Probing Teachers' Pedagogical Content Knowledge in Statistics: "How will Tom get to school tomorrow?"

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This paper provides an analysis of interview responses from 40 teachers to a student survey question based on a pictograph. Responses comprise the first segment of 30-40 minute interviews exploring teachers' pedagogical content knowledge (PCK) in statistics at the middle school level. The responses are analysed against a back-drop of previous research on how teachers integrate their understanding to plan for appropriate learning outcomes. Four components of PCK are identified and a wide range of capability observed in the group of teachers interviewed.

The notion of pedagogical content knowledge (PCK) for teachers originated with Shulman (1987) as one of his seven types of knowledge required in the classroom. Since then it has been reviewed and modified by other researchers in mathematics and statistics education and linked variously to other of Shulman's seven knowledges (e.g., Chick 2007; Hill, Rowan, & Ball, 2005). Hill et al.'s phrase "knowledge of mathematics for teaching," for example, encompasses the commonly expected mathematical knowledge of adults as well as the specialised mathematical knowledge of the classroom needed to carry out the "work of teaching mathematics." Groth (2007) translates this into "statistical knowledge for teaching." Despite such variations in terminology, PCK retains its position as a key educational phrase across a number of disciplines, such as science (Goodnough & Nolan, 2008) and technology (American Association of Colleges for Teacher Education, 2008).

The stance on PCK taken in this study emerges from the work of Watson, Callingham, and Donne (2008a). In that study the focus was on teachers' content knowledge, their knowledge of their students as learners, and their ability to use student responses to devise teaching intervention. Based on survey items used with students, teachers responded in a written survey format in three ways. First, they suggested appropriate and inappropriate student responses to items, displaying their own content knowledge through their suggestion of appropriate answers and their knowledge of students as learners through their suggestion of students' inappropriate answers. Second, teachers suggested how they would plan to address the imagined student difficulties in the classroom, displaying further knowledge of students as learners and strategies for dealing with misunderstandings in the classroom. Third, they were presented with inappropriate or incomplete student answers to other questions and asked how they would respond to the student who gave the answer. These tasks were intended to provide opportunities for teachers to show all three aspects of PCK. Rasch analysis on the hierarchically coded responses of 42 teachers suggested a single construct along which teachers could be clustered into three groups of increasing ability related to PCK.

The items that provided student answers to questions were related to proportional reasoning in chance or data contexts and were further analysed by Watson, Callingham, and Donne (2008b). Their rubric for coding responses assessed pedagogical intervention that would reflect knowledge of content and students, and increasingly complex integration of these components in suggesting responses to the students. Although a majority of teachers could at least recognise the content issues in student answers and suggest vague teaching strategies, few could suggest meaningful questioning of students, multiple aspects of the problem, proportional reasoning strategies, and/or cognitive conflict. This was

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disappointing as the capacity of a teacher to intervene consequent to an inappropriate or incomplete student response is a central aspect of PCK. In acknowledgement of the limitations of relying on written surveys as a sole research instrument it was decided to incorporate similar problems in extended teacher interviews. Teachers would be asked to speculate on the big ideas within the tasks reflecting the statistical content, suggest appropriate and inappropriate student responses, and discuss intervention for authentic student answers. This paper reports on one of the three tasks used in the teacher interviews.

