It Depends on the Students: Influencing Teachers' Beliefs About the Ends and Means of Numeracy Teaching

Kim Beswick
University of Tasmania
<Kim.Beswick@utas.edu.au>

This paper reports on the impact of a brief professional learning program for K-8 teachers of mathematics, on teachers' beliefs about effective numeracy teaching strategies and appropriate goals of numeracy teaching, for students with mathematics learning difficulties and for students generally. Evaluation data indicated that the teachers finished the program less inclined to espouse differing beliefs in relation to the two types of students, and that their final beliefs were more in line with the aims of the program.

Numeracy is accepted as having its foundations in mathematics (Australian Education Council, 1990) and is typically defined as also involving affective dimensions and the ability to use mathematics in everyday life (Australian Association of Mathematics Teachers, 1997). More recently, the Tasmanian Department of Education (DoET) (2002) stated that numeracy;

> requires the knowledge and disposition to think and act mathematically and the confidence and intuition to apply mathematical principles to everyday problems. … it also involves the critical and life-related aspects of being able to interpret information thoughtfully and accurately when it is presented in numerical and graphic form (p. 21).

The Australian Government remains committed to the goal of ensuring that all students achieve acceptable levels of numeracy (Department of Education, Science and Training, 2004), and national numeracy policies have acknowledged that some students require, and should receive, additional support for this goal to be realised (Department of Education, Training and Youth Affairs, 2000). Similarly, in Tasmania, it has been recognised that recent and ongoing curriculum reform must include access to a broad, rich and challenging curriculum for students with special and/or additional needs (Atelier Learning Solutions, 2004). The Atelier Report (2004) noted that, for a variety of reasons, commitments to equity and inclusion at a policy level are not necessarily translated into practice in classrooms. This study represents an initial step towards elucidating the role in this of teachers' beliefs about students experiencing difficulty learning mathematics compared to other students.

Mathematics Learning Difficulties

In 2002 Baker commented on the relative dearth of research on effective mathematics teaching for low achieving students. Nevertheless there is a body of literature, grounded in a psychological perspective, that details efforts to identify the causes of learning difficulties in mathematics. Prominent in this arena is Geary (2004) who defined the term "mathematical learning disabilities" as applying to students whose mathematics achievement over successive years is substantially lower than expected on the basis of IQ. Other researchers have used terms including "developmental dyscalculia" (Kaufman, Handl, & Thony, 2003), "mathematical disabilities" (Keeler & Swanson, 2001), and "arithmetic learning difficulties" (Micallef & Prior, 2004) for similarly defined constructs. Estimates of the prevalence of these conditions fall between 3% and 8% of school children

(Geary, 2004; Kaufman et al., 2003) and the students affected tend to use immature calculation strategies for longer than other children (Geary, 2004; Torbeyns, Verschaffel, & Ghesquiere, 2004), take longer to learn mathematical procedures, and have chronic difficulties retrieving basic facts (Micallef & Prior, 2004).

In addition to procedural and memory difficulties, Geary (2004) described a third subtype of mathematical learning disability characterised by difficulty in using and interpreting spatial representations of mathematical material. He acknowledged that relatively little is known about this subtype and it is unclear to what extent difficulties with tasks with a significant visual component are in fact due to other procedural and/or memory deficits. Geary (2004) observed that students with other types of mathematics learning disabilities appear to have spatial abilities comparable to those of other children. Evidence from other sources (e.g., Bobis, 1996) suggests that visualisation has an important role to play in the development of children's number sense and fact retrieval.

Studies of efforts to ameliorate memory difficulties include that of Tournaki (2003) who compared the effectiveness of drill and practice with that of strategy instruction in relation to basic addition facts, for students generally and with learning disabilities. She found that both interventions were effective for general students, but that only strategy instruction led to improvements for students with learning disabilities, and that only strategy instruction led to improvements for both groups of students in relation to tasks that required the use of basic facts in more complex calculations. Keeler and Swanson (2001) suggested that strategy instruction in relation to remembering may also be helpful for struggling students. These findings lend weight to assertion of Aubrey (1993, cited in Robbins, 2000) that "the majority of children identified as having special needs require not specialist teaching but good, high quality and effective teaching." (p. 55).

