# Task Types and Mathematics Learning

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This symposium presents results from one aspect of a project investigating the use of particular types of tasks in mathematics classes. The first paper provides an overview of the project, and the following three papers elaborate aspects of the three main task types used as the basis of the project.

Paper 1: Helen O'Shea and Irit Peled, Monash University. *The Task Types and Mathematics Learning Research Project.*

Paper 2: Barbara Clarke, Monash University. *Using tasks involving models, tools and representations: Insights from a middle years mathematics project.*

Paper 3: Doug Clarke and Anne Roache, Australian Catholic University. *Opportunities and challenges for teachers and students provided by tasks built around "real" contexts.*

Paper 4: Peter Sullivan, Monash University. *Constraints and opportunities when using content-specific open-ended tasks.*

# The Task Types and Mathematics Learning Research Project

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This paper provides a theoretical background and rationale for a project that involves an investigation of the power of different types of tasks. This background paper focuses on the role of tasks in creating a learning environment and takes into account the effect of the teacher in the process of task implementation. The Task Types and Mathematics Learning (TTML) research project investigates how four types of mathematics tasks contribute to mathematics learning in the middle years of schooling. Through professional development and data collected through observation of teachers and students, interviews, focus groups, and surveys, the project aims to describe the features of successful exemplars of each type, constraints that might be experienced by teachers, and teacher actions that can best support students' learning.

This session introduces a project entitled *Examining the relationship between the documented curriculum, classroom tasks, and the learning of mathematics*. We call it Task Types and Mathematics Learning, or TTML, and our research is investigating the nature and effect of using different types of mathematics tasks.

## The Research Rationale

Our research is based on an assumption that choice of tasks, and the associated pedagogies, are key aspects of teaching and learning mathematics (see e.g., Brousseau, 1997; Christiansen & Walther, 1986). We argue that what students learn is largely defined by the tasks they are given. For example, we assume that tasks designed to prompt higher-order thinking are more likely to produce such thinking than tasks designed to offer skills practice (see e.g., Doyle, 1998; Hiebert & Wearne, 1997). We agree with Ames (1992) and Gee (2004) that tasks are more likely to be effective when students have meaningful reason for engaging in the activity, when there is enough but not too much challenge, and that variety is important.

Tasks are a central part of classroom activity, an outcome of the teacher's choice among different routes to achieve instructional goals. This choice of tasks reflects teacher beliefs about instructional goals, while the way the tasks are formulated or modified reflects teacher beliefs about learning and teaching (Kaiser, 2006).

As described in a processing model suggested by Stein, Grover, and Henningson (1996), the implementation of the task is affected both by the teacher and by the students. According to their differing beliefs and attitudes, teachers might implement the same task in different ways. They might organise the class in different settings, and might lower the level of challenge, especially in the case of high-level thinking. The students affect the task by giving it interpretations that might be different from what the teacher intended and which depend on many factors including classroom norms, and the children's existing knowledge and schemes.

An important characteristic of task implementation involves the balance between the student's own work and teacher intervention. Some of this balance depends on the teacher, her beliefs and her teaching approach, and some depends on task characteristics. Boaler (2003) shows that teachers with similar declarations about their attitude towards reform might give students similar tasks, and yet what actually happens in their classes is divergent. One teacher might be quick to provide students with hints about solving a given problem, yet another teacher might allow more independent work while directing children to search for support within their own mathematical knowledge, and a third teacher might simply let the children wonder and give them no direction at all. Ainley and Pratt (2005) demonstrate differing needs for teacher support in relation to a variety of task types.

Recent research calls on mathematics educators to give greater attention to promoting children's creativity by encouraging multiple solutions and a variety of strategies. This would allow more focus to be given to open-ended tasks or tasks that encourage making connections between topics, or the use of real-life contexts. For researchers such as Walls (2005) open-ended tasks might enable a more democratic classroom environment instead of the existing one where the teacher has an agenda and a sequence of tasks that lead to it in a very controlled route. Thus, from several different perspectives, open-ended tasks are fostered, expected to allow for student independent work, and viewed as having a good potential to develop flexibility and reasoning skills.