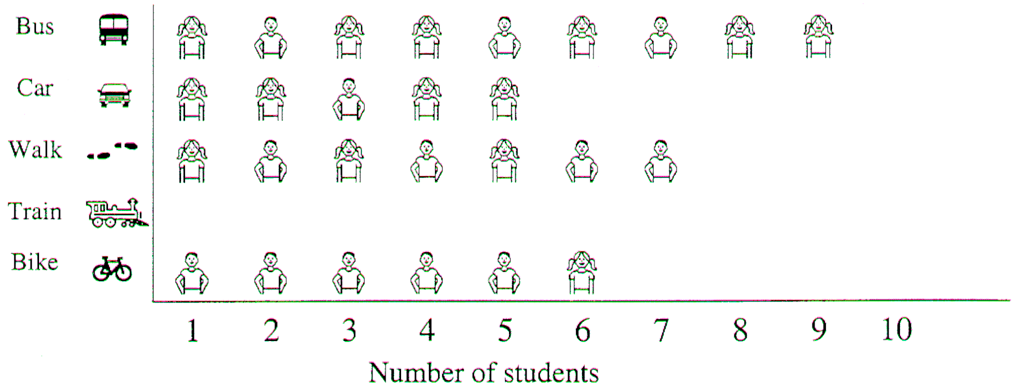
The current study moves beyond written survey responses, where perhaps time is an issue and writing a short response appears adequate. Interviews allow teachers the opportunity to stop and reflect on the questions, ask for clarification, respond to prompts suggested by the interviewer, and change their minds. The aim is to explore PCK as a dynamic process as teachers consider the purpose of a problem devised for students, how their students might answer it, and how they would intervene given specific student answers. Neat categories of teachers' knowledge are transcended as the boundaries between components become less significant and the interconnection between components is heightened. The emphasis of the investigation moves from assessing the "fixed" responses of teachers to dissecting the working elements of their responses. It encourages and allows a further definition and refinement of the working components of PCK, with the ultimate aim to determine its value in directing professional learning for teachers.

Methodology

Sample. Forty teachers from three Australian states were interviewed for this study, with 14 from each of two states and 12 from a third. They were involved in a professional learning project in statistics for the middle school years. Teachers taught in grades 5 to 12 across the three states, had teaching experience ranging from 2 to more than 25 years, and had a wide-range of previous tertiary study in mathematics and statistics. The task used in this study is not judged to require statistical knowledge dependent on tertiary study.

Protocol. The task chosen for this analysis of PCK is basic to the foundations of the chance and data curriculum. It requires the beginning of inferential thinking to consider the information and variation present in a pictograph and then make a prediction of expectation couched in uncertainty but reflecting observed frequencies in terms of probability. The student survey problem upon which the interview protocol is based arose from earlier student interviews (Watson & Moritz, 1999). A moveable bar chart on "how children get to school" was used to probe students' basic understanding of graphs, frequency, and likelihood in predicting future outcomes. The survey item was devised later for large scale testing in a pictograph format (see Figure 1), where boys and girls could be distinguished, and six questions were asked (Watson & Kelly, 2003). The six questions ranged from graph reading to interpretation and prediction, the last question about "Tom" being the basis for the teacher interview protocol used in the current study. Watson and Kelly reported that the Tom question was very difficult for students in grades 3 to 9 with only 2% of grade 9 students able to make a prediction including uncertainty. The majority of students at all grade levels responded without specifically explaining an interpretation of the pictograph, often with idiosyncratic explanations or saying "I don't know." Figure 1 presents the student survey item, the questions and prompts used during the teacher interview, and the three student answers presented to teachers. The pictograph and Tom question and the three student answers (bold font in Figure 1) were printed in large font on four sheets that were then laminated. The cards were placed in front of the teacher during the interview for ease of interaction between the teacher and interviewer.

How children came to school one day



Tom is not at school today. How do you think he will come to school tomorrow? Explain your answer.

Q2T. What are the big statistical ideas in this problem?
(Probe: What answer would you give?)

Q3T. Please can you give an example of an appropriate response and an inappropriate response that your students might give.
(Probe: Can you explain why it is appropriate/inappropriate?)

Q4T. What opportunities would this problem provide for you teaching?
(Probe: Where would you place it in your lesson sequence? Or in your school's curriculum sequence?)

Bike, because the majority of boys ride to school.

Q5T. A student gave this answer. How would you move this student's understanding forward?
(Probe: What would be the next step in learning?)

Tom will come to school by train because there is no-one next to the train so it must be him.

Q6T. [Same as Q5T]

Bus, because there is a pattern and the next one is a boy.

Q7T. [Same as Q5T]

Figure 1. Task used in teacher interviews.

