Connecting the Points: Cognitive Conflict and Decimal Magnitude

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This paper reports on an investigation into managing cognitive conflict in the context of student learning about decimal magnitude. The influence of prior constructs is examined through a brief review of the literature. A micro-genetic approach was used to capture detail of the teaching intervention used to facilitate development in student thought. A framework for considering cognitive conflict in lesson design is presented, and a case is made for the use of measurement tasks to generate data.

Proficiency with decimal numbers is essential for progress in school mathematics and for financial and statistical literacy in adults. Student difficulties with decimal numbers are well documented. In New Zealand for example, recent reports showed that only 50% of Year 8 students could identify tenths and hundredths and even fewer were able to correctly order decimal numbers (Flockton, Crooks, Smith & Smith, 2006; Young-Loveridge, 2007). This paper reports on part of a larger study into how students engage with the cognitive demands of new material that contradicts their previously-held schemata. A specific focus of decimal magnitude has been selected because while the causes of student misconceptions have been clarified, an ongoing lack of student achievement indicates a need to address further the factors that will help design more successful teaching interventions (Okazaki & Koyama, 2005).

Background

In *variation theory*, learning is seen as attending to those features in novel problems that are similar to, or vary from, existing knowledge. Variation theory occupies a niche within Piaget's equilibrium model of learning. Points of both connection and distinction need to be recognised by students when encountering new situations. This enables the self-perception that one's thinking needs re-organisation (Runesson, 2005). *Cognitive conflict* is a term used to describe the tension created when new evidence is recognised by the student as contradicting previous knowledge. The agency is with the learner as the teacher can only provide situations of *potential conflict*. Resolution of that conflict may result in new learning, as points of both connection and difference are recognised and responded to. This process has been termed *re-constructive generalisation* (Harel & Tall, 1991).

A failure to recognise the disjunctive elements of new problems may lead to the mis-application of previous understandings about number. This process has been termed *expansive generalisation* (Zazkis, Liljedal & Chernoff, 2008). With regard to errors concerning decimal magnitude, researchers have found almost complete correspondence between student answers and their underlying schema (Nesher & Peled, 1986). This is evidence that these answers are not mistakes in terms of the students' perspective but indicative of their conceptual understanding.

There are two common expansive generalisations regarding the magnitude of decimals. One is where students think that the decimal point serves to separate two, whole-number systems and is often termed *whole-number thinking*. Students transfer the whole number truth that 'longer is larger' to decimals. To decide relative magnitude, students view the

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whole number sections first and make a decision if these are unequal. If not, they then look at the length of the section after the decimal point e.g. 1.23 is regarded as smaller than 2.5, but larger than 1.8. Another is where a 'shorter is larger' system is applied, resulting from students misapplying their understanding of denominators. A larger denominator indicates a smaller fraction (given identical numerators). The system results in 0.6 being interpreted as sixths, and therefore larger than 0.65, as this is interpreted as sixty-fifths. Students who make errors with decimal magnitude typically apply one of these systems or a sub-variant of them (Steinle & Stacey, 1998).

It is not the existence of prior constructs per se that is the problem, rather it is their durability in spite of teacher-provided evidence to the contrary, the 'obstinacy factor'(Harel & Sowder, 2005). Research suggests that primitive schemata are deeply embedded and difficult to change (McNeil & Alibali, 2005). Counter-examples to prior constructs are not necessarily effective catalysts for change as the novelty of new, externally-provided information can be unrecognised, compartmentalised or disregarded by students (Zazkis & Chernoff, 2008). Disequilibrium is avoided when students fail to recognise any contradiction and simply assimilate new material into their previous way of thinking. They operate with systems that are consistent with their internal schemata and are confused as to why some of their answers are regarded as incorrect by the teacher. Cognitive conflict is also avoided when students uncritically adopt the algorithms advocated by teachers. For example, some students are told to line up the decimal points or to add zeroes (sic) to make decimals of equivalent length in order to ascertain magnitude. These students may temporarily comply with a procedure but may subsequently revert to behaviours consistent with their prior construct (Siegler, 2000).

The diagram below serves to summarise possible learning experiences and educational outcomes.

