Softly, Softly: Curriculum Change in Applications and Modelling in the Senior Secondary Curriculum in Queensland

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Applications and mathematical modelling have been a distinctive part of the senior secondary curriculum in Queensland for two decades. Findings from an on-going longitudinal study of this initiative are reported. Twenty-three teachers and curriculum figures from across the state were interviewed and artefacts collected from teachers. A deep understanding of mathematical modelling has alluded some teachers sometimes leading to their not seeing the purpose of gentle nudges from monitoring panels to move forward. A softly, softly approach to implementation has led to some progress on all fronts with approaches described as ranging from minimalist to very rich.

Reviewing the existing situation regarding applications and mathematical modelling in education worldwide, Niss, Blum, and Galbraith (2007) identified several issues presenting challenges, and articulated specific questions within each that require ongoing research and development. All but one of these issues have been addressed explicitly in Australasian work as indicated in Stillman, Brown, and Galbraith (2008), where according to this source, specific research efforts are pursuing answers to questions such as the following:

1. How are different versions of the modelling process being applied within particular contexts and levels of education, and what are their achievements and ongoing challenges?
2. What pressures do teachers indicate are significant in deciding when and how they will incorporate applications and modelling tasks in their teaching? What purposes do they have in using such tasks?
3. To what extent have teaching and assessment approaches and teachers been influenced through the inclusion of applications and modelling material in courses?

Questions such as the above do not lend themselves to definitive answers achieved from single shot designs. Rather they will be informed through the accumulation of evidence and data from a variety of programs and initiatives. The present paper sets out to provide such a contribution by an examination of one substantial longitudinal initiative – mathematical modelling in Queensland senior secondary mathematics courses.

Mathematical Modelling and Applications in Queensland

We consider the implementation of applications and mathematical modelling within senior mathematics curricula in Queensland, where the initiative was first introduced in 1990-1 in a limited number of trial/pilot schools. A particular emphasis in the paper is on the development of the initiative from the perspective of implementing teachers, and key curriculum figures responsible for its introduction and ongoing character. For this purpose structured interviews were conducted with twenty-three individuals (9 at the end of 2005 and 14 at the end of 2007) as part of an on-going study of *Curriculum Change in Secondary Mathematics* (CCiSM).

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Syllabus Objectives

We begin by considering syllabus statements relating to the field. The objectives of the 1989 Trial/Pilot Syllabuses (e.g., Queensland Board of Senior Secondary School Studies [QBSSSS], 1989a, 1989b) included the identification of the assumptions and variables of a mathematical model, formulation of a model, derivation of results from a model and interpretation of these in terms of the given situation. Mathematics C, the most challenging Mathematics subject, also required the modification and validation of the model. While syllabus refinements have occurred in the interim (e.g., QBSSS, 1992, 2000; Queensland Studies Authority [QSA], 2009), the essence of the mathematical modelling component has remained essentially stable for over a decade although different emphases in wording and placement of modelling and application aspects in organising categories for general objectives has brought to the fore different aspects in implementation over this time frame.

We take as a representative focus the current *Mathematics B senior syllabus 2008* (QSA, 2009) for a subject taken by all students intending to undertake serious mathematical study beyond secondary school, noting that similar general objectives apply across all three mathematics subjects. *Modelling and Problem Solving* features specifically as a category of objectives, and another category, *Communication and Justification,* includes aspects that also are considered essential to the proper implementation of applications and mathematical modelling. Representative syllabus objectives include the following:

- apply problem-solving strategies and procedures to identify problems to be solved, and interpret, clarify and analyse problems
- identify assumptions (and associated effects), parameters and/or variables during problem solving
- represent situations by using data to synthesise mathematical models and generate data from mathematical models
- analyse and interpret results in the context of problems to investigate the validity (including strengths and limitations) of mathematical arguments and models
- develop and use coherent, concise and logical supporting arguments, … when appropriate, to justify procedures, decisions and results. (QSA, 2009, pp. 4-5)