The focus of this project is on the role of task *characteristics* rather than teacher choices and task formulation. We focus on the opportunities task characteristics create for student mathematical thinking, and on the diversity of performances exhibited. Although we focus on the tasks, and have given teachers a set of already formulated tasks, it is still inevitable that teacher beliefs and knowledge influence task implementation, and therefore we take this into account in the analysis of task effects.

The TTML project focuses on the following four types of mathematical tasks:

Type 1: Teacher uses a model, example, or explanation that elaborates or exemplifies the mathematics.

Type 2: Teacher situates mathematics within a contextualised practical problem to engage the students but the motive is explicitly mathematics.

Type 3: Students investigate specific mathematical content through open-ended tasks.

Type 4: Interdisciplinary investigations.

## Research Design

Our goals are to describe in detail how the tasks respectively contribute to mathematics learning, the features of successful exemplars of each type, constraints that might be experienced by teachers, and teacher actions that can best support students' learning.

The model underlying the design and data collection in the project involves five sets of variables: teachers' knowledge; beliefs, attitudes and self-goals; situational and other constraints; teachers' intentions; and teacher actions and student learning. The first three of these influence one other, and collectively they influence the fourth. A fifth set of variables, teacher actions and student outcomes, completes the model for the collection of data.

We are working with middle years' teachers (Years 5 to 8) from volunteer clusters of schools in the inner and outer suburbs of Melbourne and its semi-rural surroundings. These three regions represent a spread of socio-economic student backgrounds and include both government and Catholic sectors.

The TTML project has actively supported teachers in their teaching of the task types. Meetings with teachers have provided professional development in using the task types while enabling us to collect data on teachers' experience of teaching them. Each of the principal researchers has taken responsibility for one task type, moving from one school cluster to another after one or two school terms. In this way, each cluster of teachers has received professional development in each of the task types, including the creation or sourcing of tasks matching the teachers' curriculum.

Teacher development has focused on the nature of each of task types one to three, the associated pedagogies, ways of addressing key constraints, and student assessment. (in type four tasks, teacher use a combination of task types in an extended interdisciplinary project.) For each task type we set the teachers a goal of using at least one task of the relevant type in one lesson per week, with the goal that teachers would eventually generate their own tasks. Regular cluster meetings and conferences combining the three clusters allowed teachers the opportunity to share experiences of teaching the tasks.

Our intention has been to create optimal conditions for the successful implementation of each task type by ensuring that teachers have access to high-quality task exemplars and by supporting teachers on associated pedagogies. We also intend that the process of task creation and use is self-sustaining. One way we hope to achieve this is through the development of a website on which we have posted around 40 reports of lessons developed according to task type, trialled by participating teachers, and incorporating their experiences of teaching them. In addition, we are preparing to upload plans for whole units of work, incorporating lesson plans and comments. The website is currently accessible to the approximately 50 teachers involved in our project, but we plan to make this resource more widely available on completion of our research.

## Data Collection

The TTML project combines the interpretive analysis of teaching and teacher development with quantitative data collection and analysis (see Knobel, 1999). Data have been collected in several phases, reflecting teachers' training in the task types and their teaching of exemplary tasks, with each phase following a cycle of researcher input through professional development, trialling by teachers including their responses and researchers' observations, review, further trialling, and so on. During these phases, which covered the first half (18 months) of the project's duration, we also refined the instruments used for the observation and interpretive analysis of teaching and student learning, including an observation schedule and rubric for interpreting teacher actions.

In the following phase of more intensive data collection, we worked with teachers in several schools to plan a whole unit of mathematics lessons comprising exemplars of the four task types. We then observed each of the lessons, interviewed teachers before and after lessons and observed students' work. Additional data were collected from assessment tasks following the completion of the unit of work.

The teachers were observed using a structured schedule to gather data on their classroom actions. The focus of the observations was the type of tasks used, the pedagogies associated with the task (especially questioning), the match between teachers' intentions and their actions, and any apparent constraints. Teachers were also interviewed before and after observed lessons to review their intentions and actions, including perceived constraints. Teachers not being observed reported on their task use indicating their intentions, outcomes, constraints and advice to other teachers.

Both when teachers worked alone and when researchers observed their teaching of tasks, data on the nature of the mathematics that students produced was gathered in the form of selected work samples, photographs, interviews with students questioning their mathematical thinking, and tests generated by teachers and/or researchers.