In this study the term "mathematics learning difficulties" (MLD) was used and its meaning negotiated with participating teachers to apply to the 10% or less of students who experience greatest difficulties with mathematics. Since the study was concerned with teachers' beliefs (defined as anything they held to be true) about students, the IQ of students was not considered. It is likely, therefore, that the students the teachers had in mind as they participated included some with low IQ. Despite this difference from most of the studies cited above, discussions with teachers throughout the project indicated that the students of concern to them exhibited the kinds of difficulties described in the literature. Specifically these students were substantially behind other students in their mathematical development, had difficulty learning procedures and were still struggling with recall of basic facts at the end of primary school and beyond.

Teachers' Beliefs

Teachers' beliefs have long been regarded as critical to the reform of mathematics education (Cooney & Shealy, 1997) and the ineffectiveness of reform efforts has been attributed to failure adequately to address them (Battista, 1994). The gap between policy and practice identified by the Atelier Report (2004) may well be attributable, at least in part, to a disjunction between policy and the beliefs of teachers that underpin their practice. Very little has been written about the teachers' beliefs in relation to students who experience difficulties in learning mathematics, but teachers' disagreement with inclusion policies generally have been documented (e.g., Coates, 1989, cited in Shade & Stewart, 2001) There is also evidence that high teachers' expectations of students in relation to academic tasks are associated with improved achievement (Schoen, Cebulla, Finn, & Fi, 2003). Such expectations are likely to be underpinned by positive beliefs about student capabilities which Beswick (2004) found, even for a teacher with beliefs broadly consistent with a constructivist view of mathematics learning, to be associated with classroom practice more aligned with recent and ongoing mathematics education reform efforts. This study sought to measure the extent to which the participating teachers held differing views concerning appropriate teaching approaches and goals of numeracy teaching for students whom they perceived as having difficulty with mathematics and students more generally.

One element of Green's (1971) description of belief systems related to the observation that beliefs may be held either on the basis of evidence or for non-evidential reasons such as the perceived authority of the source of information, or the fact that a particular belief fits with other centrally held beliefs. Evidentially held beliefs are, by definition, susceptible to change in the light of contrary evidence. The professional learning program that formed the context of this study aimed to present teachers with evidence likely to challenge negative beliefs about the capabilities of students with MLD and the appropriateness for these students of innovative curricula, such as that being implemented in Tasmania (DoET, 2002), that emphasise the importance of deep understanding. Consistent with the literature, visualisation and strategy instruction aimed at conceptual understanding were emphasised.

The Study

The study comprised part of the evaluation of a professional learning program aimed at improving the numeracy education of students with learning difficulties. The program consisted of three spaced half days of interactive workshops and was based upon the following beliefs concerning mathematics/numeracy:

1. All students are entitled to a rich, broad and challenging mathematics curriculum (Atelier Learning Solutions, 2004).
2. All students are able to learn mathematics (Ollerton, 2001).
3. A belief that mathematics makes sense is an essential part of being numerate (Van de Walle, 2004).
4. All students should experience mathematics teaching aimed at the development of deep conceptual understanding.

In addition the program was designed, to the fullest extent possible, to embody characteristics of effective professional learning including: meeting the immediate perceived needs of participants (Atelier Learning Solutions, 2004); addressing both teachers' practice and beliefs (Wilson & Cooney, 2002); relating theory and practice, and including an expectation that participants would trial new ideas in their classrooms (Guskey, 1995); and providing opportunities for teachers to share ideas and experiences (Franke, Carpenter, Levi, & Fennema, 2001).

The program began with discussions aimed at eliciting the teachers' beliefs about the needs and capacities of students with MLD in relation to numeracy, and about the appropriateness of innovative curricula for various students. The responses of the participants to questions relating to these issues determined the specific content of the program. The program thus provided participants with specific ideas relating to teaching mathematics topics that they considered problematic yet crucial to the development of numeracy, as well as opportunities to discuss a range of issues related to the program's aims. The topics and issues nominated by the participants and addressed (however briefly) by the program are shown in Table 1. Asterisked items were treated in somewhat more detail than the others and many of the issues raised were recurrent themes in the teachers' discussions as various topics were addressed.

