Pre-Service Primary Teachers' Judgements About the Probability of Everyday Events

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A group of pre-service primary teacher students were asked to evaluate the probability of some everyday events, by (i) ranking the events in order of likelihood, (ii) assigning each event a probability word, and (iii) giving each event a numerical probability. Mismatches between ranking and numerical values were found, some students had difficulty estimating values for events with probabilities close to *certain* or *impossible*, and the ambiguity of some probability words was revealed. Students indicated that they thought that making such evaluations was associated with numeracy understanding, but not to the same extent as a more traditional computational number question.

As Konold (1991, p.139) suggests, probability is a "particularly slippery concept." Fundamental principles are susceptible to misconceptions, numerical values can be expressed in fundamentally different ways (e.g., part-whole representation of fractions, and part-part representation of odds), and interpretation of its language can be open to debate.

In 1983 Green (p.41) suggested that teachers need to "give attention to educating pupils in the normally accepted usage of various terms which indicate varying degrees of likelihood." Current primary school mathematics curricula (see, e.g., Board of Studies, 2000, pp. 35, 55) explicitly include the language of chance as a learning outcome and many classroom activities have been suggested to assist in teaching the meaning of chance words (e.g., "Starting points", 1998). Nevertheless, as pointed out by Hope and Kelly (1983), everyday expressions of probability are highly ambiguous, making the success of such activities doubtful. One aim of this study is to examine the use of such words.

The use of numerical probability values is also encouraged in curriculum documents (e.g., Australian Education Council, 1991). Very little work has been done to examine choices made in assigning numerical probabilities to everyday events, and the numerical interpretation of the meanings of words (see Watson & Moritz, 2000, for an example of the latter). This study also aimed to investigate aspects of these issues.

Finally, the idea of *statistical literacy* (Watson, 1997) is relatively recent, and has been argued to be an essential aspect of numeracy understanding. A final question of interest for the study is to determine the extent to which probability is regarded as part of numeracy.

Method

A group of 89 pre-service primary teacher students was involved in this study. The students were in the final year of a two-year Bachelor of Teaching degree, a pre-service program for graduates. Prior to the study, the students had completed 2 semesters of study in mathematics education, with some mathematics strands still to be covered in their final single semester course. Some students would have done pre-tertiary mathematics courses in Years 11 and 12.

The students responded to a 45-minute "Numeracy Questionnaire" which contained 23 basic mathematics questions from all areas of the primary mathematics curriculum. The probability question that provided most of the data for this research was the twelfth question. In this question students were asked to consider seven events. Part (a) required the students to rank the events from least to most likely. Part (b) asked students to give the likelihood of each event using words like “possible”, “probably”, “impossible”, “certain”, “very likely”, etc. Finally, part (c) asked for a numerical probability value for each event. The seven events (paraphrased) were as follows.

- Rain in Melbourne during the week of Easter (RAIN)
- The Prime Minister will be selected on the Aust. Women’s Netball Team (PM)
- Rolling a 6 with a normal die (DIE)
- A letter is posted and doesn’t arrive at a local address in 4 days (MAIL)
- Tossing a coin and getting tails (COIN)
- The price of unleaded petrol will go above $1.05 by the end of 2001 (PETROL)
- The sun will rise tomorrow morning (SUN)

One other question on the questionnaire was of interest for this research. The final question (CORDIAL) asked students to determine how much concentrate is used to make 200ml of cordial if cordial is made by mixing concentrate and water in the ratio of 1:4.

In order to gauge their numeracy beliefs, students were asked to indicate, on a 5-point Likert scale, their level of agreement with the statement that the question they were responding to was a “numeracy question”. For the purpose of this study “numeracy” was defined to be *the mathematical knowledge and understanding that adults need and should be able to use in everyday life without specific revision*. This definition was explained to students and it was on each page of the questionnaire.

Data was entered into a spreadsheet for ease of data analysis, although some analysis had to be done by hand. Judgements about acceptable terminology and numerical values were made after discussion between the authors. Some of the events have a clearly defined objective probability, while others had to be estimated or were more subjective.