Procedure. All interviews took place on school premises during or after school. The task presented here represents about 25% of the interview content in an interview taking between 25 and 40 minutes. There were four interviewers across the three states with two in each state. The first two authors each conducted interviews in two states, while the other two interviewers conducted interviews in one state. The number of interviews was distributed as evenly as was practical: A, 12; B, 7; C, 11; and D, 10. All interviewers undertook joint interviews before conducting their own individual interviews. Although a limitation of the study was having four interviewers, each had knowledge of the overall project and had had previous contact with the teachers, so was not a stranger. Observing teachers in the classroom presenting the Tom problem might have offered greater opportunity to capture the working elements of PCK than the interview material, but this was not feasible. The interview study, however, moved further than the previous survey study in providing a rich description of how teachers would plan to approach the Tom problem in their classrooms.

Analysis. Aware of the previous analysis of teachers' written survey responses and of the desire to mine the richness of the interview responses for aspects of PCK as teachers revealed them, a holistic approach was taken initially with the transcripts. Beginning with suggestions from Shulman (1987) and others and the previous research related to the current study that influenced the creation of the overall task, all transcripts were read and a clustering technique (Miles & Huberman, 1994, p. 248) employed to discern an appropriate set of components for analysis. This process led to four initial headings, and all interviews were then read by a different author, with detailed categorisation of comments by teachers to the questions across the protocol. These were then discussed by all authors and refinements made to the four components and their distinguishing characteristics. Throughout this process, teachers were assessed in relation to a rough continuum reflecting the degree to which the authors believed low to high qualities of emerging PCK were displayed. This was again a holistic process, with links amongst components being the significant feature rather than graded individual evaluations of the four components. Minor discrepancies of assessment did not alter agreement on the teachers' placement on three levels of PCK as revealed in their interviews. Because the results of this study reflect both the authors' refinement of components characterising PCK and the subsequent classification of teachers' responses based on the four components identified, both are presented in the Results section.

Results

Refinement of PCK. The framework that emerges from the responses of 40 teachers to the protocol presented in Figure 1 comprises four non-hierarchical components which, although independent of each other, become integrally linked as the teacher's response progresses. The optimal features that characterise each component are presented in Table 1. The first two components – Recognises Big Ideas and Anticipates Student Answers – reflect the previously recognised bond between content knowledge and knowledge of students as learners. They also are usually, although certainly not always, documented through teachers' responses to the early questions in the protocol. The last two components – Employs Content-specific Strategies and Constructs Shift to General – include the ingredients of pedagogical practice as foreshadowed by the teachers to progress student understanding. These components are exemplified mainly, but again not exclusively, through teachers' responses to the three student answers to the Tom question. Employs Content-specific Strategies refers to appreciating the nature of the student answer, beginning at that point, and suggesting appropriate strategies with respect to the answer that demonstrate opportunity to move the student forward. Constructs Shift to General indicates an appreciation of the many statistical ideas that are related to the initial task and the ability to explore and expand these with the class based on the opportunities provided by student responses, either anticipated by the teacher or introduced through the protocol.

Several extracts from the interviews are used to illustrate the characteristics of the framework's components listed in Table 1. Extracts are biased primarily to Components 3 and 4 that encompass elements of PCK not clearly distinguished in prior literature. In a secondary manner, this selection also demonstrates the lack of hierarchy in the framework. Although the quotations illustrate a high representation of a characteristic representing a particular component, in themselves they do not necessarily equate with overall high PCK. Space does not permit all characteristics to be exemplified, and some quotations display multiple characteristics. Identifiers are assigned to quotes for reference later in the Results.

Table 1
Framework for a Refinement of PCK

Component	Description
1. Recognises Big Ideas	Articulates sequence of related concepts Infers meaning through specific response to the Tom problem
2. Anticipates Student Answers	Canvasses wide range Distinguishes clearly between appropriate and inappropriate answers Demonstrates understanding of student reasoning
3. Employs Content-specific Strategies	Encourages questions that clarify and explain student answers Offers alternatives by introducing parallel data sets or changing scenario Constructs a sequence of questions sourced from a personal understanding Formalises a discussion related to a specific interpretation
4. Constructs Shift to General	Reveals difference between the pictograph as a statistical model and a vehicle representing real data Links to related statistical ideas Explores concept of majority Exposes limitations of data collection Experiments with alternative data representations Introduces an awareness of language

Recognising the Big Ideas in the student survey task is succinctly stated by a teacher who said it is about "trying to make predictions from previous experiences … what they think is a likely outcome … to see if they can interpret different representations of data without having something they've just become accustomed to" (T1). Few teachers specifically articulate the Big Ideas encompassed by the Tom problem, but most imply an understanding that reveals itself over the length of the interview. For the second component, if a teacher readily Anticipates Student Answers and also demonstrates an understanding of student reasoning, as is partially illustrated in the next quotation, there are greater opportunities for connecting with other components and maximising the learning potential of the task.

