Using Developmental Frameworks to Support Curriculum Outcomes

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Curriculum documents in Australia are designed around outcomes and related standards. Teachers need to provide opportunities for students to learn the content that will allow them to meet the expectations defined in the curriculum. After undertaking professional learning sessions about the SOLO model, mathematics teachers in six high schools hypothesised developmental pathways for several key mathematical ideas. These theorised pathways were compared with Australian and State curriculum outcomes. The implications of using this approach for supporting teachers are discussed.

The change in curriculum emphasis to focus on the outcomes of learning rather than inputs to schooling is part of a pressure and support approach to educational reform espoused by a number of governments over the past decade (Fullan, 2000). Outcomes-based curriculum approaches demand that teachers are more intentional in their work with the assessment of outcomes being integrated into teaching. Teaching and learning become inextricably linked and assessment is embedded within the teaching process (Pellegrino, Chudowsky & Glaser, 2001; Shepard, 2000). The integration of curriculum (what is taught), teaching (how it is taught), and assessment (what has been understood by the learner) is termed curriculum alignment (Biggs, 1996).

Aligning curriculum objectives with teaching practice and assessment of outcomes is important, if schools are to achieve improved student learning. Unless, however, teachers make the necessary connections among students' responses, students' underlying conceptual understanding and the demands of the subject and the curriculum, they are unlikely to be able to use curriculum materials effectively (Manouchehri & Goodman, 1998). Concerns such as these have led to a considerable research agenda around pedagogical content knowledge (Shulman, 1987), mathematical knowledge for teaching (Hill, Sleep, Lewis & Ball, 2007) and teachers' mathematics content knowledge (e.g., Ma, 1999). Consistently, the research literature suggests that teacher practices in classrooms are what contribute most to students' outcomes (Ingvarson, Beavis, Bishop, Peck & Elsworth, 2004). Curriculum outcomes, however, typically describe key ideas needed by students to progress, rather than the smaller building blocks used by teachers to plan their programs and provide targeted intervention for their students. Teachers are hence left with little curriculum support for their day-to-day work.

From the early work of Piaget (e.g., Piaget & Inhelder, 1969) to more recent developments in the area of neuroscience (Goswami & Bryant, 2007), there is an acceptance that learning is gradual, building on prior experiences mediated through language. As Goswami and Bryant put it "Incremental experience is crucial for learning and knowledge construction" (p.20) and teachers must provide the necessary opportunities. To plan effective programs for their students, teachers need to recognise developmental pointers, and these may not be present in curriculum documents. In this situation, one solution is to consider general developmental frameworks that can be applied to students' mathematics learning.

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One such framework is provided by the SOLO model (Biggs & Collis, 1982, 1991). SOLO (Structure of the Observed Learning Outcome) is characterised by identifiable levels of response that are categorised by the complexity of the language used. These levels occur in cycles of Unistructural (U), where use is made of a single piece of information, Multistructural (M), where information is used in a stepwise process, and Relational (R) where information is synthesised into a coherent explanation or generalized to new situations. These cycles occur within modes of response: kinesthetic, iconic, concrete-symbolic and formal. Two U-M-R cycles have been identified in many situations, especially in the concrete-symbolic mode, which is the target mode for most school curricula (Pegg, 2003).

In the study reported here, teachers were introduced to the two-cycle approach to SOLO (Callingham, Pegg & Wright, 2009). As part of the professional learning sessions, teachers of mathematics chose to theorise developmental U-M-R cycles for some common topics in mathematics as a way of helping themselves to understand students' development. From this background, the research question reported in this paper is:

To what extent do theorised developmental sequences used by teachers to identify students' understanding match curriculum outcomes?