Results and Discussion

Ranking the Events

Part (a) of the probability question asked students to rank the events in order from least likely to most likely. The most appropriate order was PM, MAIL, DIE, COIN, RAIN, and SUN, with any position between PM and SUN judged acceptable for the event PETROL since there was no long-term experience on which to base a judgement. In all, 20 students (22%) successfully ordered the events. The remaining 69 students (78%) usually had PM and SUN at the extremes but incorrectly ordered the remaining events. A total of 32 students (36%) considered that the mail being late was *more* likely than rolling a 6. This may reflect Tversky and Kahneman’s observation (1973) that judgements about probability can be clouded by the memorable occurrence of an event. This is known as the *availability heuristic* (see also Shaughnessy, 1993). As an example, the fact that an item of registered mail did not arrive springs to mind more readily for the first author than the fact that the family newsletter has arrived on time every week for the past two years.

Word Usage

In discussing part (b) and the use of probability words it must be acknowledged that the sample probability words supplied in the statement of the question may have influenced students' choices. This needs to be borne in mind during the discussion.

Student responses	Event PM	Event MAIL	Event DIE	Event COIN	Event RAIN	Event SUN	Event PETR.	Authors' view of numerical probability
Impossible	73	0	0	0	1	0	0	Probability = 0
Never	2	0	0	0	0	0	0	
Not possible	1	0	0	0	0	0	0	
Extremely unlikely	1	0	0	0	0	0	0	Probability closer to 0 than to 0.5 0 < Prob. < 0.25
Highly improbable	1	0	0	0	0	0	0	
Highly unlikely	1	0	0	0	0	0	0	
Very unlikely	5	1	0	0	0	0	0	
Not likely	0	3	1	0	0	0	0	0 < Prob. < 0.5
Unlikely	3	18	2	0	0	0	0	
Improbable	0	1	0	0	0	0	0	
50/50	0	0	0	2	0	0	0	Probability = 0.5
Even	0	0	0	1	0	0	0	
Even chance	0	0	0	1	0	0	0	
Even odds	0	0	0	1	0	0	0	
Fifty % chance	0	0	0	1	0	0	0	
Likely	0	1	4	12	6	0	11	0.5 < Prob. < 1
Probable	0	8	24	21	21	1	20	
Almost certain	0	0	0	0	1	1	1	Probability closer to 1 than to 0.5 0.75 < Prob. < 1
Highly likely (not certain)	0	0	0	0	0	1	0	
Highly probable	0	0	0	0	0	0	1	
Most likely	0	0	0	0	0	0	1	
Very likely	0	3	5	25	22	7	26	
Very likely (in my experience)	0	1	0	0	0	0	0	
Very likely / probable	0	0	1	1	0	0	0	
A certainty	0	0	0	0	0	1	0	Probability = 1
Certain	0	1	0	0	0	65	2	
Certainly	0	0	0	0	0	2	0	
Certainty	0	0	0	0	0	1	0	
Definite	0	0	0	0	0	5	0	
Definitely	0	0	0	0	0	1	0	
Dead set certainty	0	0	0	0	0	1	0	
Possible	1	51	50	21	33	2	24	Probability is not impossible or certain 0 < Prob. < 1
50/50 but "tails never fails"!	0	0	0	1	0	0	0	Probability meaning ambiguous
Anyone's guess	0	0	0	0	1	0	0	
Likely/possible	0	0	0	0	1	0	0	
Maybe	0	0	1	0	1	0	0	
Possible and likely 50/50 chance	0	0	0	1	0	0	0	
Quite likely	0	0	0	0	1	0	1	
Quite possibly	0	0	0	0	0	0	1	
Not answered	1	1	1	1	1	1	1	not answered

Figure 1. Frequency of students' choices of words to indicate the likelihood of events (N=89). Shaded regions indicate those words that are clearly acceptable descriptors for the events.

One student did not respond to the question. The remaining 88 students used a total of 39 words or phrases to describe likelihood when assigning words to the seven events, as shown in Figure 1. This suggests that some students, at least, did not feel restricted by the given sample words. Figure 1 also shows the number of students giving each response (and this is summarised in Table 1), while the shading indicates those words that are clearly acceptable descriptors for each event.