At this point it is appropriate to locate this approach to mathematical modelling within a variety of versions that use the descriptor in curricular implementations. At least five different emphases have been identified (Stillman, Brown, & Galbraith, 2008), of which four are represented in the Australasian context. An approach that uses *contextualised examples to motivate the study of mathematics*, for example, as explored by Pierce and Stacey (2006), is employed by some teachers to engender positive feelings towards mathematics. Often goals tend to be affective rather than cognitive, with actual mathematics learning a subservient goal. Modelling as *curve fitting* (Galbraith, 2007) reduces the concept of model development to the exercise of regression options on graphical calculators, and CAS alternatives. This effectively distorts the modelling process by over emphasising one (albeit important) possibility within the solution process. At other times mathematical modelling is used as a *vehicle* for teaching mathematical content. Here the emphasis is on using practical situations as settings within which to situate required curricular material (Zbiek & Conner, 2006).

By contrast treating modelling as *content* (e.g., Ikeda, Stephens, & Matsuzaki, 2007; White, 2005) is motivated by the desire for students to develop and apply skills appropriate to obtaining a mathematically productive outcome for a problem with genuine real-world connections. A program driven by this view of mathematical modelling focuses on enabling students to use their mathematical knowledge to solve a range of real world related problems, while simultaneously scaffolding their development as modellers over time. It is this version that the generic objectives above encompass.

Implementation in Schools

The Queensland system of school based assessment means that the production of assessment tasks and the award of levels of achievement is in the hands firstly of individual schools, with panels at district and state levels performing critical reviewing roles to assure comparability of outcomes across schools and regions. In keeping with the school-based nature of the Queensland context, individual schools and teachers design individual work programs (including assessment tasks) under the syllabus umbrella. Hence a key element is the translation of the general objectives into specific criteria for the teaching, learning, and assessment of school based activity. An illustrative example follows of how an individual school has translated aims and purposes into action statements for classroom implementation.

As part of the 'data' for this paper we present the assessment criteria as developed by a school in the Wide Bay region for a task (summarised in Figure 1) based around an agricultural context using information from the Department of Primary Industries (DPI).

> Heinrich Schultz, a farmer in the South Burnett, has traditionally grown peanuts and sorghum during summer, and wheat in winter. He is investigating whether it is possible to increase his profits. His farm has an area of 500 hectares (of Red Scrub soil) and he has $300 000 available to cover the variable costs involved in all aspects of crop production for 1 summer and 1 winter season. (Profits in any season are already committed to repaying debts so cannot be used as capital for the next season). Prepare a report, using the information from the DPI*, which uses linear programming techniques to advise Heinrich of his best options.
> * Since obtaining this information there has been a 30% increase in seed and fertilizer costs for maize, and a 30% decrease in harvesting costs for navy beans.

Figure 1. Crop Task.

The task was elaborated further to vary the number and type of crops that Heinrich might consider. To evaluate the student modelling efforts, criteria (including the following) were designed as a basis for awarding credit:

- explanation of crops chosen and why,
- discussion of risks,
- extent to which DPI recommendations were met,
- mathematical solutions obtained,
- explanation of crop alternatives,
- comparison of advantages and disadvantages of different scenarios,
- evaluation of conclusions, and recommendations.

This illustration is but one example of how a school has turned the general objectives of the syllabus into precise criteria for application to the assessment of student work. For this to be successful a viable pathway between syllabus statements and school implementations must exist, and be capable of validation by moderating panels. Of interest is the extent to which schools and teachers in general find this essential process to be workable and valid in terms of the expectations of the syllabus, given the other administrative and curricular pressures that impact on teaching. This key aspect is one indicator for evaluating the introduction and continuing implementation of an initiative such as the mandating of modelling in syllabus requirements. To pursue this goal more broadly an on-going longitudinal study is being conducted into this curriculum initiative.

