Developing Ten Facts With Prep Grade Students: A Teaching Experiment

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A teaching experiment was conducted with a small group of prep children considered 'at risk' mathematically, to explore the effect of explicit instruction on the development of tens facts through visualisation of the ten frame. After the two-week instructional period, children could mentally recall all addition and subtraction tens facts without counting, and this performance was considerably higher than the class average. In this paper, the sequence of instruction is documented, highlighting the influence of instruction upon tens fact development.

Numeracy, as a basic goal of education, refers to providing students with key skills to enable them to successfully participate in schooling and to equip them for life beyond school (DETYA, 2000). One of the basic components of numeracy is mental computation (Steen, 1999). One of the pre-requisites for mental computation facility is instant recall of basic facts (Sowder, 1992).

The process of acquiring basic facts to the point of instantaneous retrieval is not clearly reported in the literature. There appears to be two bodies of research that draw almost opposing implications for teaching. On the one hand, extensive research into the growth of children's addition and subtraction knowledge indicates a developmental sequence of strategies for addition and subtraction, predominantly emerging from sophistication in counting skill, that becomes automatic through experience with and exposure to a variety of addition and subtraction situations (as summarised by Fuson, 1992). Following such research, strategy and problem solving methods have been advocated for developing basic facts (e.g., Baroody, 1985; Wright, 1996). Through the provision of rich learning environments, where explorations of number combinations and arrangements are encouraged, children spontaneously derive their own strategies for basic fact combinations (Baroody, 1985; Fuson, 1992) as well a develop number sense through exploration of number relationships (Wright, 1996). A second body of research has shown that many children do not develop efficient strategies for basic facts. Poor mental computers in primary grades predominantly use inefficient counting strategies (McIntosh & Dole, 2000; Mercer & Miller, 1992; Steinberg, 1985). A recent cross-sectional study reported that mathematically 'at risk' children in Grade 6 still use primitive and errorful counting strategies for addition and subtraction in a manner similar to Grade 2 children that also were identified as mathematically 'at risk' (Ostad, 1998). In terms of instruction for such children, explicit teaching of strategies for particular groups of basic facts has been shown to facilitate children's recall of such facts and these facts have been applied meaningfully in problem situations (e.g., Mercer & Miller, 1992; Rightsel & Thornton, 1985; Steinberg, 1985; Thornton, & Smith, 1988). The question is, to what extent should teachers refrain from providing children with explicit instruction on basic fact strategies so that children can develop their own strategies for mental computation? For children experiencing learning

difficulties in mathematics, the issue of explicit instruction versus strategy self-discovery is a serious one.

Teaching Basic Addition Facts

Research into addition and subtraction has found that children frequently will use knowledge of facts (that is, those that are retrieved automatically) to derive answers to addition and subtraction problems involving quantities related to known facts (Hierdsfield, 1999; Steinberg, 1985). For example, to solve 8+5, a child may say 8+2 is 10, plus 3 more is 13, clearly relying on knowledge of tens facts and counting on. Such as strategy has been defined as the derived fact strategy (DFS) (Steinberg, 1985). By looking at DFSs it can clearly be seen that particular known facts serve to scaffold development of other facts, thus suggesting the hierarchical nature of groups of basic facts. Sequencing instruction based on strategies that relate to particular groups of facts has been the basis of previous research (e.g., Rightsel & Thornton, 1990; Steinberg, 1985; Thornton & Smith, 1988) and such research clearly reflects the logic of instruction based on targeting facts grouped according to strategy type and consolidating particular groups of facts to enable DFSs to be applied.

One particular group of facts that has been identified as necessary to both assist DFSs and mental computation of larger amounts is the combinations of ten facts (Ruthven, 1998; Steinberg, 1985). For developing tens facts, the use of a ten-frame has been suggested (Van de Walle, 1988). A ten frame is a 2 by 5 grid with grid cells large enough to accommodate a singe counter or dot to represent numbers and number combinations to ten, and is a powerful means for promoting visualisation of numbers and combinations to ten (Bobis, 1996; Van de Walle, 1988). According to Bobis (1996), visualisation enables children to develop rich understandings of mathematics. Her research with young children (kindergarten) revealed the value of the ten frame in assisting children recognise part-whole relationships and to develop benchmarks of 5 and 10.

