The Relationship between the Number Sense and Problem Solving Abilities of Year 7 Students

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This paper reports on a component of a large yearlong study in three Year 7 classes in three different schools. The aim of this research component was to determine the relationship between students' number sense and their problem-solving ability by means of paper-and-pencil tests, classroom observations, and interviews of students and teachers. The results revealed a strong correlation between these two aspects of school mathematics, with important implications for classroom teachers.

This paper reports on one aspect of a much larger study conducted in three schools in the Perth Metropolitan over a one-year period (Louange, 2005). The three schools were an independent boys' school, an independent girls' school and a coeducational public school. One Year 7 primary school teacher, identified by tertiary mathematics educators as being an effective mathematics teacher, was selected in each school for the study. The main study investigated the relationships between, teaching style, learning style, number sense, and problem solving ability in the three Year 7 classes through testing, interviews of students and teachers, and extensive periods of classroom observation over the year's mathematics lessons. The purpose of this paper is to determine the relationship between number sense and problem solving as a facet of the larger study just described.

Background

Both number sense and problem solving are promoted as two of the major areas of emphasis in mathematics education, as evidenced in major curriculum documents (Australian Education Council, 1991; National Council of Teachers of Mathematics, 2000; Curriculum Council of Western Australia, 2005). Thus, there is an urgent need to answer questions such as, "How do they relate to each other in terms of how they are taught, learnt and utilised in solving mathematical problems?" While there have been many studies relating to the two aspects independently, there is a lack of research which focuses on the relationship between the two.

Unfortunately, one possible reason for lack of a combined number sense and problem solving research could be due to the difficulty of divorcing one from the other. Both the terms 'number sense' and 'problem solving' have suffered many diverse definitions. The controversy stems mainly from the fact that number sense is akin to common sense (McIntosh, Reys, Reys, Bana, & Farrell, 1997), which is a necessary tool for solving any problem. Compared to problem solving, number sense seems to have proved to be the most difficult to define, as McIntosh et al. (1997) state:

> Like common sense, number sense is a valued but difficult notion to characterise and has stimulated much discussion among mathematics educators (including classroom teachers, curriculum writers, and researchers) and cognitive psychologists. (p. 3)

Thus, while there seems to be general agreement by mathematics educators on a definition of a problem, it is not possible to define number sense in such a straightforward manner. A problem is generally regarded as a situation where both the solution and method

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of solution are not obvious. However, number sense is best described by a comprehensive list of characteristics. Without listing them here, we would venture that number sense could be considered as 'the ability to make sound estimates', since this ability necessarily incorporates the list of number sense attributes that are usually identified.

In the publication edited by McIntosh and Sparrow (2004) mathematics educators from the UK, Europe, the USA and Australia who are recognised as leaders in the issue of number sense, provide considerable insight for researchers, curriculum developers and mathematics teachers at all school levels. There is an abundance of other material by mathematics educators and researchers on the teaching of number sense, with publications by Anghileri (2000) and Askew (2002) being two examples. Considerable research has been undertaken on problem solving, and the teaching of mathematics through problem solving continues to be a focus of mathematics educators (Schoen & Charles, 2003; Nisbet & Putt, 2000). However, investigations into the links to number sense are lacking. Despite their evident connection, problem solving and number sense are not necessarily synonymous because while number sense is inherent in problem solving, many problems are solved without recourse to number sense (Hiebert et al., 1997). Most definitions of number sense incorporate a sense of problem solving (Denvir & Bibby, 2002), which serves to show that it is virtually impossible to separate the two. Although in these definitions the intention weighs more towards number sense inherent problems, it could also be that number sense ability is intricately linked to mathematics problems, which are devoid of number sense (Anghileri, 2000). Yet there is a lack of research to elucidate the relationship between problem-solving ability and number sense.

In the context of this present research any problem which necessarily requires knowledge and skill in number, to arrive at an acceptable resolution, will involve number sense; while any question for which the solver has no immediate and apparent way of solving will constitute a problem (Thiessen & Trafton, 1999; Reys & Yang, 1998). This study aims to explore what relationships exist between Year 7 students' number sense and their problem-solving ability.