## Students' Experience of Learning Mathematics

An innovation in the project's design is the inclusion of data from students. This includes a survey of about 1000 middle year students at schools participating in TTML project on their experiences of learning mathematics. In addition, we have collected data from students participating in the units of work. These include a survey of the tasks they enjoyed most and those they felt they learned most mathematics from, as well as an open-ended question about their ideal mathematics class.

## Research Findings

Meetings with teachers and particularly our intensive data collection throughout units of work at two schools have yielded rich data about student preferences and performance in lessons developed using the task types, as well as the opportunities and constraints experienced by teachers.

In the papers that follow, our colleagues elaborate on the rationale for each of the task types on which our research focuses, the definitions they have developed for each of these types and research findings relating to teachers' experience of teaching them.

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# Using Tasks Involving Models, Tools and Representations: Insights from a Middle Years Mathematics Project

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As part of the Task Types for Mathematics Learning Project (TTML), teachers developed and used a range of tasks which focused on tools, models or representations. Data were collected on the ways in which middle years teachers described these tasks, their preferences among particular tasks, the opportunities they saw them as providing, and the constraints they observed during their use. There was general agreement that these tasks form an important part of a balanced mathematics curriculum.

## A Rationale for Explicit Tasks

There have been a number of researchers who have attempted to classify mathematical tasks. As part of the Quantitative Understanding: Amplifying Student Achievement and Reasoning Project (QUASAR), Stein, Smith, Henningsen and Silver (2000) classified tasks into four categories, which can be summarised as: Memorization involving reproducing previously learned rules or facts; Procedures without connections, Procedures with connections; and Doing mathematics. The task type which which is the basis of this paper comes under the latter two categories.

The effective use of models, tools and representations is a key component of effective mathematics teaching (Clarke & Clarke, 2003). Appropriate models and representations, in the hands of capable teachers, support children's conceptual development and can build skills. What tasks do teachers use to introduce and implement these tools, models and representations? The Task Types for Mathematics Learning (TTML) project included such *explicit* tasks (referred to as *Type 1* by project participants). A key focus in this research is to examine the opportunities and constraints experienced by teachers when using the various task types. It would appear that there are powerful constraints operating which discourage their use, because it appears that in many Australian classes the tasks used are generally routine and unlikely to lead to successful learning (see Hollingsworth, Lokan, & McCrae, 2003). Such *explicit* tasks are associated with good traditional mathematics teaching (see Watson & Mason, 1998), but their use is not always evident in the regular classroom. The term explicit is used here to emphasise that the mathematics was made explicit in the use of the task, not to imply that the teacher was "telling" the student what to do, without any student decision making.

## The Explicit Tasks that are the Focus of our Project

In describing these tasks, we are not referring to exercises but to explicitly focused experiences that engage children in developing and consolidating mathematical understanding. An example is a teacher who uses a fraction wall to provide a linear model of fractions, and poses tasks that require students to compare fractions, to determine equivalences, and to solve fractional equations. The fraction wall simplifies the mathematical complexity of the concept by providing a tangible and clear model of fractions that can be otherwise abstract.

In the initial TTML professional development, teachers were provided with the following definition of these types of tasks:

> The teacher commences with an important mathematical idea, and proposes tasks which involve

> models or representations or tools, which help students to understand the mathematics. There is no attempt to link mathematics to its practical applications. For example, the use of a fraction wall in a chance game can assist in developing an understanding of equivalence, improper fractions, and simple operations with fractions.
>
> Following student work on the task, the teacher leads a discussion on the mathematics which has emerged from the task, and seeks to draw out commonalities and generalisations.

After developing, adapting and using explicit tasks, the teachers filled out a survey with a series of open prompts (*n* = 31) focusing on the specific task type. There were three groups of teachers with varying experience with other types of tasks in the context of the project when the survey was completed. However, there was little difference between these groups in the patterns of responses, except that they seemed to provide more extensive responses and included more comparison references when they had experienced the use of other tasks types. This is not surprising, as they were then in a position to note appropriate contrasts. The results from this survey are presented in the following sections.