Method

Mathematics teachers in six NSW public high schools were involved in the study. All 11 teachers were very experienced, with the average number of years of teaching being above 15. They chose to develop SOLO sequences for some common mathematics topics including percent, congruence and Pythagoras' theorem as a way of understanding SOLO and also identifying students' development. They used these theorised sequences to construct assessment and teaching tasks. Generally, the initial sequence was developed within one school to address a specific need within that school's context. The SOLO sequence was then brought to the next project meeting, sometimes with some student work samples, and discussed with other project participants and the researchers. Hence the final hypothesized sequence was the result of collaboration among experienced teachers and researchers. Although not always fully validated due to time constraints in the project, teachers were happy to accept the SOLO U-M-R cycles as a guide for planning and assessment.

These theorised U-M-R cycles were then compared with two curriculum documents. The first was the NSW syllabus for Years 7 – 10 (NSW Board of Studies, 2003), which was the curriculum used by the teachers in the project. The second was the new draft Australian Curriculum (AC) (Australian Curriculum Assessment and Reporting Authority (ACARA), 2010). The comparison identified the key ideas identified by teachers and located them in relation to the stage or grade of the curriculum document. Results are reported for two theorised sequences: percent and Pythagoras' theorem. Both of these ideas are important in middle-years mathematics, and require good understanding of underpinning concepts before they can be successfully learned by students.

Results

The tracking of the hypothesised SOLO levels against the two curriculum documents for percent is shown in Table 1.

Table 1
Comparison between SOLO Levels and Curriculum Documents

Hypothesised SOLO level	NSW Syllabus	Australian Curriculum
U1 Knowing the symbol % and that it means 37 parts out of 100	NS2.4 (end Year 4) Students learn about: • recognising that the symbol % means 'percent'	No direct equivalent but implicit in M5NA3 (Year 5) Content description Solve problems involving making comparisons using equivalent fractions and decimals and everyday uses of percentages, relating them to parts of 100 and hundredths.
M1 Understanding that 50% = ½ and 25% = ¼	NS2.4 (end Year 4) Students learn about: • equating 10% to , 25% to and 50% to	M5NA3 (Year 5) Elaboration Representing decimal and percent equivalents of familiar fractions by using equipment, such as percentage strings, metre rulers, number lines, and 10 × 10 grids (e.g., counting 25 beads on a string of 100 to show 25⁄100 or 25% to demonstrate that this is one quarter of the beads, and that it can also be written as 0.25 or ¼).
R1 Finding 50% or 25% of very easy numbers (without formal procedures) e.g., 50% of $80	Outcome NS3.4 (end Year 6) Students learn about: • calculating simple percentages (10%, 20%, 25%, 50%) of quantities.	M5NA3 (Year 5) Elaboration Using equivalences with fractions to calculate 50%, 25% and 10% of quantities (e.g., recognising that 25% of 80 is the same as ¼ of 80 which is 20)
U2 Finding a % of an amount, (using % 100 amount) OR I lost 14% of my money, what % is left?	No direct equivalent	M7NA3 (Year 7) Elaboration Solving problems using fractions and percentages, such as those that require calculating fractions or percentages of quantities.
M2 Finding a discount by finding a % and deducting it. Increasing by a %. Solving a question like: Find the original price if after 25% off, you pay $84.	NS4.3 (End Grade 8) Students learn about: • Increasing and decreasing a quantity by a given percentage • Expressing profit and/or loss as a percentage of cost price or selling price	M8NA1 (Year 8) Elaboration Expressing profit and loss as a percentage of cost or selling price, comparing the difference, and investigating the methods used in retail stores to express discount; Applying the unitary method to solve real life problems involving ratios and percentages, including those where the whole is unknown (e.g., 'after a discount of 15%, an MP3 player was worth $183. What was its value before the discount?')
R2 Solving a problem like: If you increase 100kg by 10%, and then reduce by 10%, what do you have?	No direct equivalent, similar to NS4.3 (End Year 8) Students learn about: • Interpreting and calculating percentages greater than 100%	No direct equivalent
Formal: Solve a question like: A leather handbag was discounted by $x and then sold for $y. Find the % discount in terms of x and y.	No direct equivalent	No direct equivalent

The U, M, R refers to Unistructural, Multistructural and Relational and the subscript identifies whether it is located in the first or second U-M-R cycle in the concrete-symbolic mode. Statements from the two curriculum documents were selected where they best matched the SOLO description. In some instances these were part of the outcomes or key ideas (NSW), or the content description (AC); in others the activity statement "Students learn about …" (NSW) or Elaboration (AC) provided the best match. These latter two are provided in the curriculum documents to exemplify the nature of the expectations for classroom practice, rather than being definitive teaching points.