The word “possible”, which was used—for the most part—in an acceptable fashion given its meaning, nevertheless appears to be used as a catch-all expression (see also Table 1). Large numbers of students used it for events such as DIE and COIN whose numerical probabilities were usually known (as revealed in part (c) of the question), and yet students did not choose to use a more specific expression that reflected these values. Its use is less surprising for MAIL, especially given the difficulty that students had with numerical estimates of this event. Of greater interest, however, is the similar use of “probable”, and, to a lesser extent, “likely” and “very likely”. Both authors feel that all these words can be applied only to events with a better than even chance of occurring (see placement in Figure 1), but it seems that there may be a greater ambiguity than expected in social usage. Over a quarter of the students used “probable” to describe the probability of rolling a 6, suggesting that for some students the word may be seen as synonymous with “possible” or meaning “having a probability”. The use of the phrase “very likely” for the probability of tossing a coin and getting tails by a similar number of students seems particularly inappropriate to the authors. Further investigation is necessary to determine what students actually understand as the meaning of these words, and to investigate on what basis the words are chosen. For Table 1, however, the acceptability of students’ words was based on the authors’ view of the meanings; those with alternative viewpoints can refer to Figure 1.

Table 1
Percentages of Acceptable and Unacceptable Words Associated with Each Event (N=89)

	Event PM	Event MAIL	Event DIE	Event COIN	Event RAIN	Event SUN	Event PETROL
Acceptable words	89	26	3	7	56	85	70
“Possible”	1	57	56	24	37	2	27
Unacceptable words	9	16	39	69	3	11	2
Not answered	1	1	1	1	1	1	1

Numerical Probabilities

Part (c) of the question—requesting numerical probability values for each of the events—caused more difficulties for students than the other two parts, with 16 of the students (18%) not responding at all, and others not giving an answer for some events. As might be expected PETROL was the most avoided question, with 28% of students not responding.

Students used a range of representations for their numerical probabilities, including ratios, fractions, decimals, percentages, and expressing probabilities as “a in b” (e.g., “1 in 2” for COIN). Moreover, each individual student was not necessarily consistent among questions, often mixing representation types, for example by using fractions for DIE and percentages for RAIN. The different representations used are shown in Table 2, together with the extent to which they were used correctly. Two students used integer values in the range 0 to 10, suggesting a 0 to 10 probability scale rather than 0 to 1 (the values were often acceptable viewed in this light). The existence of this misconception is a concern.

Table 2
Percentage of Students Using Different Forms of Numerical Representation of Probability (N = 89)

	Not done	Numerical representation chosen					
		Ratio	Fraction	Percent	Decimal	A "in" B	Other
Correct		1.9	10.3	18.3	10.4	3.9	1.1
Incorrect	22.6	11.4	3.9	7.7	3.4	2.2	2.9

Note. These values were obtained by taking the average of the values for the seven parts to the question.

A number of students used an a:b ratio representation. These caused difficulty, with most students who used them seeing them as representing a part-whole relationship. This was particularly apparent on (a) the DIE question (see Table 3(a)) where 12% of students gave 1:6 and none gave 1:5, and (b) the SUN question where 11% gave 1:1 or similar. It is suspected that the correct ratio responses to other questions were happy accidents rather than indicating understanding of this representation; for example, both 1/2 and 1:2 are acceptable answers for RAIN, even though their numerical values differ. This understanding of ratios was further tested by the CORDIAL question, which was answered correctly by only 21% of students and not attempted by a quarter of them. The students who gave correct responses to the cordial question appeared to be (a) more likely to respond to the numerical probability question and be correct, (b) more likely to use decimals (and be correct), and (c) less likely to use ratio or percentage, than those who were incorrect or did not attempt it. Other comparisons are more difficult to make because the numbers involved are small. Interviews might reveal more about students' understanding of ratio, if only to determine how they pronounce "a:b" (e.g., as "a to b" or "a in b").

Looking at responses to individual questions reveals some interesting results. As shown in Table 3(a), only around half of the students were able to provide an accurate numerical probability for DIE, although an additional 12% "knew" but had problems because of their choice of ratio as a representation. A couple of the numbers included in "Other %" were close values like 15%. Even allowing for these, however, only around two-thirds of the students knew the numerical probability of rolling a 6.