## *The Teachers' Definitions of the Tasks*

The first survey prompt was "*If you were explaining to a group of teachers about how to use tasks of this type, how would you describe this type of task?*"

The importance of the model and the explicit focus on the mathematics were the most common components of teachers' responses. The linking of the model or tool explicitly and directly to the mathematical concept was highlighted. Sample responses included:

> Using Models/tools representations to explicitly focus on a particular mathematical idea or concept. Often takes the form of teachers introducing a mathematics idea and students play a game or complete an activity. Follow up discussion on understanding/learning with students.
>
> The use of a model or representation to assist student understanding of a particular mathematical concept to be used as a reference for further student work on the concept.

These teachers had been provided with some exemplars of tasks as well as worked in school- and cluster-based teams to develop and trial tasks. There responses were consistent with the intentions of the researchers.

## *Some Examples of the Tasks that Teachers Valued*

In the survey, teachers were asked *"of the tasks of this type that you have tried in your class this year, which worked best?"* They were then asked to list two more of the best tasks.

Not only did no particular task emerge as the most popular, but the most striking feature of the responses was the diversity of tasks that were valued, with 17 different tasks identified as best across the 31 teachers. The Chocolate Fraction task (Clarke, 2006) where the sharing of chocolate represents both an engaging context and a model for the development of the concept of fractions as division was identified by a number of teachers in their best three.

The reasons that the teachers gave for selecting the Chocolate Fraction task included:

> Gave students something they could see. They were interested in the chocolate so it remains in most of their minds.
>
> …was so effective in engaging the children and representing fractions as division.

The teachers were teaching in the middle years (Years 5 – 8, with student ages from around 10 to 14), with the vast majority in upper primary. An important curriculum focus in these years is fractions and decimals, and the majority of the best explicit tasks focused on children’s development in these areas. This would seem to be due in part to curriculum importance but also to the nature of the content and the availability of effective models and tools. Of the 81 nominations by the teachers as their top 3, 51 were focused directly on learning in Number. This was despite the fact that in later professional development an attempt was made to present and encourage teachers to try explicit tasks in other content areas. Teachers tended to trial and identify tasks that they had experienced during project meetings or developed as a team within the school.

There was limited justification for the teachers’ preferences but the key themes appeared to be the engagement of the children followed by the importance of the model/tool/ representation in enabling mathematics learning.

The following quote was from a teacher responding to why a specific task was successful:

> Concept that hasn’t been introduced was made explicit through the use of this model. Students could see clearly what maths happens when you divide/multiply by a number larger/smaller than one.

### *The Advantages of Explicit Tasks as Seen by the Teachers*

To gain insights into the opportunities and advantages of the specific task type, the prompt was “*What do you see as the advantages of using this task type in your teaching*?”

The most common feature in the teachers’ responses was the value of these tasks for developing student understanding. There were also many who commented on the engagement of students both in the sense of participation but also in the way the model (sometimes referred to as “visual” or “hands-on”) allows engagement with the mathematics.

Increasingly during the phase of the project where the different task types were trialled, the discussion of the teachers involved the role of different types of tasks and the value of a range in their planning including for the range of students. The following quotes about the advantages of explicit tasks were from teachers who had trialled open-ended (see Sullivan in this volume), context-based (see Clarke and Roche in this volume) and explicit tasks:

> Yes as a starter to teach new maths that then can progress to Task type 3 [open-ended].

> In particular areas of maths eg- using operations-using this task allows students to learn the maths skills required before moving into applying it in a variety of contexts.

> It can support the concrete understanding of a maths concept for students for whom more abstract mathematical understanding may not develop as readily.

### *The Constraints on the Use of Explicit Tasks as Seen by the Teachers*

The following prompt was included to provide insights into the constraints that teachers identified: “*What makes teaching using this task type difficult? What are the challenges in using this type of task?*”

The difficulties that appear to be related directly to the tasks types include the difficulty in identification of explicit tasks within particular content area (e.g., chance and data) and the time required to prepare the materials.

The following are representative of the range of responses and issues:

> Sometimes finding the task. For me sometimes deciding which task is actually a type 1 task. Finding the resource and preparing it for use with a grade can be difficult ie time needed to copy, laminate, cut, etc.

> Ensuring each student has sufficient background knowledge and skills. At times I found it difficult to make the task relevant to the maths program.
>
> Understanding the model/representation is most effective when there is a purpose, ie our opportunity to apply their understanding of the model in a meaningful context. Also, particular maths concepts are easier to find models for.
>
> Sometimes modifying for lower students.

