scheme first looks at how significant the use of CAS is (primary vs. secondary), then at how well the student needs to know how to use the CAS (routine vs. advanced)—see Table 1.

Table 1
Kutzler's 2-Dimensional Classification Scheme

	Routine CAS use	Advanced CAS use
Primary CAS use	Primary Routine (PR)	Primary Advanced (PA)
Secondary CAS use	Secondary Routine (SR)	Secondary Advanced (SA)
No CAS use	-	-

Thus applying Kutzler's classification scheme produces the following five categories:

- *Primary Routine CAS use questions (PR)* are problems for which CAS use is the major activity although only superficial knowledge of the tool suffices.
- *Primary Advanced CAS use questions (PA)* are problems for which CAS use is the major activity but in-depth knowledge of the tool is required.
- *Secondary Routine CAS use questions (SR)* are problems for which CAS use plays only a minor role in solving the problem and only superficial knowledge of the tool is required.
- *Secondary Advanced CAS use questions (SA)* are problems for which CAS use plays only a minor role but in-depth knowledge of the tool is required.
- *No CAS use (NC)* are problems for which CAS use is of no assistance.

Kokol-Voljc ranks these categories in the following descending order in terms of their value for testing mathematical abilities: SR and NC (equal), SA, PA, PR.

MacAogáin's (2000) classification scheme for examination questions is less elaborate than those of Kokol-Voljc and Kutzler, and can be more easily applied to determine whether CAS users would have an advantage over graphics calculator users. MacAogáin's categories are as follows:

- *CAS trivial (CT)* questions that reduce down to two or three steps, such as enter the expression and differentiate, with CAS and so are no longer suitable.
- *CAS easy (CE)* questions where CAS use significantly reduces the difficulty, although some substantive mathematical knowledge is still required to answer them.
- *CAS difficult (CD)* questions retain (most of) their level of difficulty although CAS provides some help to answer them.
- *CAS proof (CP)* questions where CAS is of minimal or no use.

In measuring the overall impact on an examination, MacAogáin calculates a *CAS index*, ranging from 0 to 10, as a measure of the advantage of using CAS in an examination when it is not permitted. Letting x% = score obtained by correctly answering all CT and CE questions (only), the CAS index is calculated using the formula $(100 - x)/10$ and rounding to the nearest whole number. In his analysis of the 1999 Irish Leaving Certificate mathematics papers (graphics calculators not allowed), MacAogáin found that 81% of the Paper 1 questions were trivial or easy with CAS, which converts to a CAS index of 2, while paper 2 had a CAS index of 7. The main topics on Paper 1 were algebra and calculus; on Paper 2, the main topics were geometry, trigonometry and probability and statistics.

Analysis of Mathematical Methods 3/4 Examinations

There are currently two end-of-year examinations for Mathematical Methods 3/4. In 2000, Examination 1 (Facts, skills and applications) consisted of 27 multiple-choice questions (MCQ) each worth 1 mark (Part I) and eight short-answer questions (SAQ) worth a total of 23 marks (Part II). Examination 2 (Analysis Task) consisted of four extended-answer questions worth a total of 55 marks. Of these four multi-part questions, three were functions and calculus questions and one was a probability distribution question.

As noted previously, the examinations for Mathematical Methods 3/4 (CAS) will have the same format as those for the non-CAS subject, but will assume that students have access to an approved CAS calculator. With this in mind, each question on the 2000 VCE Mathematical Methods 3/4 Examinations 1 and 2 (Board of Studies, 2000) was classified by the authors according to the three schemes described in the previous section, but assuming that students would continue to have access to a graphics calculator.

First, both authors classified each question on the respective examinations working independently of each other. We then met and vigorously debated the merits of each question to come to a consensus classification in each scheme. This was much easier for some questions than for others. In particular, whether questions that require both conceptual understanding *and* algebraic manipulation for their solution should be classified as Primary or Secondary Routine according to Kutzler's scheme, caused ongoing difficulty. As Kokol-Voljc (2000) notes:

> … with any classification scheme, there is no clear-cut dividing line between the categories, because the reality is continuous—not discrete. Hence, for some exam questions it may appear arbitrary to put them into one or the other category. (p. 70)

Results and Discussion

Tables 2 and 3 summarise the results of applying the Jones and McCrae, MacAogáin, and Kutzler classification schemes to the 2000 Mathematical Methods 3/4 Examinations 1 and 2. They also illustrate the relationships that emerged between the different categories used in these three schemes.