The Study

A teaching experiment was undertaken to explore the effect of explicit instruction on the development of prep children's facility with tens facts, using the ten frame to promote visualisation of combinations to ten. The ten facts were selected as the focus of this teaching experiment as the strategy of visualising the ten frame was considered to be quite different to the counting-on strategy for count-on facts.

Subjects

Four children from a Prep class, identified by their classroom teacher as displaying 'at risk' behaviours in mathematics, participated in the study. Although not a consideration for selection, the group was evenly represented by gender comprising 2 girls and 2 boys. The study was conducted in the final weeks of the school year, thus the children had effectively completed one full year of school.

Instruments

Number knowledge and basic facts interview. The interview consisted of items

categorised under three themes: number sense, concepts of addition and subtraction, and basic facts. In the interview, the students were asked to count to 20, to visualise the relative size of 15 Smarties held in a hand; to describe 4 in relation to 7, to state the effect of addition and subtraction upon collections, and to mentally calculate a variety of facts, categorised as ten facts, count-on (1, 2, 3, 0) facts, doubles, and also their related subtraction facts.

Teaching. Three episodes within the teaching component were planned: (1) orientation to the ten frame and visualisation of numbers 1-10; (2) development of connections between number combinations to 10 through focusing on the number of counters on the ten frame and number of counters missing (commutativity); and (3) subtraction facts to ten through relating to known addition facts.

Post-instruction assessment. Two post-instruction instruments were used. The first was a basic addition and subtraction fact test. This test consisted of 35 items organised into four categories: (1) addition tens facts (11 items); (2) subtraction tens facts (11 items); (3) doubles using digits 2-9 (7 items); and (4) addition count-ons using digits less than 10 with counting on 0, 1, and 2 (6 items). The second instrument was an individual oral test consisting of tens facts (both addition and subtraction) in symbolic form printed on individual cards.

Procedure

The study proceeded in three steps, commencing with an individual interview conducted with each child in the study. For each item, students were prompted to elaborate on their responses through being asked such questions as the following: How do you know?, How did you work that out?, Did you see any visual images to help you find the answer? An individual record of each child's responses was developed to provide an overview of number sense, conceptual understanding of addition and subtraction, and basic fact knowledge.

The second step of the study was the teaching component. The students participated in a series of nine teaching sessions, of approximately 15-20 minutes duration, over a period of 2 weeks. Teaching took place in a corner of the children's normal classroom as the rest of the class were undertaking group activities directed by their classroom teacher. The planning of each session was conducted by the two researchers. The teaching was implemented by one researcher while the other researcher acted as an observer. The outcomes of each session were reflected upon by the researchers to inform the planning of subsequent episodes. The development of the instructional sequence followed an action-research cycle of planning, acting, observing and reflecting (Kemmis & McTaggart, 1990). The effectiveness of the ten frame was the focus of analysis during the teaching experiment. Children were encouraged to verbalise their thinking throughout the instruction. Following Hiebert and Wearne's (1991) methodology for studying learning to inform teaching, the researchers sought to analyse the relationship between instruction and cognitive change through identifying key contributing factors to change.

The third step in the study was the administration of the basic addition and subtraction facts test to the whole class. For test administration, students were required to listen to

each fact and write the answer in a space on the answer sheet. The oral test of tens facts was administered on an individual basis only to the four subjects after the whole class test.

Results

Pre-Interview

A summary only of results from the interview and teaching is reported here. From interview data, all four children could competently count both forwards and backwards, and displayed understanding of the value of numbers to 10, but not beyond that (i.e., they had difficulty visualising the size of 15 smarties). Concepts of addition and subtraction varied amongst the children with only one child stating that addition resulted in a larger amount and subtraction a smaller amount. In terms of basic facts, counting strategies were predominantly used, often in conjunction with fingers as concrete materials.

Teaching

The pre-planned three episode instructional sequence extended to seven distinct episodes in light of students' reactions and progress during instruction. At particular points in the teaching sequence, changes in direction occurred. In the first episode, it was found that children could immediately recognise the numbers 0 through 4 on the ten frame, without reference to the way the counters were arranged on the ten frame. However, once children knew that the ten frame consisted of ten squares, the ten frame was seen to assist children develop recognition of numbers five through ten. The focus of instruction was on numbers five through ten in the following order: 10 – a full frame; 9 – one missing; 8 – two missing; 5 – the top line filled; 6 – five and one more; 7 – five and two more. Children experienced most difficulty with 6 and 7, so more time was devoted to these numbers with children counting to check. Practise visualising the numbers 5 through 10 became the focus of the next episode and children were encouraged to close their eyes to visualise these numbers.