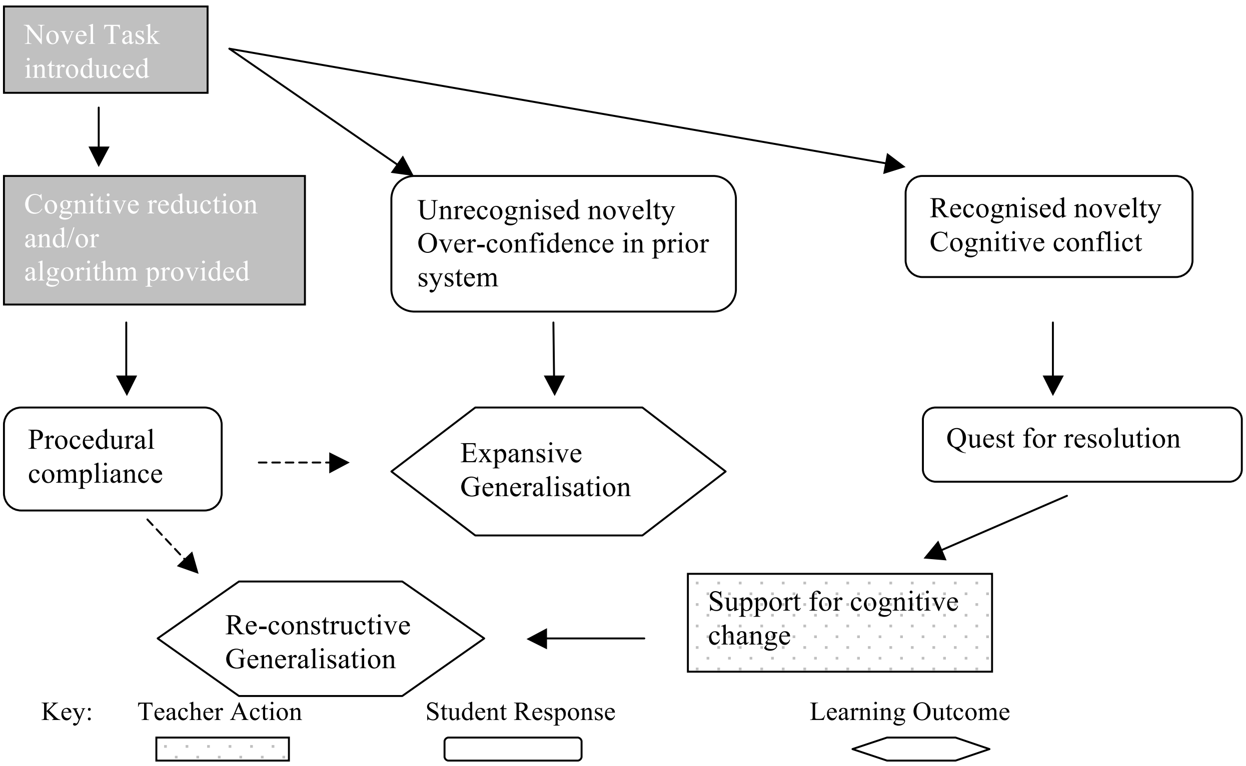

Figure 1. A Framework for Considering Cognitive Conflict in Lesson Design (Adapted from Moody, 2008)

Understanding decimal magnitude requires integration of the place-value convention of recording digits in columns with fractional understanding of denominators. Existing student constructs need not be regarded as problems but as sites to anchor new meaning. The linkage of new symbols and systems to concrete referents is seen as an important first step in understanding new concepts (Goldin & Shteingold, 2001). Equipment may faithfully represent the mathematics from the teacher's understanding, but it is the perspective of the student that will determine its efficacy as a learning tool (Stacey, Helme, Archer & Condon, 2001).

Situations where students have been creators of the evidence that produce cognitive conflict are seen as having great potential to initiate change. If students anticipate a particular result but are subsequently confronted with one that is unexpected, it may initiate deeper consideration of the prior construct, an activity that has been termed 'reflection on activity-effect relationships' (Simon, Tzur, Heinz & Kinzel, 2004). Some studies have shown that real-life experiences, whether enacted in the classroom or recalled from outside it, have facilitated shifts in thinking about decimals (e.g. Irwin, 2001). Measurement offers a powerful means of engaging students with number because relative magnitude is transparent. Students have a concrete reference for the symbol used to describe the quantity (Sophian, 2008). Use of metres and centimetres does not always help with decimals however as the common use of language (e.g. 1 metre and 45 centimetres for 1.45m) may reinforce the whole-number expansive generalisation.