Table 2
Classification of the 2000 VCE Mathematical Methods 3/4 Examination 1

Jones & McCrae	MacAogáin	Kutzler	MCQ	SAQ	Total
CAS No Impact (CNI)	CAS Proof (CP)	No CAS (NC)	59.3	47.8	54.0
CAS Impacts: Unchanged (CIU)	CAS Difficult (CD)	Secondary Routine (SR)	7.4	0.0	4.0
	CAS Easy (CE)		3.7	0.0	2.0
CAS Impacts: Omit (CIO)		Primary Routine (PR)	7.4	21.8	14.0
	CAS Trivial (CT)		22.2	30.4	26.0

According to these results, CAS availability would not impact on 54% of the total marks allocated for Examination 1 — that is, 54% of the marks are CAS proof or CAS insensitive (though not necessarily graphics calculator neutral). For Examination 2, 38% of the available marks are CAS proof. This indicates that a smaller proportion of marks were allocated to algebra or calculus techniques on Examination 1 than on Examination 2 — a surprising finding given that they are respectively subtitled 'Facts, skills and applications' and 'Analysis task'. MacAogáin's CAS index is the same (6) for both examinations because the combined percentages of CAS-trivial and CAS-easy questions are similar.

Table 3
Classification of the 2000 VCE Mathematical Methods 3/4 Examination 2

Jones & McCrae	MacAogáin	Kutzler	Total
CAS No Impact (CNI)	CAS Proof (CP)	No CAS (NC)	38.2
CAS Impacts: Unchanged (CIU)	CAS Difficult (CD)	Secondary Routine (SR)	20.0
	CAS Easy (CE)		3.6
CAS Impacts: Omit (CIO)		Primary Routine (PR)	7.3
	CAS Trivial (CT)		30.9

Functions and calculus questions accounted for 63% (17 marks out of 27) and 52% (11 out of 21) of the CAS-proof questions in Examinations 1 and 2 respectively. An example

of a question from this CAS-proof category is shown in Figure 1. (Note that the five optional answers are not shown for any of the multiple-choice items reproduced in this paper.)

The graph whose equation is $y = \sqrt{x}$ is reflected in the x-axis and then translated 2 units to the right and 1 unit down. The equation of the new graph is …

Figure 1. VCE 2000 Mathematical Methods 3/4 Examination 1, Part I, Question 2.

At the other extreme, questions worth a total of 26% of the marks in Examination 1 and 31% in Examination 2 would be trivialised or devalued by the availability of CAS and hence would require omission from a CAS-neutral (and, in some cases, even from a CAS-active) examination. As with the Irish papers, questions testing knowledge of algebraic and calculus procedures were the most affected with all devalued questions, not surprisingly, coming from this category in both examinations. The question part from Examination 2 shown in Figure 2 is a typical example. In part (a), students were required to express the volume, V, of a right circular cylinder with hemispherical caps in terms of the height h cm and radius r cm.

The total volume of the capsule will be 8000 cm3. Show that [ ].

Figure 2. VCE Mathematical Methods 3/4 Examination 2, Question 3b(i).

Table 2 shows that, to make Examination 1 CAS-neutral, we would omit questions worth a further 14% of the total marks. For Examination 2, table 3 shows that questions worth a further 7% would need to be omitted. Typically, these are questions that are not trivialised by CAS, but in our opinion are made much easier by its availability. An interesting example is given in Figure 3. Students using a CAS calculator will obtain a general solution, involving a parameter, to trigonometric equations. To determine all solutions within a given domain, particular integer values for the parameter must be substituted into the general solution. Despite the routine nature of finding the solution by hand, the use of a CAS calculator makes this question easier but does not trivialise it. This question may still be suitable for a CAS-active examination because students need an understanding of multiple solutions expressed in terms of a parameter.

Find the exact solutions of the equation $\sin(2x) = \sqrt{3}\cos(2x)$, $-\pi \leq x \leq \pi$.

Figure 3. VCE 2000 Mathematical Methods 3/4 Examination 1, Part II, Question 5.

Figure 4 shows the only question from Examination 1 that we classified as being made easier by the availability of CAS, but not sufficiently so to warrant its omission. The correct answer amongst the five alternatives given is the unsimplified form obtained by

direct application of the quotient rule. However, each of the CAS calculators gives a (different) simplified version of the answer that results after a number of extra steps.

If y = (tan x) /x then dy/dx is …

Figure 4. VCE 2000 Mathematical Methods 3/4 Examination 1, Part I, Question 18.

The question shown in Figure 5 is an example of an item that is only slightly impacted by the availability of CAS since the by hand algebra involved is very simple. However it retains its level of conceptual difficulty and so need not be omitted from a CAS-neutral examination. A similar question with more difficult algebra (arising from more complex probability terms) would need to be omitted, but could be part of a CAS-active examination with the availability of CAS changing the question's focus to mainly conceptual.

The probability distribution for the discrete random variable X is given by

X	0	1	2	3
$\Pr(X = x)$	k	$2k$	$4k$	$8k$

The value of k is …

Figure 5. VCE 2000 Mathematical Methods 3/4 Examination 1, Part I, Question 23.