Methodology

As indicated above, the subjects were three Year 7 primary classes in three different schools, each of which had a full-time teacher identified by a number of tertiary mathematics educators to be an effective mathematics teacher. The study involved both quantitative and qualitative approaches. Pre-tests in both number sense and problem solving were administered to all students at the beginning of the school year, and the same tests were given again at the end of the year. On both occasions the tests were administered over two days, with one test each day in the normal mathematics time-slot. Each item was read to the class to help overcome any possible reading difficulties. During the year, 30 mathematics lessons were observed in each class, which was one lesson per week for each teacher over most of the data collection period. Both teachers and students were interviewed.

The number sense instrument consisted of 45 of the items used by McIntosh et al. (1997) in their international study of number sense in three countries. A sample item is included in the Appendix. The problem-solving instrument consisted of items selected from problem-solving competitions conducted by the Mathematical Association of Western Australia (2000, 2001, 2002). The problem-solving instrument was refined from a pilot study and consisted of eight items – four of which involved number sense and four that were devoid of number sense. A sample of each type is in the Appendix. Each problem

was presented on a single page and students were required to document all their processes and the strategies used. It was considered most important that the process as well as the product be taken into account for each problem. A scoring scale was adapted from Charles, Lester, and O'Daffer (1987) that allocated a score of 0, 1, or 2 for each of the categories Understanding the Problem, Planning a Solution, and Getting an Answer. These three scores were then added to give an overall score for each of the eight problems.

A total of 64 students completed all the assessments and 45 of these were formally interviewed. The 45 were selected in such a manner as to be representative both of the three classes and also their performances on the two initial paper-and-pencil tests. This interview involved each student working aloud to complete two problems of each type, with follow-up questions by the interviewer as appropriate, and using stimulated recall. Half the 64 students were also interviewed informally during the classroom observation sessions over the one-year period. Semi-structured interviews were conducted with each of the three teachers, and there were informal interactions on the occasions of the observations of the mathematics lessons. Observations of significance to the study were recorded during the observed mathematics lessons by the researcher. All data gathering was undertaken by the one researcher.

Results

The results for both the pre- and post-tests of number sense (NS) were combined to give one score. The same was done for the pre- and post-tests of problem-solving (PS) performance to give one basis for comparing the two aspects. The combined scores for number sense were categorised as High (H_{NS}), Medium (M_{NS}), or Low (L_{NS}). The combined scores for problem solving were categorised in the same way. Using the three levels for each skill, the grid of nine cells in Table 1 gives a good insight into the relationship between number sense and problem solving.

Table 1
Number and Percentage of Students within each Score Category for NS and PS

	H_{PS}	M_{PS}	L_{PS}	Total
H_{NS}	12 (18.8)	6 (9.4)	1 (1.6)	19 (29.7)
M_{NS}	5 (7.8)	15 (23.4)	6 (9.4)	26 (40.6)
L_{NS}	1 (1.6)	6 (9.4)	12 (18.8)	19 (29.7)
Total	18 (28.1)	27 (42.2)	19 (29.7)	64 (100)

Note: N = 64, with percentages of students shown in parentheses.

The diagonal of matching categories demonstrates a very strong relationship between number sense and problem-solving ability, with 12 students rated high in both, 15 rated medium in both, and 12 rated low in both aspects. There are 11 students rated high on one

aspect and medium on the other, and there 12 rated medium on one and low on the other, so that the relationship for both these sets could be considered as fairly strong. This leaves only two students who scored high on one and low on the other. These results clearly demonstrate a strong relationship between the two aspects overall.

The problem solving instrument consisted of four number-sense-inherent problems (NSIP) and four devoid-of-number-sense problems (DNSP). The 64 students were asked whether they preferred NSIP or DNSP types. Figure 1 shows a scatter plot of the relationship between problem-solving performance and number sense, and also gives each student's preference for either NSIP or DNSP types. A two-tailed Pearson Correlation was applied to the pre- and post- PS and NS combined scores, resulting in quite a strong correlation of 0.77 at the 0.01 level. The coefficient of determination indicates almost 60 percent shared variance, which implies that number sense helps to explain nearly 60 percent of the students' scores on the problem solving test. Although the converse could also be true, triangulation of data obtained from the various forms of data collected, especially those from the interview involving stimulated recall with the solving of four problems, show greater support for a theoretical framework in which problem solving ability level depends more on number sense than vice versa. For instance, for the 45 students interviewed, there was a correlation between their PS scores at the interview and their PS test performance scores ($R = 0.31$, $p < 0.04$). There was also a significant correlation between the 45 students' interview PS scores and their NS performance scores ($R = 0.55$, $p < 0.005$).