Table 3(a)
Percentage of Students Giving Indicated Numerical Probability Values for DIE

Probability value	1/6	1 in 6	16%-17%	0.16-0.17	1:6	Other ratio	Other %	Other decimal	3/7	Not done
Percentage	31	12	4	3	12	3	8	3	1	20

Performance on COIN was better, with 64% of students giving a correct version of 0.5, and an additional 15% using an incorrectly expressed ratio like 1:2. Three students' answers appeared to be incorrect, even allowing for idiosyncratic representations.

The PM question concerned an event whose likelihood is impossible (or, at best, *extremely* low). Just over half the students wrote 0 for the numerical probability, and an additional 11% chose non-zero values less than or equal to 1 in a million (with 1:1000000 also allowed). A couple of these latter values appeared to incorporate the size of Australia's population. The magnitude of very small non-zero probabilities seems difficult for some students to comprehend, with 14% of students suggesting probability values larger than 1 in a thousand for PM (see Table 3(b)). This difficulty was also apparent for very large but not certain probabilities. For example, on the SUN question two students appeared to allow for the (presumably remote) possibility of some cataclysm, by giving values of 99% or 99.9% for the probability of the sun rising in the morning.

Table 3(b)
Percentage of Students Giving Indicated Numerical Probability Values for PM

Probability value	0	1 in 1 million or less	Up to 1%	More than 1%	1 in infinity	Other	Not done
Percentage	51	11	8	6	1	1	22

Three questions had less clear-cut probability values: RAIN, MAIL, and PETROL (see Tables 3(c), 3(d), and 3(e), respectively). Nearly half the students judged that the probability of RAIN in the week of Easter was 50% or more; answers judged appropriate given the location, the time of year, and the week-long "window of opportunity" for rain. A few students were clearly influenced by the week idea or the fact that days were involved and so gave fractions with denominators 7 or 365. Most of those out of 7 were appropriate values; the other values—3/365, 4/365, and 7/365—were not, although the numerators may reflect the students' guesses of the number of days on which it will rain.

Table 3(c)
Percentage of Students Giving Indicated Numerical Probability Values for RAIN

Probability value	Perc. >74	Perc. 50-70	Perc. <50	Out of 7 or ratio to 7	Out of 365	0	Other	Not done
Percentage	25	24	29	9	3	1	1	21

The results from the MAIL question shown in Table 3(d) should be of particular concern to Australia Post's public relations division! Only 9% of students suggested that fewer than 1 in 100 letters are not delivered on time, and a staggering 49% believed that more than 1 in 10 letters are delivered late. Again these results may be influenced by the availability heuristic.

Table 3(d)
Percentage of Students Giving Indicated Numerical Probability Values for MAIL

Probability value	0.1% or less	0.1%-1%	2%-10%	11%-50%	>51%	Other	Not done
Percentage	7	2	16	42	7	3	24

Table 3(e) gives the students' results for the least clear-cut question, PETROL, with most students suggesting values between 11% and 90% for the probability that petrol prices will increase to $1.05 a litre. Only five students gave an answer of 50%, suggesting that, on this question at least, few students' choices of values were influenced by *equiprobability bias*, the tendency to treat chance outcomes as equiprobable by nature (see Lecoutre, 1992; Williams & Amir, 1995). With only two outcomes available—the price of petrol going above $1.05 or not—the equiprobability bias would suggest that the probability of event PETROL is 50%.

Table 3(e)
Percentage of Students Giving Indicated Numerical Probability Values for PETROL

Probability value	1%-10%	11%-50%	51%-90%	More than 91%	Other	Not done
Percentage	4	26	37	2	2	28

Ranking and Numerical Values

Students' responses were examined to see if the rank order given in part (a) matched up with the order induced by the choice of probability values in part (c). For the purposes of the comparison the event PETROL was not considered because its status was less clear-cut. Only a quarter of the students achieved this. Further research will be needed to determine if students appreciate the existence of a connection between the two parts.

Beliefs About Numeracy

The students' judgements about the numeracy status of the questions used in this study are summarised in Table 4.