Kutzler Classifications

By definition, Kutzler's 'No CAS use' category applies to all (and only those) questions classified as being CAS no impact/CAS proof by the other schemes. Questions worth only 6% of the total marks in Examination 1 compared with 24% of the total marks in Examination 2 were classified as Secondary Routine (SR). Examples of questions of this type are shown in Figures 4 and 5. In general, these questions retain their conceptual difficulty even though CAS use may make the manipulative part of the question easier. A further example of such a question from Examination 1 is shown in Figure 6. Though impacted by the availability of CAS, this question retains its relative difficulty because it can be solved in CAS-style on a graphics calculator ($Y1 = V$, $Y2 = (Y1(10) – Y1(0)) / 10$). The primary objective of Question 16 is to test whether the student understands what is meant by 'average rate of change'

Rainwater is being collected in a water tank. The volume, V m3, of water in the tank after time, t hours, is given by V = 2t2 – 3t + 2. The average rate of change of volume over the first ten hours in m3 per hour is …

Figure 6. VCE 2000 Mathematical Methods 3/4 Examination 1, Part I, Question 16.

All probability questions that CAS impacted upon were Secondary Routine. Examination 2 contained a far higher proportion (20% of the total marks) of Secondary

Routine questions that maintain their level of difficulty (CIU/SR/CD) than Examination 1 (4% of the total marks). This confirms Kokol-Voljc's high ranking of SR questions for testing mathematical abilities and illustrates the intended different nature and emphases of the two examinations with Examination 2 more demanding conceptually.

It was determined that we would omit all Primary Routine (PR) questions present in both examinations from a CAS-neutral examination. Questions categorised as PR ranged from being trivialised at one extreme to retaining some of their (conceptual) difficulty, though made easier, at the other extreme. No questions from either examination were judged to require advanced knowledge of CAS. Given the examination format (multiple-choice and short-answer), this may still be the case for the Mathematical Methods (CAS) Examination 1, but it would probably be appropriate for Examination 2 to include some questions requiring advanced use of CAS.

Brand-Neutral Assessment

Another issue being researched in the CAS-CAT project is whether fair brand-neutral assessment is possible (Stacey, McCrae, Chick, Asp, & Leigh-Lancaster, 2000). Of concern here, is the extent to which the different CAS capabilities of the available calculators affect the difficulty of questions. Two examples arose in our analysis from Examination 1 and one from Examination 2. Two of these questions have already been discussed. For the question in Figure 2, two of the calculators give the answer as a single combined fraction, whereas the other one gives it in the required form. Similarly, for the Figure 4 question, the answer given by one of the calculators is much closer to the required form than the answers given by the other two calculators. These two questions illustrate *brand differences in output*, whereas the question in Figure 7 is an illustration of *different brand capabilities*. It can be solved with one instruction (solve($dy/dx > 0$, x)) by two of the calculators, whereas the other calculator appears to only solve linear inequalities.

> For the curve with equation y = –x3 – x2 + 2x + 2, the subset of R for which the gradient of the curve is positive is closest to …

Figure 7. VCE 2000 Mathematical Methods 3/4 Examination 1, Part I, Question 15.

CAS Calculators Versus Graphics Calculators

As previously emphasised, in assessing the impact of CAS calculators on the two examinations we assumed that students already had access to graphics calculators. Accordingly, questions that could be answered using only the graph drawing capabilities of a CAS calculator were classified as CAS no impact/CAS proof/No CAS. Further, questions that could be answered most efficiently this way, even if there was a CAS alternative method, were similarly classified. Question 4(d) of Examination 2 is an example of the latter type in which the minimum value of a given trigonometric function, $x(t)$, has to be found correct to 3 decimal places. This can be tackled on one of the CAS calculators by setting the derivative to zero and then solving for t with an appropriate restriction ($t > 0.8$ and $t < 1$). This method was considered inefficient compared to finding the minimum value

directly from the graph of $x(t)$. However, if such a question required an exact solution, a CAS Impacts: Omit (CIO) classification would be appropriate.

The fact that a graphics calculator can be used 'CAS-style' to solve some types of questions was also taken into account in assessing the impact of CAS — see the discussion of Question 16 (Figure 6) earlier. It should also be noted that the CAS calculators used in the project were *not* as useful as a common graphics calculator in answering some of the probability distribution questions because they do not have in-built functions to calculate binomial and normal probabilities.

Conclusion

About 40% of the VCE 2000 Mathematical Methods 3/4 Examination 1 and about 38% of Examination 2 would need to be changed to ensure that a student using a CAS calculator would not have a potential advantage over a student using a (non-CAS) graphics calculator. For Examination 1, about 80% of the questions from the algebra and calculus areas of study would have to be modified or replaced (CIO/CT or CE/PR), while for Examination 2 about 62% would need modification or replacement. Some of these questions, however, would be suitable for CAS-active examinations, where access to a CAS calculator could be assumed, such as are proposed for Mathematical Methods (CAS) 3/4. Kutzler's (Kokol-Voljc, 2000) scheme could be used to check that CAS use was appropriately tested in such examinations, but a typology akin to that developed by Kemp, Kissane and Bradley (1996) for the use of graphics calculators, is likely to be more useful to the examination designer. Variations in the CAS capabilities of calculators warrant close attention when setting CAS-active questions.

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Barry McCrae is a Principal Fellow in the Department of Science and Mathematics Education at The University of Melbourne.

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