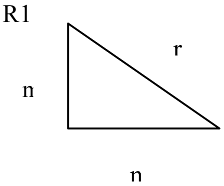
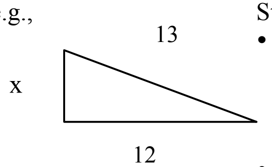
For the percent concept, the lower SOLO levels were identifiable in both curriculum documents, although the U_1 level (recognising %) was implicit rather than explicit in the Australian curriculum. The development of the percent concept was similar in both the theorised SOLO sequence and the two curriculum documents and spanned the middle years of schooling. In NSW, however, the recognition of the percent symbol (U_1) and understanding of familiar fractions as percents (M_1) was expected by the end of Year 4 whereas in the AC this occurred in Year 5. In contrast, calculation of familiar percents (R_1), such as 25% and 10%, was also expected in Year 5 in the AC but not until the end of Year 6 in NSW. There was no explicit mention of percent in Year 6 in the AC but in Year 7 students were expected to calculate simple percents beyond familiar fraction equivalents (U_2). By the end of Year 8 in both NSW and AC, the expectations were similar of fluent use of percent for complex computations such as discounts, including inverse problems (M_2). Two further SOLO levels were theorised, both more abstract in nature, neither of which had a direct equivalent in the curriculum documents. The R_2 level hypothesised involved problems that changed the basis for the percent calculation and a further Formal mode level was entirely algebraic.

The project teachers theorised the SOLO sequences in order to help them understand the development of the concept of percent. This detail was present in the curriculum documents but was often buried in the activity statements. The outcome statements (NSW) and content descriptions (AC) were too dense to be useful in identifying the small steps necessary for teachers to plan for development. In addition, some of the SOLO levels were compressed. The whole of the first cycle ($U_1 – M_1 – R_1$), for example, was placed in Year 5 in the AC, and the M_2 and R_2 levels were both expected by the end of Year 8 in NSW. The AC addressed a single grade in its statements in contrast to the NSW document, which described outcomes in two-year blocks, implying a two year period for the development of ideas. Experience from other studies suggests that students need time to consolidate lower levels of development and often making the shift from a Multistructural to a Relational level of thinking can be difficult (Pegg, 2003). The expectation for percent development in Year 5 in the AC may, on this basis, be unrealistic.

A somewhat different picture emerges when Pythagoras' theorem is considered. All of the theorised SOLO levels were expected by the end of Year 8 in NSW, and most were also located in Year 8 of the AC. Little explicit attention was given in either curriculum document to the development of essential underpinning ideas such as recognition of key parts of a right triangle (U_1 and M_1), although some of this is implied in the Year 5 content description "Make connections between different types of triangles and quadrilaterals using their features, including symmetry and explain reasoning" in the AC. The R_1 SOLO level (recognising the Pythagorean relationship) is explicit for the end of Year 8 in the NSW activity statement but not in the AC Year 8 statements, where the emphasis is on using the relationship.