The next episode was aimed at linking number pairs through a focus on recognising a number on the ten frame and stating (without counting) the number of dots required to fill the frame. This was assisted through children recording number sentences in the following form: ❑ and ❑ makes 10. Visualisation of the number of missing dots was easier for numbers greater than five, and this caused a change of emphasis in teaching. The following number facts were targeted, listed here in ascending order of difficulty for the children: 10 and 0, 9 and 1, 8 and 2, 5 and 5, 6 and 4, 7 and 3. Consolidating these links was through providing children with opportunities to visualise these fact pairs.

The focus of instruction up to this point had been on encouraging children to think of "what goes with" particular numbers. The symbolic recording to match this language is the missing addend form (e.g., 7 + ❑ = 10). Introducing this representation to children followed recording of their number sentences. Number facts of this form printed on cards were used to practise facts. Children were encouraged to think of the ten frame if they were unsure. The children readily stated that this was what they thought of in such situations.

Using commutativity to complete the tens facts was the focus of the next episode. In an effort to make this meaningful, four different representations were shown to the

children. Firstly, the ten frame was filled with two different coloured counters. However, this representation led to children counting each set of counters. Next, two ten frames displaying complementing number pairs were used. This also appeared to invoke counting as children transferred counters from one frame to the next. Thirdly, individual cards with numbers 0 to 10 in symbolic form were laid out in sequence going from ten to zero, and the matching pair placed next to each number. Finally, known facts in symbolic form (e.g., 7 + ❑ = 10) were placed in front of the children, and after stating the missing number, the 'spin-around' fact card was presented (e.g., 3 + ❑ = 10). During this activity, children appeared to make links with number pairs, with one child saying "oh, you just think of the other one." For one other child in the group, the patterning activity appeared to be useful as he acknowledged that he used the pattern to assist in the recall of some number combinations.

The last episode in the teaching sequence focused on the development of subtraction facts. The ten frame was referred to, and children removed particular counters to determine how many were left. The symbolic form of each tens fact was displayed at the same time. Children were encouraged to reflect on what they had done with the counters and ten frame, and to consider what the symbolic representation was asking. Once again, the strategy "what goes with" was used. In this teaching episode, children at first appeared unsure, and the Researchers considered that the addition facts may not have been consolidated to make this a simpler task. Children were given their own sets of fact cards, both addition and subtraction, and engaged in pair-testing with each other. Surprisingly, by the end of the two lessons in this episode, all children were answering all facts correctly.

The original three-episode sequence was thus expanded into a seven-episode sequence with visualisation sessions a specific focus of the sequence. The seven episodes developed as a result of this teaching experiment are listed and briefly summarised below:

1. Orientation to the ten frame (describing the representation of the numbers as the following: 10 is a full frame; 9 is one missing; 8 is two missing; 5 is the top line full; 6 is 5 and one more; 7 is 5 and 2 more).
2. Visualisation of numbers 1-10 (encouraging sight recognition rather than counting, reinforcing the ten frame representation).
3. Linking number pairs (focussing on numbers 10 through to 5 and asking children to identify the number of dots and the number missing).
4. Visualisation of number pairs (encouraging recall of mental pictures developed in episode 3).
5. Symbolic representation (missing addend representation, encouraging children to visualise the ten frame and think of "what goes with" particular numbers).
6. Commutativity (linking numbers 0 through 4 with "known" number pairs involving numbers 10 through 5).
7. Subtraction facts (linking to known addition facts).

Posttest

On the posttest, collectively the four students performed well on both the addition (82%) and subtraction tens facts (64%). On the doubles facts, performance was extremely low (7%), and scores for add-on facts also were low (33%). In comparison to the

performance of the rest of the class, the four students' scores were higher for the addition and subtraction facts, but considerably lower for the doubles and count-ons facts. Class mean percentage scores for each section of the test were 41%, 47%, 39% and 64% respectively. The four students' individual scores (as a percent) for each section of the test are presented in Table 1, together with collective scores and class scores.