In order to investigate the mechanism of conceptual change, intense collection of in-situ data of student engagement with a situation of potential conflict is required. Standard cross-sectional studies lack the temporal resolution to capture evolving (rather than evolved) competence and the subtlety of interactions as learning occurs (Lamon, 2007; Seeger, 2001). These considerations led to the adoption of micro-genetic methods (Siegler, 2007).

Method

Six students who were 'at, but not above' national expectations in mathematics were involved after consultation with the classroom teacher, the parents and the students themselves. They could order unit fractions but had received no formal teaching of decimals. In the study I was both the teacher and the researcher.

A design experiment model was used (Cobb, Confrey, Di Sessa, Lehrer & Schauble, 2003). Baseline data were collected via a personally modified version of the Decimal Comparison Test (DCT) designed by Stacey & Steinle (1999). The DCT has 30 pairs of decimals for comparison by magnitude. A group interview was also conducted. From these data, 5 intervention sessions were planned of approximately 45 minutes each. While a likely sequence of events was mapped, this methodology allowed for ongoing reflexive interaction between student responses and teacher initiatives (Gorard, Roberts & Taylor, 2004).

The enacted plan had sessions that focused upon the iteration of non-unit fractions including tenths, an introduction to decimal notation via equipment use, practical measurement tasks, games using decimals, and a brief exposure to additive tasks. No notes or formal procedures were given to the students. Instead, tasks were presented and conversations arose as students engaged with them, sometimes between pairs of students, as well as individual and group discussions with me. All dialogue was recorded using audio-tape and supplemented with collections of student work and personal field notes. These data were complemented by pre-and post-intervention interviews and written tasks.

Much of the practical work in sessions 2 and 3 centred upon the use of a commercial product known as Pipe Numbers. Pipe Numbers are a set of plastic tubing cut to scale with 1, 1/10 and 1/100 pieces. They are a linear model of the number system and conceptually identical to the Linear Arithmetic Blocks (LAB) described by Helme and Stacey (2000). The portability of Pipe Numbers enhances their suitability for use in measurement tasks.

Results

This section documents evidence of the initial and final thinking of all of the participants and especially tracks the learning of Grace and Wini (as representatives of the two common expansive generalisations) via samples of conversation and written work.

Table 1
Details of Participants

Pseudonym	Gender	Ethnicity	Age	System Descriptor	Consistency Score*
Mary	F	Pakeha	9	No pattern	n/a
Ripeka	F	Maori	9	Longer larger	100
Tame	M	Maori	9	Longer larger	100
Grace	F	Pakeha	10	Longer larger	100
Wini	F	Maori	10	Shorter larger	93
Aroha	F	Maori	10	Shorter larger	87

* The consistency score is the percentage of answers that conform to the system descriptor.

During session 1, the students made models of non-unit fractions, including using the Pipe Numbers to model tenths. They were introduced to the convention that a fractional quantity involving tenths could also be represented in decimal notation, e.g. 3/10 as 0.3. The following excerpt comes from session 2 of the intervention. Students were again making models of numbers using the Pipe Numbers equipment but were given a new challenge.

Teacher: See if you can make this one, 0.12.
Wini: That's twelve!
Teacher: OK, see what you will make. [Not giving validation or refutation, but simply asking that the task be carried out].
Wini: Twelve! Got it! (Showing a model that used twelve tenths).
Teacher: So you've put twelve of those tenths on, OK, what does that symbol tell us? (Pointing to zero).
Wini: Zero.
Teacher: How many ones is that? (Pointing to 0.12 written on the board).
Wini: Zero.
Teacher: But your one (meaning her model) is bigger than 1. (Wini went away and returned a few minutes later with a new model).
Teacher: OK, you're using some of those little ones. You've got ten tenths and two of those little ones. (Wini was making another model using twelve pieces). So you've made me a whole one and those little ones. You've made me 1.02.
Wini: I don't get it!
Teacher: That's OK, I never said this one would be easy, it is hard.