> …a few in my class would say come by bike because he's male and there's males that come by bike, which shows they're getting a little bit of the understanding of the trends of it. If they went further and said … but there's also a chance he came by bus because there's a lot more came by bus than anything else. … Probably a very common one that students would say…would be they come by train cause nobody else has … somebody would have to eventually … Another one might be, they would say … any type of transport… (T1)

With regard to the third component, two descriptors are illustrated. "Offers alternatives by introducing parallel data sets or changing scenario" is seen in the following extracts.

> We could talk about examples in our class of … the different countries that students could come from … just because the options there, doesn't mean you have to use it… (T2)

> …if I put down lists of the fruit that everyone's got in their lunchbox, and I added pineapple to, a whole pineapple to the list, and your name isn't there, you must be the person with the whole pineapple … ok, let's look at the [train] question again. (T3)

> …And she said but one of my fish has died, so can I change the number of fish that I have got. … And I said, no because that was how many fish you had when we surveyed and so a survey is actually a static thing, it is a snapshot, it is a moment in time. (T4)

The next two quotes illustrate the descriptor "Formalises a discussion related to specific interpretation."

So perhaps get them to think about other modes of transport and how we could put them there and there would be nothing beside them as well. (T5)

And from that discussion would come … if we don't know where he lives, can we make an assumption about how he comes to school? (T6)

As a newly defined component, all six of the characteristics of Constructs Shift to General are exemplified. The first is "Reveals a difference between the pictograph as a statistical model and as a vehicle to represent real data."

…to be able to find that actually looking where data *isn't, is*, tells you a lot. … So it's about the connectivity between the maths and really what its purpose is for. (T7)

And sometimes if they were to just statistically look at it, and look at the probabilities or how often these people are coming, they'd give a mathematical answer but then they like throwing in a curly one and they'd say, oh he'd be the only one on the train cos he lives next to the train station." (T8)

In the following quotation the teacher makes a "link to a related statistical idea," in this case the uncertainty of prediction.

You could ask, would you be certain that he, how certain are you that Tom will ride his bike tomorrow. Can you be that certain? Maybe justify your answer… (T9)

"Exploration of the concept of majority" is illustrated in the following.

What they *mean* is the majority of people riding a bike are boys! … I would talk about the fact that one doesn't necessarily mean the other … So all cats have 4 legs but not all 4-legged things are cats … (T4).

Well of the bike riders the majority are boys but, 14 of the 14 boys, 8 are not bike riders, so in fact the majority of boys do not ride a bike to school, so we could sort of look at that for a start to say, you know, that's actually not an accurate statement. If he'd switched that around to say, of the bike riders, the majority are boys, that would be fine. So probably just looking again at the importance of language, and being very specific and of, yeah, really looking again at the data that you have. (T2)

The following extract illustrates "Exposes limitations of data collection."

…I would try and make it more real for them so maybe ask them a question about … or just try and bring it back … to the reality of what the data is showing rather than just try to look at the graph… (T10)

And they're all coming up with these elaborate answers, where did you get that from and so learning that there's only so much information in isolation that the graph can show. (T3)

…we would have to look at the fact that sometimes when we put out a question we are expecting responses that we don't always get. (T4)

"Introduces an awareness of language" is shown in the following.

…but you have said, Tom will come to school by train because there is no-one next to the train … sounds definitive … so I would probably look at the way that's been written and try to say … instead of saying "will" how could you change that so it opens possibilities … so I'd probably look at the way it's been written and try to change that idea… (T11)

"Experiments with alternative data representations" is a feature displayed by a number of teachers in response to the student answer "Bus, because there is a pattern and the next one is a boy."