When the four children were individually tested on recall of the tens facts using cards on which the fact was printed, all children responded appropriately and in an instant manner (well within the 2 second limit that has been typically used to determine automatic recall of facts – Thornton & Smith, 1988).

Table 1
Individual Scores and Collective Totals for the Four Children Together with Class Mean Scores for Each Test Section

Student	Tens facts addition (11 items)	Tens facts subtraction (11 items)	Doubles (7 items)	Count-ons (6 items)
Gemma	100%	64%	0%	17%
Kerrie	64%	55%	29%	17%
Aaron	100%	82%	0%	67%
Colin	64%	55%	0%	67%
Group mean	82%	64%	7%	33%
Class mean	41%	47%	39%	64%

Discussion

The students in this study were rated by their classroom teacher as exhibiting mathematical learning difficulties. It would be assumed that these four children were not performing well on maths tasks in the classroom in comparison to their peers. The data from the pre-interviews appears to confirm the teacher's view and justifies her cause for concern. At the conclusion of this study, all four children were able to recall both addition and subtraction tens facts without resorting to counting when tested individually, and scored high scores for such facts when tested in a whole class situation.

The instructional sequence, aimed at developing mental recall of these facts, appeared to have achieved its goal. The teaching experiment served to highlight the value of the ten frame for building visualisation of numbers 1 to 10, and also linking number pairs. This finding supports suggestions in the literature in relation to the ten frame (Bobis, 1996; Van de Walle, 1988). Analysis of instruction in light of students' responses throughout the teaching episodes also enabled particular teacher actions that appeared to promote children's thinking and understanding of the tens facts to become transparent. Although not trialed with a full class, this study has highlighted the concrete and symbolic representations, and accompanying language as well as particular key teaching points, that enabled children to develop visualisation skills and recall of tens facts. As a result of this study, a sequence of instruction for explicit teaching of tens facts has been provided.

Whilst it is acknowledged that this study is only a small study with a limited number of participants and a focus on only one category of basic facts, results of this study raise many issues associated with instruction in basic facts, addition and subtraction strategies, mental computation, and children with learning difficulties in mathematics. The students in this study were identified as experiencing difficulties in mathematics at an early age. Assuming that this will be a continuing trend throughout their school life, as recent research has suggested (Ostad, 1998), these children's future performances in areas such as addition and subtraction would be hindered considerably by their limited access to strategies of computation. Further, as research has shown that recall of basic fact combinations is one of the pre-requisites for successful mental computation (Sowder, 1992), limited recall of basic facts for these children would hinder their performance in mental computation as they move through the grades.

Explicit teaching for the development of basic facts has been suggested as a needed focus for research, particularly for students with learning difficulties (Ostad, 1998). This study has revealed the remarkable achievements that can be made, in a relatively short time frame, by students experiencing difficulties in mathematics, through explicit teaching of one particular group of facts.

What has not been revealed from this study is the extent to which these children can apply these facts to solve addition and subtraction problems, and the permanence of these facts over time. Such issues are beyond a small study such as this, but could be the focus of future research in this area. Also as the tens facts were the group of facts under investigation in this study, direction for future basic fact instruction is still unclear. However, as inability to recall tens facts hinders children's ability to use 10 as the basic for deriving solutions to other facts (for example, calculating 8+5 as by linking 8 with 2 to make 10 and then counting on 3 more) (Steinberg, 1985), the importance of consolidating these facts before introducing other particular fact groups is apparent. In addition, this study has clearly shown that instruction in tens facts does not required facility with the counting-on strategy.

Concluding Comments

The focus of this study was on developing tens facts with prep children experiencing difficulty in mathematics through visualisation. Results of this study indicated the positive gains children made in tens facts development through explicit teaching. The pedagogic value of this study lies in the development of a suggested sequence for tens facts that is readily accessible to classroom teachers, and that can clearly assist children experiencing difficulties in mathematics to build strategies for basic facts beyond counting.

In an era of testing and outcome-based performance, we are becoming aware of many children's limited mental computation ability and recall of basic number facts as they move through the grades in school. Research conducted over 15 years ago showed the benefits of grouping facts according to type (e.g., tens facts, count-on facts, doubles, etc.) and focusing instruction in this direction. Perhaps it is time to revisit such research to develop comprehensive guidelines for basic fact teaching so that children who do not spontaneously acquire basic facts and develop efficient strategies for mental computation are not educationally disadvantaged.

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