Table 2

Comparison between SOLO levels and Curriculum Documents for Pythagoras' Theorem

SOLO	NSW Syllabus	Australian Curriculum
U1 Recognise hypotenuse	MS4.1 (End Year 8) Key Idea: Apply Pythagoras' theorem Students learn about: • identifying the hypotenuse as the longest side in any right-angled triangle and also as the side opposite the right angle.	Not mentioned
M1. Identifies parts of the right-angled triangle		Implicit in M5MG1 (Year 5) Content Description Make connections between different types of triangles and quadrilaterals using their features, including symmetry and explain reasoning.
R1  Recognises $m^2 + n^2 = r^2$	MS4.1 (End Year 8) Key Idea: Apply Pythagoras' theorem Students learn about: • establishing the relationship between the lengths of the sides of a right-angled triangle in practical ways, including the dissection of areas.	M8MG7 (Year 8) Content Description: Use Pythagoras' theorem to solve simple problems involving right-angled triangles. Elaboration: Using Pythagoras' theorem in right-angled triangles: $a^2 + b^2 = c^2$, where a and b represent the lengths of the shorter sides and c represents the length of the hypotenuse.
U2 Calculates hypotenuse length.	MS4.1 (End Year 8) Key Idea: Apply Pythagoras' theorem Students learn about: • using Pythagoras' theorem to find the length of sides in right-angled triangles	M8MG7 (Grade 8) Elaboration: Solving problems involving the calculation of unknown lengths in right-angled triangles
M2 Calculates short side or identifying triad.	MS4.1 (End Year 8) Key Idea: Apply Pythagoras' theorem Students learn about: • identifying a Pythagorean triad as a set of three numbers such that the sum of the squares of the first two equals the square of the third.	M8MG7 (Grade 8) Elaboration: Solving problems involving the calculation of unknown lengths in right-angled triangles
R2 Decision making and Reversibility e.g.,  Find x.	MS4.1 (End Year 8) Key Idea: Apply Pythagoras' theorem Students learn about: • solving problems involving Pythagoras' theorem, giving an exact answer as a surd (e.g.,) and approximating the answer using an approximation of the square root • using the converse of Pythagoras' theorem to establish whether a triangle has a right angle.	M8MG7 (Grade 8) Elaboration: Applying understanding of Pythagoras' theorem to determine if a triangle is right angled
Formal. Using Pythagoras, single part of a 3-D object, Bearings etc.	SGS5.3.1 Deductive geometry • proving Pythagoras' theorem and applying it in geometric contexts • applying the converse of Pythagoras' theorem	M9MG2 (Grade 9) Content Description: Solve problems involving right angled triangles using Pythagoras' theorem … and justify reasoning

Discussion

A number of issues are raised by the mapping exercise described in the last section. The SOLO pathways were theorised by a group of experienced teachers, and were not formally validated. Nevertheless, the hierarchies were accepted as reasonable representations of the development of the notions of percent and Pythagoras' theorem. The same kinds of understandings were evident in the curriculum documents as well, suggesting that the nature of the knowledge that students need to develop is widely agreed upon.

A major difference, however, was in the developmental aspect, particularly over a period of time. The concept of percent in the curriculum documents did have some sense of growth across time, with the earliest notions located in the later years of primary school, and the more complex ideas situated in the lower secondary years. Some aspects of percent, however, which were identified as having different levels of complexity in the SOLO sequence, appeared within a single grade in the curriculum documents. For example, the M_1 (Understanding that 50% = ½ and 25% = ¼) and the R_1 (Finding 50% or 25% of very easy numbers) SOLO levels both appeared in Year 5 Elaborations in the Australian Curriculum. The first of these elaborations is explicit about the use of concrete materials such as using percentage strings or 10x10 grids and is like the M_1 SOLO level, but the second is considerably more abstract in expecting the use of fraction equivalents, similar to the theorised R_1 SOLO level. This finding would suggest that the first elaboration might be better addressed earlier in Year 5 and the more abstract idea revisited later in the year but the curriculum document does not provide any indication of sequencing within a particular year level. In the NSW syllabus document, the M_1 SOLO level appeared at Stage 2, that is at the end of Year 4 and the R_1 level was identified in Stage 3, the end of Year 6. The two-year time frame suggests that the curriculum allows for growth over time, and implicitly acknowledges students' developmental needs.