As the teacher, I resisted the urge to 'help' her and thus bypass her personal agency to learn. Wini adjusted her model by replacing one of the one-hundredth pieces with a one-tenth piece. Her model still involved twelve pieces, but a new piece of feedback was possible.

Teacher: That is 1.11

Wini: (After a pause) Oh!

She then adjusted her current model to the correct one, gave me an expectant look, and then received a nod. She had used the information received from my responses (1.02 and 1.11) and reinterpreted how the task would be completed. Her responses to subsequent tasks showed that she had not merely interpreted the physical model but was engaging with the underlying place value concept.

In session 3, students measured and recorded lengths of objects in the room. Recording and presenting each set of measurements on A3 paper allowed the students to see and discuss each other's data.

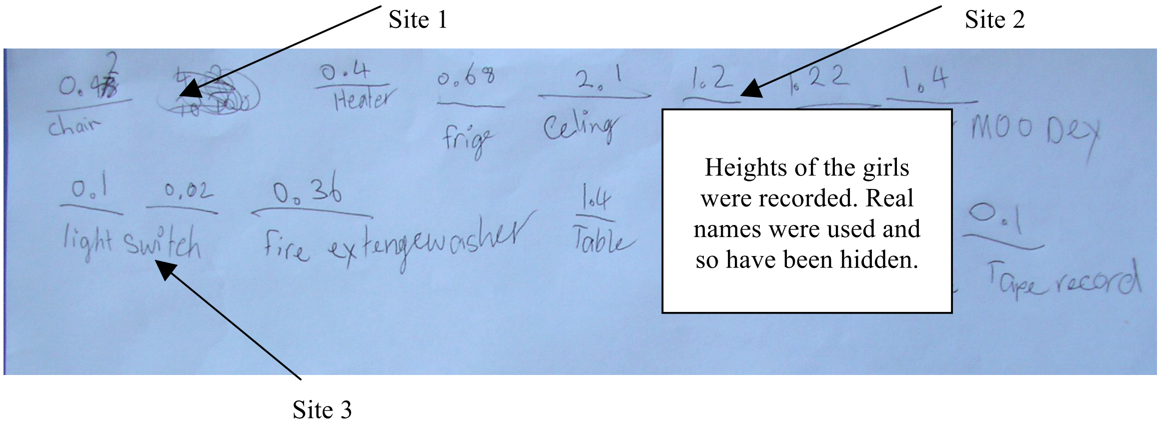

Figure 2. Photograph of Student Work by Grace and Wini

Table 2
Comparison of Students' Initial and Final Responses to the DCT

Pseudonym	Initial System Descriptor	Consistency Score	Final System Descriptor	Consistency Score
Mary	No pattern	n/a	Correct to 2dp	90
Ripeka	Longer larger	100	Correct	93
Tame	Longer larger	100	Correct	97
Grace	Longer larger	100	Correct	97
Wini	Shorter larger	93	Correct	100
Aroha	Shorter larger	87	Correct	100

Each student was then directed to three pairs of examples from their two DCT scripts and asked to explain why they had made changes. Their scripts were unmarked so as not to provide external validation of either response. The student's new answer is underlined.

Wini: (0.55 and <u>0.555</u>) Five thousandths more. (Previously a 'shorter is larger' system was used.)

Grace: (0.75 and <u>0.8</u>) It's larger, it has an extra tenth; when I first started I thought that (pointed to 0.75) was the highest because of seventy-five. (Previously a 'longer is larger' system was used.)

The students' knowledge of decimals was re-assessed after 6, then 16 months and found to be secure.

Discussion

The initial data showed that 5 of the students were consistently working from secure prior constructs as predicted by earlier research (e.g. Nesher & Peled, 1986; Steinle & Stacey, 1998). These constructs can be described as expansive generalisations as the procedures arising from them can be explained as the misapplication of a previously observed rule (Harel & Tall, 1991; Zazkis et al, 2008).