Research Methods

Syllabus and review documents from the late 1980's up to the latest syllabuses to be implemented in 2009 were examined. In addition purposeful samples of 5 key curriculum figures [QKCG] (e.g., non-teacher members of expert advisory committees, curriculum officers of the state education department, or statutory board or authority officers overseeing syllabus implementation), 6 secondary mathematics teachers in key implementation roles [QKTG] (e.g., state or district review panel chairs or state review panel members), and 12 secondary mathematics classroom teachers [QCTG] were selected. These teachers were representative of several school districts and of the state, Catholic and independent schools system. A series of interview questions covering the period of introduction, and later periods of widespread implementation and modification were asked of these 23 interviewees. In addition, practising teachers were asked to provide artefacts, usually in the form of tasks, that typified their use of real world applications and modelling in teaching and assessment, and their use of technology in these contexts. In order to identify emergent themes within the interview responses, and the teaching and assessment artefacts, these data were entered into an NVivo 8 database (QSR, 2008) and analysed through intensive scrutiny of the data to develop and refine categories related to these themes. Specifically this paper will address emergent themes related to responses to the following questions:

1. While the general objectives of the 1992 syllabuses were for mathematical modelling, it could be argued the implemented syllabuses were mainly concerned with mathematical applications. Does that distinction have meaning for you?
2. Do you believe that mathematical modelling is an established part of upper secondary school practice in Queensland? Why or why not?

Findings

Implications of Level of Understanding of Distinction between Applications and Modelling

A review of the trial/pilot experiences (Peckman, 1992) highlighted that teachers did not share a common understanding of the difference between mathematical applications and problem solving. It was thus decided not to use the term "problem solving" but instead use "mathematical applications" in life-related situations or purely mathematical contexts in the 1992 syllabuses. According to Gray, Brown and Abbey (1993), the objectives of this category encouraged "development of problem solving and mathematical modelling skills" (p. 12). The latter focus was largely ignored in implementation (Stillman, 1998), however, there were teachers who saw the need to change as evidenced by this response from a head of mathematics speaking about his staff's uptake of modelling at the end of the third year of his school's involvement in the implementation:

> Well, it is taking a while because we are moving away from the direct presentation, chalk and talk approach, but as I said, most people, what we are now doing is setting a problem and then saying what mathematics do we need to solve the problem; so, the modelling is coming slowly but there is still a fair way to go. (Teacher interview, 1996, Stillman, 2002)

Most interviewees reflecting back on the 1992 implementation agreed that it was implemented as applications more than modelling with the general objectives largely being ignored except by the enthusiastic few. The main reason for this was seen as being that mathematical applications were much closer to what teachers were already doing whilst mathematical modelling was not well understood and central to teachers' pedagogy.

> QKTG2: The other thing was too hard and people in schools didn't understand it. The applications thing was you could get away with tinkering … your old questions, and writing a few words around them and you'd survive and didn't get into too much trouble with panels. Unfortunately that's where a lot of people stayed. The full blown modelling thing was too hard for most people and that's why it stayed at that level of application. (November, 2005, CCiSM interview)

Other reasons given for modelling being less likely to be taken up were that it was time consuming and thus usually assigned to alternative assessment and not every one considered alternative assessment to be valid and authentic.

> QKCG5: I think the applications were easier. I think modelling was time consuming. So if modelling occurred I guess it would have been given as alternative assessment. And of course there were people who didn't believe that alternative assessment was authentic. … There were hang-ups about that I guess. (December, 2007, CCiSM interview)

There was, however, a group of enthusiasts who pursued modelling and often these teachers were also technology adopters and evangelists and this resulted in an alternative route for modelling to grow in prominence through the period of the 1992 syllabuses.

> QKCG5: In some ways though, the modelling came through in a de facto way with graphics calculators. So often these in-services were being conducted and the material being used was modelling material … giving teachers some idea which way to go, how they could use the technology and develop modelling. (December, 2007, CCiSM interview)

The 2001 syllabuses, as with the 2008 syllabuses, used the terminology "modeling and problem solving" explicitly. Also, the mathematics syllabuses had to use three equally weighted criteria to paralell other subjects. *Justification,* which had been within *Mathematical Applications* in the old syllabus*,* was joined with *Communication*. *Communication and Justification* now had to be taken seriously in terms of the general objectives, as it ought to have been already if a genuine modelling approach was being implemented. Gradually more pressure was brought to bear by monitoring panels to make sure all criteria were being fully addressed.