…what if it was presented with all the girls first and then all the boys for each bus, car, walk, train, bike, like this one is the bike. What if the data was presented with all the girls together and all the boys together? And then I would ask the question again. (T12)

Teachers' Level of PCK. With respect to the protocol presented in Figure 1 and the framework in Table 1, 9 teachers' responses were judged to display a High degree of PCK in their interviews; 14 were classified as Medium; and 17 were classified as Low. Generally the illustrative extracts presented above were taken from the interviews of teachers in the High group, with three from the Medium group (T2, T4, T12) and two from the Low Group (T5, T6). Teachers in the Medium group made some comments illustrating the ideas in Table 1 but were inconsistent across the four components. One teacher in this group, for example, explained the big ideas in the task and although presenting convincing "appropriate responses" was not as competent with suggesting "inappropriate responses." Strategies for handling the student responses were mainly focused on asking specific questions related to the task and mentioning the need for students to experience data collection. Teachers' responses in the Low group often did not see the uncertainty involved in the task as a big idea. Some made quick judgements on the appropriateness or otherwise of student responses and tended to suggest "explaining" answers rather than further questioning students or suggesting alternatives. Few appeared able to extend the task from the specific response about Tom.

Discussion

In probing teachers' PCK this study confirms its significance as a concept central to educational analysis and albeit tentatively, builds on prior understanding to refine its meaning. An understanding of PCK is gleaned through 40 interview segments designed to capture how teachers move students toward an appreciation of the beginnings of inference, as represented by the Tom task. What is attempted in this study is a containment and clarification of the nebulous components of PCK. Four components become apparent. The importance of content knowledge and knowledge of students as learners is confirmed and corresponds with "Recognises Big Idea" and "Anticipates Student Answers." In focusing on the teachers' response to the students' range of answers two further components emerge. "Employs Content-specific Strategies" and "Constructs Shift to General" track the various means by which teachers tap the full potential of student answers to the statistical problem. Of these components it is the fourth one that adds a fresh dimension to the construction of PCK. Identifying the ability to embed the students' learning experience in an environment that shifts to more general understanding extends the analysis of Watson et al. (2008b).

Although the defining components of PCK in this study are not intended to be hierarchical, it must be acknowledged that some basic content knowledge will precede development of the other three components, and it would appear that parallel development should be planned from the start. In this study content knowledge is assumed in the wider concept of PCK rather than segregated as a separate entity. Its value as a springboard for teachers to enter the other components is essential to the whole. These components are now the benchmarks for the final round of the professional learning project. Further analysis of the entire protocol employed with these 40 teachers will inform more completely the modified framework.

With the four components of PCK more clearly distinguished it is important to explain the authors' decision to adopt a holistic appraisal rather than to calculate a grading derived from the sum of a teacher's performance for individual components. Indeed, it is through this explanation that a different appreciation of PCK is revealed. Apart from the difficulty of detecting sharp boundaries to the components in the unfolding "action" of the interview, it becomes apparent that the integrated and robust adoption of all four constitutes High PCK. As stated earlier, teachers in the Medium group make some comments illustrating the ideas in Table 1 but are inconsistent across the four components. Although the distinctiveness of each component facilitates debate about PCK by extending our analytical language, it is the dynamic interplay of all four components that more adequately expresses the complex practice whereby a teacher engages with students as critical learners. Analysing the interview material further will contribute to an understanding of PCK as a creative process and it is hoped, will broaden the opportunities for professional learning.

Initial suggestions for professional learning may be usefully based on tasks and processes outlined in this study. The progression based on considering a task for which student responses are available and display a range of understanding, would appear to be realistic and meaningful to teachers. In a collaborative learning session, teachers would have the opportunity to discuss the big ideas behind the task, share expected responses from their students, develop strategies to address specific content issues and articulate opportunities to shift student understanding to a more general context. Although the authors have incorporated some of these elements in their previous work with teachers, the components described in this study will provide the explicit framework for future interactive professional learning sessions.

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