## Conclusion

In the trialling phase of this project, there was a number of issues that arose in the teaching of explicit tasks. The teacher quotes and summary comments above provide some insights into some those issues. They can be summarised as follows:

- While these tasks are not contextualised, there is sometimes a “hook” that helps to engage the students. The chocolate fraction task is an example of this.
- Some content areas, particularly Number, seem to provide more opportunity for successful explicit tasks.
- Extensive exposition is not necessarily required. The provision of the model or representation can enable the students to generate the mathematical ideas and justification.
- The model, representation or tool needs to be linked closely to the mathematical concept begin developed.
- The mathematical focus is pivotal and it seems that teachers might be less willing to deviate from the intent than with contextualized tasks.

The teachers in this project were able to articulate the purpose, opportunities and constraints of explicit tasks. Such tasks are an important component of curriculum, but teaching them well is not simple. However, as one of the teachers pointed out, *it is important to be reminded why* we need to include them.

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# Opportunities and Challenges for Teachers and Students Provided by Tasks Built Around "Real" Contexts

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Following professional learning sessions which focused on developing and using tasks in middle years classrooms which began with "real" contexts, teachers trialled such tasks in their classes, and then completed a survey, the results of which are reported here. Teachers were able to articulate the features of such tasks, see potential benefits, and articulate opportunities and constraints in their use. Secondary teachers saw greater constraints in using such tasks than did primary teachers.

## Introduction

There is a strong consensus in the research literature that the nature of student learning is determined by the type of task and the way it is used (Kilpatrick, Swafford, & Findell, 2001). "Instructional tasks and classroom discourse moderate the relationship between teaching and learning" (Hiebert & Wearne, 1997, p. 420).

When teachers pose higher order tasks, students give longer responses and demonstrate higher levels of performance on mathematical assessments (Hiebert & Wearne, 1997). The greatest gains on performance assessments, including questions that required high levels of mathematical thinking and reasoning, are related to the use of instructional tasks that engage students in "doing mathematics or using procedures with connection to meaning" (Stein & Lane, 1996, p. 50).

The provision of meaningful and challenging mathematical tasks remains an issue in middle years' mathematics in Australia. For example, the Executive Summary of *Beyond the Middle* (Luke et al., 2003), a report commissioned by the Australian Commonwealth Department of Education, Science and Training, and involving a literature review, a curriculum/policy mapping exercise, and system, school and classroom visits, claimed:

> There needs to be a more systematic emphasis on intellectual demand and student engagement in mainstream pedagogy. … This will require a much stronger emphasis on quality and diversity of pedagogy, on the spread of mainstreaming of approaches to teaching and learning that stress higher order thinking and critical literacy, greater depth of knowledge and understanding and increases in overall intellectual demand and expectations of middle years students. (p. 5)

## What are Type 2 Tasks?

When using Type 2 tasks, teachers situate mathematics within a contextualised practical problem where the motive is explicitly mathematics. This task type has a particular mathematical focus as the starting point and the context exemplifies this. The context serves the twin purposes of showing how mathematics is used to make sense of the world and motivating students to solve the task. It is intended that the context provide a motivation for what follows and dictates the mathematical decisions that the students make in finding a solution. Although the contexts are in some cases contrived, it is important to distinguish Type 2 tasks from *word problems* (e.g., Fennema, Franke, Carpenter & Carey, 1993), which are only contextualised in a very basic way.

Hodge, Visnovska, Zhao, and Cobb (2007) studied the use of a range of contextualised tasks with seventh-grade students in the United States, with a focus on the extent to which these tasks supported students' mathematical engagement and their developing mathematical competence. Most tasks involved comparing two data sets in order to make a decision or judgement (e.g., deciding whether the installation of airbags in cars impacts on car safety, exploring the impact of a treatment program for AIDS patients). During the design experiments, the authors found that issues, which were of a personal or societal relevance, were the most effective in engaging students. They attributed this to "adolescents' growing interest in their place in society and their sense of power in affecting [sic] change on society and their immediate community" (p. 398).

Peter-Koop (2004) summarised many of the difficulties which students face when solving context-based problems, including comprehension of the text, and the identification of the mathematical core of the problem. We need to be careful about the use of problems which have little in common with those faced in life, Maier (1991) describing them as school problems coated with a thin veneer of "real world" associations.