Table 1
Topics and Issues Addressed by the Program

Topics	Issues
Place value*	What survival life skills do students need?
Mental computation*	When to use which concrete materials
	Mathematical thinking and reasoning*
Visualisation*	Developing metacognition*
Time	Meaning of numeracy – application
Money	Affective responses of students
Fractions*	Time implications of teaching for understanding/with concrete materials*
Estimation	Retention of knowledge*
Decimals*	Repetition of content
Rational numbers	Engaging older students with concrete materials
	Promoting student autonomy
	Role of language in mathematics
	Encouraging students to verbalise their thinking*
	Moving from concrete to abstract reasoning
	The place of calculators/spreadsheets

In each of the second and third sessions several teachers brought along examples of relevant materials and programs that they had found useful in the past. Text-based resources were made available for all to inspect and discuss at specific times during the workshops, while a set of Linear Attribute Blocks (Stacey, Helme, Archer & Condon, 2001), useful in facilitating the development of students' understandings of decimals, that one participant had made were demonstrated and discussed in some detail. Many teachers implemented ideas from the earlier sessions with their classes and reported on these experiences in subsequent sessions. A set of six readings was also provided as a further stimulus to discussion.

The evaluation of the program included an examination of the extent to which teachers' beliefs about appropriate goals of mathematics teaching and approaches to teaching mathematics, differed according to their perceptions of the students' mathematics learning abilities at both the beginning and end of the professional learning program.

Subjects

The 22 teachers who participated in the professional learning program were the subjects of the study. Five identified as early childhood teachers, eight as primary teachers and nine indicated that they taught middle school grades, meaning the lower grades of secondary school in this context.

Instrument

The survey, *Numeracy for Students with Mathematics Learning Difficulties* (NSMLD), comprised three sections, the last of which is reported on in this paper. This section comprised 22 items, many of which had been used in earlier work on teachers' beliefs (e.g., Beswick, 2003) concerning approaches to teaching mathematics and the goals of mathematics instruction. Each item required responses on two five-point Likert scales, one relating to students generally (labelled, 'All students') and the other relating to students with mathematics learning difficulties (labelled 'Students with MLD'). Responses were scored from one for "strongly disagree" to five for "strongly agree".

Respondents were asked to use a code name in order to allow the initial and final surveys to be matched while preserving the respondents' anonymity.

Procedure

Subjects completed the NSMLD at the beginning of the first professional learning session and again at the end of the last. Ideally two versions, dealing respectively with beliefs about students with MLD and students generally, would have been administered on separate occasions but time did not allow this. As it was, teachers were fully aware of the extent to which they were distinguishing between all students and those with MLD and this may have reduced the differences reported.

Results and Discussion

There were statistically significant differences in relation to the two groups of students for the items shown in Table 2.

Table 2
Items Eliciting Significantly Different Responses for All Students and Students with MLD

	Mean (all students) n=22	Mean (Students with MLD) n=22	Mean diff. (All-MLD)	Std Dev.	Sig. (2-tailed)	Effect size
3. Conceptual understanding is an appropriate goal of mathematics students.	4.09	3.81	0.29	0.56	0.030*	0.51
3. *Conceptual understanding is an appropriate goal of mathematics students.*	4.43	4.24	0.19	0.40	0.042*	0.47
8. Students should not rely on concrete material rather than thinking, for solving mathematics problems.	2.05	1.64	0.41	0.67	0.009**	0.61
11. Providing students with 'survival' mathematical skills is an appropriate goal of mathematics instruction.	3.27	4.18	-0.48	1.15	0.001**	0.79

*$p<0.05$. ** $p<0.01$.

Higher mean scores indicate greater agreement with statement, and italics indicate differences that were obtained on the second administration of the survey. The effect sizes were calculated by dividing the mean difference by the standard deviation of the differences to provide an indication of the relative size of the difference in means in relation to the general variability of responses (Burns, 2000). The effect sizes obtained were medium in the case of Item 3 at both administrations of the survey and medium and large for Items 8 and 11 respectively.