> QKCG4: I think some people in the old syllabus [1992] had pretty much ignored *Justification* from the comments people had made when they now had to do it as a separate criteria [sic]. And I'm thinking, 'To me, well this isn't really a huge difference. I was expecting it before but now I tick it off over here'; whereas for some people it was a big thing and they really hadn't been expecting the students to do that. And I think then gradually over the years the Panel started focusing on different objectives and through that some change has happened. (December, 2007, CCiSM interview)

Some teachers embraced the new changes after an initial period of caution as expressed by the following teacher:

> QKTG5: I was very nervous at first but I really like the approach and I think it is really important for the students. I think it has enormous benefits to their understanding if we have the time to do it well. From my point of view I think it gives us, with the technology that we have got on hand, a chance to do mathematics that has more meaning for the students so you can use real life data… . It has given me great insight into how students actually think. (December, 2007, CCiSM interview)

However, not all change is necessarily progress and sometimes these efforts by panels to assure better teaching of the general objectives as a whole have had unexpected outcomes. Ensuring students address particular elements such as "extending and generalising from solutions" in the standard for, say, an A in the *Modelling and Problem Solving* criterion has resulted in some mechanistic approaches as the following demonstrates.

> QCTG6: Our kids will ask us, 'Have I got enough G's? Have I generalised enough?' because we actually count how many times. I don't suppose a lot of schools do that. And we think. 'Oh, we might have to put a G on the next test' or we might have to put a 'refine the model', something which gives them the opportunity to demonstrate what we call A attributes in that criteria [sic]. So I just feel sometimes that they are jumping through hoops. (November, 2007, CCiSM interview)

In some cases this could be construed as a response by schools to a perceived desire by the panels for schools to show explicit evidence of developing particular modelling or problem solving competencies in tasks targeting these specifically (see Figure 2) rather than holistically through open or structured modelling tasks allowing the demonstration of a variety of competencies associated with modelling. However, it could also be due to a lack of differentiation between applications and modelling persisting to this day.

The following information was obtained from the Australian Bureau of Statistics regarding the population growth of the greater Brisbane area:

Year	Population	Year	Population	Year	Population	Year	Population
1856	5345	1881	48136	1911	146991	1947	402030
1861	8240	1891	104276	1921	220371	1954	502320
1871	26382	1901	122210	1933	299748	1961	656612

Population demographers at the time decided that the trends in the population growth could be best described by the quadratic function $y = 60x^2 - 764x + 20496$. Using the same set of data, develop cubic, quartic, exponential and logistic models to represent this population growth. Discuss the strengths and limitations of each of these models with reference to the accuracy of depicting the given data set. Reference should be made to specific ranges of data points for each model which gives a good representation.

Figure 2. Task for exploring strengths and limitations of models and refining a model (Source: QCTG8).

QCTG5, for example, was on the teaching staff of the school where the head of department in 1996 indicated "modelling is coming slowly". She did not, however, see a distinction in 2007 between mathematical applications and modelling despite this long involvement in the implementation. Not surprisingly, she did not recognise the essential elements in the assessment criteria that distinguished modelling from applications. She described the panel's focus on such elements of the criteria as "nit picking".

> QCTG5: What we see now is nit picking particularly in *Modelling and Problem Solving* and in the *Communication and Justification*. It says here 'the student must consistently demonstrate recognition of the effects of assumptions used, evaluation of the validity of arguments', right? And if you don't have students showing that three quarters of the time they cannot get an A ... *Modelling and Problem Solving*, "exploring the strengths and limitations of the model", once again, why are we doing this? I really don't think that all of these little tiny things, all of which have to be shown and demonstrated, have helped it at all. (November, 2007, CCiSM interview)