In accordance with the application of variation theory (Runesson, 2005), the similarities and differences between prior knowledge and new evidence arose in meaningful ways for the students. The cognitive conflict this awareness created was managed by the teacher in order to address the issue of resistance to change (McNeil & Alibali, 2005; Zazkis & Chernoff, 2008). For example, in the transcript excerpt from Session 2, Wini was confronted with two 'truths' that could not simultaneously co-exist. 0.12 must represent twelve tenths according to her whole-number schema, but the knowledge that ten tenths was equivalent to one whole was also known to be true from her understanding of fractions. The conflict arose from the unexpected result (Simon *et al, 2004*) and resolving this tension is a vital part of re-constructive generalisation according to Harel & Tall (1991). The benefit of simultaneously using concrete referents (Stacey et al, 2001), realistic problems (Irwin, 2001) and measurement tasks (Sophian, 2008) can be seen when examining three items from the work sample of Grace and Wini shown in Figure 2.

At site 1, the chair's measurements were originally recorded as 4/10 and 2/100. This indicated that the students were initially thinking in fractional terms from using the Pipe Numbers and then applying their new proficiency with decimal notation. They could interact with one place value column at a time. The common pronunciation of "point four two" does little to convey meaning. Encouraging students to decode the symbol as four tenths and two hundredths may help reinforce the connection between the new symbol and the more familiar expression of that quantity. As Goldin and Schteingold (2001) suggested, developing clear links between quantity, vocabulary and symbol is critical for new understanding.

At site 2, the girls knew that they were very similar in height but in 'longer is larger' terms, 1.22 is much bigger than 1.2, while in 'shorter is larger', it's much smaller. This was a meaningful context where two decimals can be demonstrated as being of similar size despite there being a different number of digits used to represent the quantity.

At site 3, the length of the switch (0.02), was seen as smaller than the entire light fitting (0.1), and thus attention was drawn on the place value of the digits used to record the measurement. This stands in contrast to the large number of Year 8 students in the NEMP study who could not distinguish 0.7 from 0.07 (Flockton et al, 2006).

Decimals in the context of games and in tasks that did not use pipe numbers were important in establishing whether place-value thinking was changing or simply mastery of a new manipulative was being exhibited. It appeared that the students were able to see past the materials to the mathematics in that they were applying place-value language to make decisions concerning magnitude and were extending such discussions beyond tenths and hundredths, the limit of the physical representation. For example, in the student-initiated discussion of thousandths, the students had begun to consider the decimalisation process as generalisable. They started to discuss the equipment in terms of what it *could be*, rather than what it *was*. It is these glimpses of insight into how small experiences can lead to conceptual change that micro-genetic study is especially suited to capture (Siegler, 2007).

The final DCT data indicated that all students had improved in their ability to order decimal numbers. It was inferred that this was due to a reconstruction of their place value schemata as they had not been given any new procedures to adopt. The high consistency scores showed that the students were working from stable schema. This did not imply that their previous schemata had been totally eliminated from their thinking but that currently the new conception was clearly dominant (Siegler, 2000).

Conclusion

This study investigated the mechanism of conceptual change with respect to decimal magnitude. This was in response to continuing reports of student difficulties (e.g. Flockton et al, 2006) and in recognition that the change process is complex and required further investigation (McNeil & Alibali, 2005). Its findings demonstrate that it is possible to stimulate cognitive conflict by involving students in practical tasks and providing them with feedback on the contradictions that arise between new evidence and prior thinking. It is thought that student production of evidence was an important factor in initiating steps towards conflict resolution. The need to unambiguously communicate measurements may have helped the students to appreciate why they should discontinue use of an expansive generalisation. Self-realisation of the reasons for change is a factor in countering the durability of primitive schema (Harel & Sowder, 2005). As Zazkis and Chernoff (2008) suggested, students who exercise personal agency when faced with potential cognitive conflict are more likely to respond to counter-examples with new learning than those where expert opinion is simply presented to them.

In agreement with Sophian (2008), the findings of the study also showed that the use of a measurement-based system to represent numbers has much to commend it. Combining this system with practical activities allowed for student engagement with issues of decimal notation as these arose in context. Their resolution could proceed at the pace of student thought and at student-chosen moments without the teaching agenda being compromised. As Siegler (2007) suggested, the use of a micro-genetic approach allowed for the capture of important details of student learning. These provided insight into the thoughts of students as learning occurred. It is suggested that further studies are undertaken into the mechanisms whereby situations of potential cognitive conflict result in student re-conceptualisation, particularly in areas of known learning difficulty.

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