Table 4
Students' Judgements of the Numeracy Status of the Questions (N = 89)

	Number showing numeracy	Percentage of students with given degree of agreement with numeracy for those showing numeracy status					Mean score
		SA (5)	A (4)	N (3)	D (2)	SD (1)	
(a) Ranking	82	19.5	45.1	20.7	11.0	3.7	3.66
(b) Words	80	22.5	38.8	25.0	10.0	3.8	3.66

(c) Numbers	74	12.2	43.2	25.7	14.9	4.1	3.45
Cordial ratio	62	35.5	48.4	9.7	6.5	0.0	4.13

Assigning values of 5 to "strongly agree" (SA) down to 1 for "strongly disagree" (SD) allowed the computation of a mean level of the degree of agreement with the statement that the question was a "numeracy question". For the ranking question (part (a)) and the assignment of probability words question (part (b)) the means were identical at around 3.66 (95% confidence interval (3.4, 3.9)), suggesting that students generally agreed that these were numeracy questions. The mean for part (c), assigning numerical probability values, was slightly lower at 3.45 (95% CI (3.2, 3.7)), but still on the agreement side of neutral. There was no significant difference between the three values for the three parts of the probability question.

When the remaining data from the questionnaire is analysed it will be interesting to compare students' judgements about the probability questions with their judgements about other questions, to see if any question received a lower numeracy evaluation. To highlight that this could be an issue, consider the CORDIAL question, concerned with ratios, not probabilities. Here the mean level of agreement was 4.13 (95% CI (3.9, 4.3)), which was significantly different from the judgements about the probability questions (one-way ANOVA, p=0.0011). Students clearly believed that this was a better example of a "numeracy question", perhaps because of its more computational, better-defined nature.

Conclusion

This study has revealed that some adults have difficulty making probability judgements of everyday events. In particular some have a limited vocabulary of probability words, and there are ambiguities associated with their meaning. Another area of misunderstanding is the meaning of numerical values for probability, especially in using ratio/odds notation, but also in appreciating the magnitude of values and in estimating quantities by thinking carefully about what can inform an estimate of an event's likelihood. Further study could examine the relationship between the word and the numerical value chosen for each event.

The study also showed that students judged the probability questions to be within the realm of mathematical knowledge required for everyday life, but not to the same extent as a more traditional mathematics question. Until further analysis of the data is conducted it is not clear the extent to which probability understanding has really entered people's consciousness as "mainstream numeracy".

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References

Australian Education Council. (1991). *A national statement on mathematics for Australian schools.* Melbourne, VIC: Curriculum Corporation.

Board of Studies. (2000). *Mathematics: Curriculum and Standards Framework II.* Melbourne:Author.

Green, D.R (1983). School pupils' probability concepts. *Teaching Statistics*, *5*(1), 34-42.

Hope, J.A., & Kelly, I.W. (1983). Common difficulties with probabilistic reasoning. *Mathematics Teacher, 76*, 565-570.

Konold, C. (1991). Understanding students' beliefs about probability. In E. von Glasersfeld (Ed.), *Radical Constructivism in Mathematics Education*, pp.139-156. Dordrecht, The Netherlands: Kluwer.

Lecoutre, M. (1992). Cognitive models and problem spaces in "purely random" situations. *Educational Studies in Mathematics*, *23*, 557-568.

Shaughnessy, J.M. (1993). Probability and statistics. *Mathematics Teacher, 86*, 244-248.

Starting points: The language of chance. (1998). *Australian Primary Mathematics Classroom, 3* (4), 15.

Tversky, A., & Kahneman, D. (1973). A heuristic for judging frequency and probability. *Cognitive Psychology, 5*, 207-232.

Watson, J.M. (1997). Assessing statistical thinking using the media. In I. Gal & J.B. Garfield (Eds.), *The assessment challenge in statistics education* (pp. 108-121). Amsterdam, The Netherlands: IOS Press.

Watson, J.M., & Moritz, J.B. (2001). *The development of appreciation of chance language in context: Evaluation and interpretation.* Manuscript submitted for publication.

Williams, J.S., & Amir, G.S. (1995, April). 11-12 year old children's informal knowledge and its influence on their formal probabilistic reasoning. Paper presented at the Annual Meeting of the American Educational Research Association, San Francisco, CA. (ERIC Document Reproduction Service No. ED 387 256)