Boaler (1993) was also critical of these kinds of problems, particularly those extracted from the adult world (e.g., wage slips and household bills) with an assumption that students could identify with these. She also criticised the misconception held by some that "mathematics in an 'everyday' context is easier than its abstract equivalent" (p. 13). Boaler also noted that "one difficulty in creating perceptions of reality occurs when students are required to engage partly as though a task were real while simultaneously ignoring factors that would be pertinent in the 'real life version' of the task" (p. 14).

## Where Was This Photo Taken? An Example of a Type 2 Task

A number of teachers in the TTML project used what we have come to call the Signpost Task. In using the task, the teacher asked students whether, during family travels, they had ever seen a sign at lookouts or at other tourist places which showed how far and in which direction a number of key places were from their current location.

*Figure 1.* The sign presented to students in the Signpost Task

The teacher then explained that today's lesson would involve the students working, in pairs, on trying to find out the location of the signpost. A number of teachers reported that several students needed what Sullivan, Mousley, and Zevenbergen (2004) termed *enabling prompts*—appropriate variations on the task or suggestions to students, which might help those who are having trouble making a start. One helpful enabling prompt was to suggest to students that they pick a city named on the sign and find out how far on the map it would be from the sign's location and therefore which "mystery city" might contain this signpost.

A number of writers (e.g., Brown & Walter, 1993) have stressed the importance of problem posing by students. Several teachers extended the work on the task, by encouraging students in groups to create their own signposts with cities of their own choice, and then to pose their problems to another group. Incidentally, the photograph above was taken inside Auckland International Airport in New Zealand.

Teachers were encouraged to use the tasks provided by the project team as models for developing their own tasks. The two below were rated most highly by teachers:

*Maps for the commander* (Downton, Knight, Clarke, & Lewis, 2006). Here, students are presented with two views drawn by spies of a city surrounded by a circular wall—one drawing from the West, one from the South. The students are challenged to draw the view, which the third (missing) spy would have drawn from the North-east.

*Land proportions.* Students are presented with a copy of a real email sent to the authors by a person seeking some help. The letter read as follows: "If, on paper, a block of land is 2 cm x 5.8 cm, and the overall dimensions are 4768 square metres, how do I work out the actual length and width of the block?"

## Project Teachers' Descriptions of Type 2 Tasks

Teachers were asked to describe Type 2 tasks as they would if they were explaining them to another teacher. The prompt was "If you were explaining to a group of teachers about to use tasks of this type, how would you describe this type of task?" Their explanations included the following:

- A mathematical problem embedded in a real situation.
- Questions which allow/require investigation through use of materials data gathering, testing and calculation. The tasks are based in authenticity.
- The mathematical problem is contextualised, but with an explicit maths focus.
- Contextualised maths investigations with explicit mathematical focus.
- Application tasks involving situate mathematics within a contextualised practical problem where the focus is explicitly mathematics.

### *Teachers' Views on Advantages and Difficulties in Using Type 2 Tasks*

After at least one school term of trialling a range of Type 2 tasks, teachers were asked to list "advantages of using this task type in your teaching." Typical comments were:

- More hands on.
- Some were good for the student who struggles with mathematics.
- The mathematical skills and strategies are made purposeful and meaningful by being situated in a "real world" context.
- Increases the students' ability to think.
- Allows the students to draw on a variety of understandings and topics – engaging and relevant to what they are doing.
- Engages advanced students. Combines knowledge and skills, e.g., a task may need measurement, calculation, logic.
- Each task can be taken in various directions by the students. There are different ways to solve the puzzle and are very engaging.

Teachers were also asked, "What makes teaching this task type difficult." In the comments below, "support students" refers to those students in the classes in which students of "lower ability" were grouped. Typical responses were the following:

- Some of the tasks were too challenging for support students and too long!
- The different learning needs and abilities of the students; at times some students arrived at their conclusions more quickly then others.
- Students who are less confident have very little idea of where to start if left to their own devices rather than assisted. These tasks can compound their negative feelings about themselves and maths.
- Not all the real situations are relevant to middle years students and may not fit neatly into the existing curriculum.
- You need to do some preparation with the students. Students are more interested in the answer than the process.

It is worth noting that teachers in secondary schools found using the Type 2 tasks more challenging to use generally than did those in primary schools.