Embedding of Applications and Modelling in Current Practice

Responses about whether modelling was an established practice in Queensland upper secondary schools were almost equally divided between those who agreed it definitely was and those who saw pockets of modelling well established in some schools but not others resulting in a continuum from minimalist approaches to very rich. In the main, classroom teachers agreed modelling was established. Teachers in key implementation roles were more in a position to give a broader view, whereas the classroom teachers were often influenced by what was happening in their local environment and extrapolated from there. In contrast, two curriculum figures most remote from classroom practice thought there was little modelling occurring. Some of this variation is captured below:

> QKTG2: There are pockets of it being done properly but I couldn't say it was done well across the whole state. I could say the questions I saw when I was on State panel more broadly … have improved over time. But I think we are a little way off having some genuine modelling stuff going on for us to be confident that it's really taken hold. Without the constant gentle pressure from our monitoring processes I could see a lot of people just dropping back. I think that there would always be some applications in there. I think that is here to stay. But modelling I think we are a little bit further off. (November, 2005, CCiSM interview)

> QCTG7: So, is mathematical modelling established? Well if you compared it with 20 years ago, very definitely yes, but there is still room for improvement. (October, 2007, CCiSM interview)

> QKCG4: It is still patchy. There are some schools that don't try all that hard. But there are other schools that are doing fantastic things. So I think everybody has moved a certain amount, some people have moved a large amount, others are still being pushed. (December, 2007, CCiSM interview)

The most positive interviewees saw the establishment of modelling in classroom practice as a "slow evolutionary process" (QKTG4) strongly supported by the panel system and the insistence on alternative assessment. Unlike implementations in other states in which the pace and degree of change expected immediately was more dramatic (Stillman, 2007), a deliberate tolerance for diversity in uptake of modelling has meant that there was some progression in all schools. One interviewee summed this up by saying that modelling could be an established part of teaching practice in any school "if the person teaching mathematics in that school wants it to be" (QKTG6).

Discussion and Conclusion

The tasks in Figures 1 and 2 illustrate the variety exhibited in task development. The former is much more open, with the making of assumptions, choice of mathematics, and interpretation in context essential aspects of this modelling problem. The latter essentially asks students to carry out specified mathematical calculations, and there is no assessment of whether the proposed models make sense in terms of the real problem context. It is an example of an application in which essential activities, central to modelling, are absent.

With respect to the responses of participants, we note that opportunities provided by modelling were grasped by some who welcomed the legitimacy of a syllabus requirement in supporting a desire for change. On the other hand there was a desire by others to remain within the comfort provided by interpretation of 'applications' as little different from previous activity as was found by Stillman (1998) in the early implementation of the 1992 syllabus. A number of teachers welcomed a symbiotic relationship between modelling and technology use, with these two different curricular elements mutually supportive.

The elaboration of modelling assessment criteria in a later syllabus revision were generally viewed as helpful in providing enhanced guidelines for the production of tasks, and the assessment of performance. On the other hand the reduction of rich criteria to box ticking procedures, signaled a desire by some to continue with minimalist approaches, attempting to assimilate challenging new requirements into traditional conservative practices. In this respect the impact of review panels emerged as ambivalent facilitators of change. In other words, in viewing review panels as agents for change and guardians of comparability, the appearance of implementations along the continuum from minimalist to very rich, suggests that these functions require further work. The assessment by interviewees of current practice reflected this variety. In this sense the acceptance of more conservative implementations over time than some would wish, was seen as helping to assure that at least some progress was made, perhaps more than in an environment requiring full immediate compliance where 'failure' is a real possibility. On the other hand some respondents enthused that schools were thereby enabled to do "fantastic things".

To conclude, we reflect back on the three general questions listed in our opening section. For the first, we have obtained a cross sectional view of representative implementations at senior secondary level in a state context. Our respondents have indicated pressures that apply to teachers in meeting syllabus requirements, and the challenges they feel these impose on what can be achieved. Finally, useful insights have been obtained as to ways in which changing assessment requirements have been used (or not) to facilitate the teaching of modelling at senior secondary. Many challenges remain.

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