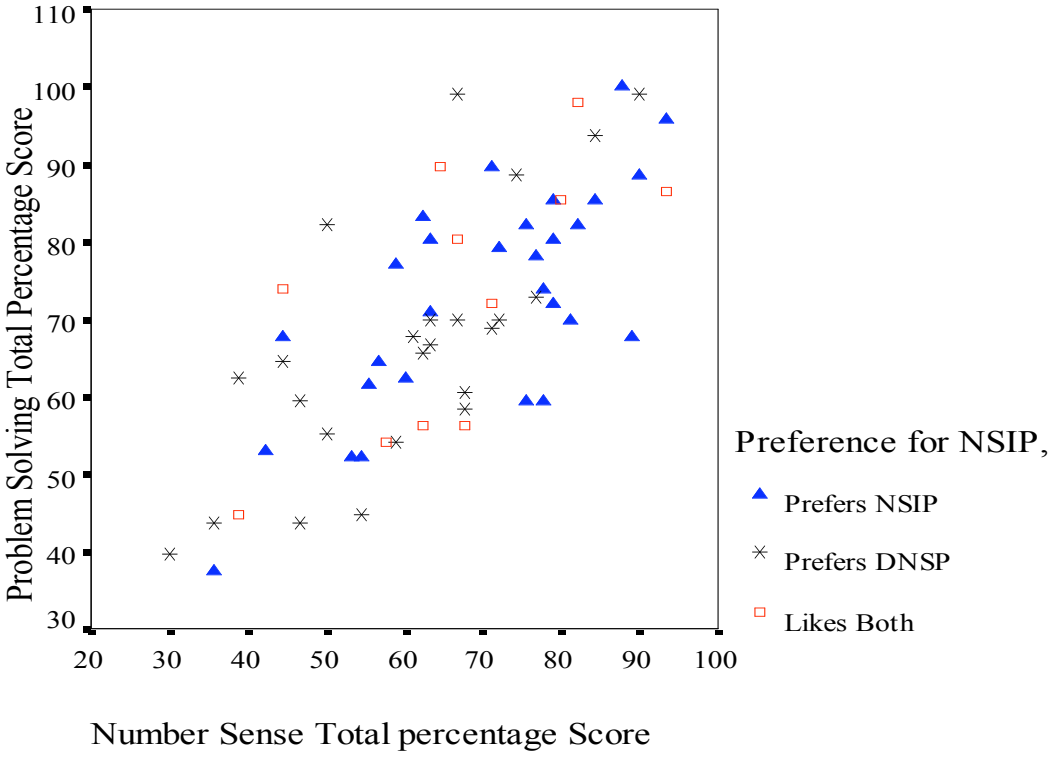

Figure 1. Relationship between number sense and problem solving, with student preferences.

Irrespective of the statistical calculations, Figure 1 clearly shows a strong correlation between the number sense and problem-solving ability of the 64 Year 7 students. Since the students preferences for solving either NSIP or DNSP or both seemed to be related to their number sense and problem solving performance, the scatter plot presented in Figure 1 also shows the distribution of the students' scores according to the type of problems they preferred to solve. Although there was no marked difference between the percentage of students preferring NSIP (45%) and those preferring DNSP (38%) it was found that student preference for solving NSIP was more closely related to number sense performance (R = 0.69). Even more striking was the higher correlation of NSIP preference and problem solving scores pertaining to NSI problems (R = 0.56) as opposed to a very low correlation between NSIP preference and performance scores for solving DNS problems (R = 0.29). These results are graphically supported in the scatter plot presented in Figure 1.