The situation in the curriculum documents with respect to Pythagoras' theorem is less developmental. There appears to be little consideration in the Australian Curriculum to building an understanding of the Pythagorean relationship before using it to solve problems, although the expectation of concrete approaches to proving the relationship is explicit in the NSW document. Understanding Pythagoras' theorem, however, is largely restricted to the Year 8 level in both documents, and includes most of the hypothesised SOLO levels.

Neither SOLO nor the curriculum documents provide any clear indication of the rate of learning; that is whether it is reasonable to expect students to progress through a number of developmental levels within a short time frame. SOLO, however, does provide some support for teachers in terms of sequencing activities, which is not evident in curriculum documents. One aspect of SOLO that the project teachers particularly appreciated was that it allowed them to understand how a concept developed, so that if students were struggling with an idea the teacher could move back to an earlier notion in a structured and informed manner. For example, one teacher stated

> It's [SOLO] sort of made me understand about that really basic level, and unless they know that, and feel comfortable with it, and understand it, they can't start linking everything together.

In addition, SOLO provided the teachers involved with approaches that informed their teaching and made it more intentional. They were able to use the complexity of students' responses to identify whether they were ready for the next stage of development, and were mindful of the need to move students through the developmental levels identified, for example:

> I'm progressing through it and seeing that the kids are at a certain level, and saying to them…and thinking to myself, at school and at home, how can I get them to a higher level?

SOLO also had an impact on other aspects of teaching, in particular the nature of the questions posed to students. The teachers in the project reported that they were no longer satisfied with questions that allowed students to demonstrate only particular skills. Instead they were deliberately posing questions that required explanations or demonstrations of understanding. For the two concepts considered here, percent and Pythagoras' theorem, there is almost no indication of students explaining their thinking in the curriculum documents. For example, students are expected to represent percents using concrete materials but not to explain why that representation is suitable. Hence, students could manipulate the materials successfully with apparent expert behaviour but without deep understanding of the concept.

It should be acknowledged that the curriculum documents do not purport to advise teachers on how to teach, only on what to teach. Without some acknowledgement of students' mathematical development and corresponding approaches to teaching, however, the curriculum documents become somewhat sterile. The NSW mathematics syllabus is explicit in having a developmental focus and having flexibility for students to achieve the standards at different times and in different ways. The NSW syllabus states "Syllabus outcomes in mathematics contribute to a developmental sequence in which students are challenged to acquire new knowledge, skills and understanding" (NSW BoS, 2003, p.147). In contrast, the Australian Curriculum has an explicit focus on increasing difficulty of the mathematics rather than student-focussed developmental pathways:

> The Australian mathematics curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought processes and problem-solving skills… (ACARA, 2010, p. 1).

Despite considerable emphasis on equity considerations in the earlier Mathematics Framing Paper (National Curriculum Board, 2009), in which principles for the writing of the Australian Curriculum were established and which acknowledged "the markedly different rates at which students develop" (p.17), the focus on developmental aspects of students' learning of mathematics is limited in the draft AC. It may be that the added flexibility of having two-year stages for which outcomes are described in the NSW Syllabus provides a stronger developmental framework than the year level expectations of the Australian Curriculum. Neither document, however, appears to set out to map a specific developmental pathway that provides sequencing information for teachers' day-to-day work. In this void, SOLO can provide a theoretical perspective to support teachers' decisions about their students' learning by identifying the small steps needed for students to progress.

Conclusion

Mapping current curriculum documents against theorised learning sequences provided some insights into the structure and nature of the mathematics curriculum. Although the kinds of understandings that students demonstrate as they develop mathematical concepts are, in general, well documented in formal curriculum documents, there is little indication of the sequence of development of such understanding. Teachers, using the SOLO model (Biggs & Collis, 1982, 1991), were able to theorise levels of mathematical development that allowed them to plan appropriate learning activities that met their students' learning needs. With the focus on learning outcomes that is apparent in modern mathematics curricula, some support to teachers in the form of a developmental model, such as SOLO, would seem to be helpful for the routine work of teaching.

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