The participants began the program significantly less inclined to see conceptual understanding as an appropriate goal for students with MLD compared to students generally. Rather, they regarded survival skills as more appropriate for these students and were more inclined to see concrete materials as supporting answer getting, rather than the development of understanding for these students. This is consistent with their conversations in the first professional learning session about the problems such students tend to have with retaining facts. There was still a statistically significant difference between participants' beliefs about the two groups of students in relation to conceptual understanding as a goal, at the end of the program but Table 2 shows that the means had increased for both groups and come closer together. The difference was slightly less significant at the end of the program (p=0.42) than at the beginning (p=0.03). Both the direction of the change and the convergence of the means are in accordance with the principles upon which the program was designed. The very significant difference in relation to Item 11 at the start of the program did not exist at the end, suggesting that participants finished the program less inclined to believe that 'survival' mathematics was the province of students experiencing difficulty learning mathematics.

Table 3 shows items for which there were significant changes from one administration of the survey to the next, in relation to either all students or to students with MLD. In this case items relating to students with MLD are italicised. Again effect sizes were calculated.

Table 3
Items Eliciting Statistically Different Responses at the two Administrations of the Survey

	Initial Mean n=22	Final Mean n=22	Mean diff. (initial-final)	Std Dev.	Sig. (2-tailed)	Effect size
4. Telling children the answer is an effective way of facilitating their mathematics learning.	2.82	2.14	0.62	0.92	0.006**	0.67
4. Telling children the answer is an effective way of facilitating their mathematics learning.	2.77	2.10	0.62	0.92	0.006**	0.67
8. Students should not rely on concrete material rather than thinking, for solving mathematics problems.	1.64	2.10	-0.48	1.03	0.047*	0.46
21. Explicit teaching in mathematics should focus on task requirements, strategies, and highlighting significant mathematical learning.	3.59	3.95	-0.38	0.80	0.0.42*	0.47

*p<0.05. ** p<0.01.

Following the program participants were less likely to believe that telling students answers was an effective way of teaching them. The change was significant and the effect size medium to large in relation to both students generally and those with MLD. Consistent with this was the change in relation to participants' opinions regarding what should be made explicit in mathematics teaching for all students. Care was taken in the delivery of the program to define explicit mathematics teaching in terms consistent with item 21 (see Table 3) and not as prescribing procedures for solving problems or performing calculations. The participants were also more inclined, after the program, to reject the notion that students with MLD should use concrete materials as a substitute for thinking to get answers.

Overall it seems the program had some success in influencing the academic expectations of teachers in relation to students with MLD in ways likely to contribute to their improved achievement (Schoen et al., 2003).

Conclusion

The results of this study need to be viewed with some caution due to the small number of teachers involved and the brevity of the intervention. Nevertheless it provides some evidence that teachers do hold differing beliefs about appropriate means and ends of numeracy teaching for students depending upon their perceptions of the students' ability to learn mathematics. In particular, they are likely to regard a skills based curriculum focussed on 'real world' survival, rather than one aimed at the development of deep conceptual understanding to be appropriate for students with MLD.

In addition, they are more likely to approve of the use of concrete material for answer getting rather than for supporting conceptual development, for students with MLD. This illustrates the point made by Askew, Brown, Rhodes, Johnson and Wiliam (1997) that superficially similar practice may in fact have quite different outcomes depending upon the underlying beliefs of the teacher. It is certainly not sufficient to mandate particular practices in hope of achieving real change in students' learning.

It seems that the problem of translating policy concerning equity and inclusion into classroom practice that was identified by the Atelier Report (2004) is at least partly due, in the area of mathematics/numeracy, to beliefs that some teachers hold in regard to students with MLD. Attention will need to be paid to teachers' relevant beliefs if inclusive policy is to have a real impact on students with MLD. This study provides some encouragement that these beliefs are evidentially held (Green, 1971) and hence susceptible to change when they are made explicit, and evidence to the contrary is presented. In this study some of this evidence was sourced from research on effective numeracy teaching for students generally and selected, mindful of what little is known about effective numeracy teaching for students with MLD, to address the immediate perceived needs of the teachers in relation to these students. In addition, evidence from the teachers' own experiences as they trialled various approaches and activities, albeit briefly seemed, anecdotally at least, to have a positive impact. These observations are consistent with the notion of a dialectic relationship between beliefs and practice in which both change together in complex ways.

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