Student interviews revealed that the students considered number sense to be a significant factor in problem solving. Although few used the actual term number sense, it was clear from their descriptions that this was the inference. Table 2 lists the three major factors considered by students to affect problem-solving performance. Lack of number sense often created difficulties, as explained by one student:

> I don't think that I did not understand what I read. I understand all these words, but there are calculations to be made, but I don't know which calculation to do. I don't always understand what the numbers, what to do with the numbers.

Table 2
Main Factors Identified by Students as being Responsible for Poor Problem-solving Performance (N = 64)

Factor	Summary of students' most common answer	Count	*Percentage*
Lack of number sense	Lack of understanding of number facts and how to apply them	45	*70*
Lack of language proficiency	Not understanding the language; not being able to read properly	36	*56*
Mathematics anxiety	*Afraid to solve any mathematics problems; lack of confidence*	*31*	*48*

One issue arising from the discussion with the students was the need to work mathematically. This was supported by the three teachers and was summarised quite succinctly through Bob's statement that "without number sense students would find it hard to work mathematically"; and according to Amanda, "it is extremely difficult to work mathematically if one has poor number sense … because this will make it even more difficult to solve most problems". Yet, as Chantal pointed out, the challenge to overcome this obstacle "is a very big one, given that mathematics is not only about number sense, but also about other concepts and mathematics sense". This notion of "making sense of the mathematics" was explained by Chantal as being "more prominent in making sense of number, as it permeates all other strands of the mathematics curriculum". Such a notion was quite widespread in both practice — through the learning experiences observed — and theory, as expressed by Amanda: "since most problems require number sense, students

with such ability have a great advantage over those with poor or no number sense, when it comes to successfully solving a problem". All three teachers and the majority (70 percent) of students believed that lack of number sense is a probable major cause of poor performance in solving mathematics problems. Clearly, the link between number sense and problem solving is very significant. Bob reiterated this point when he stated that:

> Number Sense is very much like problem solving in the sense that you have to read the problem, try to understand it, plan a way to solve it and come up with a reasonably accurate answer. All these performance components must be assessed in both number sense and problem solving if I am to encourage the students to love working with numbers, and to make sense of what they have learnt.

All three teachers favoured an assessment method that took process as well as product (the solution) into account, thus linking problem solving with assessment. Hence, it was not surprising to learn that Chantal's comment that "… number sense should be, or maybe I should say must be assessed through problem solving, since it [number sense] involves mainly how students make sense of the number components of a problem" was a view also shared by the other two teachers. Bob's view, that "assessing for number sense through a problem-based method helps me not only to gauge the student's content knowledge, but also his thinking process and solution" was a prevalent one among all three teachers.

Conclusion

This study showed that there is quite a strong correlation between the number sense and problem solving proficiency of Year 7 students. The evidence points towards a relationship in which problem solving performance depends upon number sense proficiency more than the latter depending on the former. The relationship is borne out, not only in the results of the paper-and-pencil tests, but also from the views of both the teachers and students. Teaching through a problem-based approach should be a priority for every teacher of mathematics who endeavours to enhance his or her students' number sense problem solving proficiency. As pointed out by the NCTM *Standards* (2000), both number sense and problem solving are crucial to the learning of mathematics. Number sense and problem solving are linked through assessment, which incorporates consideration of both the thinking process and the final solution by a student. The specific relationship between number sense and the solving of problems, which are devoid of number sense does need some further investigation. Finally, it is clear that teachers need to ensure that problem solving is the focus of their mathematics programs, so that students are always working mathematically.

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Appendix

Number Sense Item Sample

How many different <u>fractions</u> are there between $^{2}/_{5}$ and $^{3}/_{5}$?

Circle your answer and then fill in the blanks.

A None. Why? _____

B One. What is it? _____

C A few. Give two: _____ and_____

D Lots. Give two: _____ and _____

Problem Solving Item Sample Including Number Sense

Peter, Paul and Pat divide $120 so that Peter gets three times as much as Paul, who gets half as much as Pat. How much does Peter get?

Problem Solving Item Sample Devoid of Number Sense

Alan, Brett, Carol and Dianne went to basketball, cricket, hockey and athletics. Carol didn't go to basketball; Brett couldn't go to cricket; the girl who went to hockey would like to have gone to cricket; and the person who went to basketball was upset she couldn't go to athletics. Who went where?