Boaler (1993) provides an insight into the potential transfer of mathematical understanding when she notes that "it also seems likely that an activity which engages a student and enables her to attain some personal meaning *will* enhance transfer to the extent that it allows deeper understanding of the mathematics involved" (p. 15). She notes that "school mathematics remains school mathematics for students when they are not encouraged to analyse mathematical situations and understand which aspects are central" (p. 17).

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# Constraints and Opportunities When Using Content-specific Open-ended tasks

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After teacher learning sessions on open-ended tasks, teachers trialed such tasks in their classes, and then completed a survey, the results of which are reported here. It seems that the teachers collectively could adequately define open-ended tasks, could give illustrative examples, and could articulate both opportunities and constraints. This knowledge allows teachers to plan to take advantage of opportunities and to minimise the constraints.

## A Rationale for Open-ended Tasks

An assumption underlying each of the three tasks types in the Task Types and Mathematics Learning (TTML) project is that the nature of teaching and what students learn is defined largely by the tasks that form the basis of their actions. In this case, we argue that working on open-ended tasks (type 3 in our project) can support mathematics learning by fostering operations such as investigating, creating, problematising, communicating, generalising, and coming to understand procedures.

There is substantial support for this assumption. Examples of researchers who have argued that tasks or problems that have many possible solutions contribute to mathematics learning include those working on problem fields (e.g., Pehkonen, 1997), and the open approach (e.g., Nohda & Emori, 1997). It has been suggested that opening up tasks can encourage pupils to investigate, make decisions, generalise, seek patterns and connections, communicate, discuss, and identify alternatives (Sullivan, 1999).

Specific studies that support use of open-ended tasks include Stein and Lane (1996) who noted that student performance gains were greater when “tasks were both set up and implemented to encourage use of multiple solution strategies, multiple representation and explanations” (p. 50). Likewise, Boaler (2002) compared the outcomes from working on open-ended tasks in two schools. In one school, the teachers based their teaching on open-ended tasks and in the other traditional text-based approaches were used. After working on an “open, project based mathematics curriculum” (p. 246) in mixed ability groups, the relationship between social class and achievement was much weaker after three years, whereas the correlation between social class and achievement was still high in the school where teachers used traditional approaches. Further, the students in the school adopting open-ended approaches “attained significantly higher grades on a range of assessments, including the national examination” (p. 246).

Two aspects of our project are of interest here. First, we wanted to know what teachers took from our professional learning sessions and how they interpreted our input. Second, we were interested in what they learned from classroom trials of exemplars of the tasks.

## The Content-specific Open-ended Tasks that are the Focus of our Project

In addition to the openness described above, type 3 tasks are also *content-specific* in that they address the type of mathematical topics that form the basis of textbooks and the conventional mathematics curriculum. Teachers can include these as part of their teaching without jeopardising students' performance on subsequent internal or external mathematics assessments. The definition that we used with our project teachers was:

> Content specific open-ended tasks have multiple possible answers, they prompt insights into specific mathematics through students discussing the range of possible answers, An example is:
>
> > A group of 7 people went fishing. The mean number of fish caught was 7, the median was 6 and the mode was 5. How many fish might each of the people have caught?
>
> Such tasks allow unambiguous focus on specific aspects of mathematics while still allowing opportunities for creativity and active decision making by students with the advantage that one task can be applicable to wide levels of understanding.

The project is/has been exploring the nature of the learning based on such open-ended tasks, the opportunities that such tasks offer to students, and the constraints that the tasks create for teachers. After the teachers had worked with the respective task types, they completed a survey which asked them questions on these issues. Their unstructured responses were inspected, categorised, and summarised, and are reported in the following.

### *The Teachers' Definitions of the Tasks*

We were interested to determine how the teachers interpreted the experiences provided by their participation in the project. On a survey, completed after working with type 3 tasks, the teachers were asked:

> If you were explaining to a group of teachers about to use tasks of this type, how would you describe this type of task?

Nearly all of the teacher responses referred to the possibility of multiple answers using terms such as "multiple answers-multiple methods", "there are a numbers of strategies for finding an answer", "not only one answer" and "explore a variety of outcomes".

Many responses also referred to the ways the tasks can be suitable for students of differing readiness, such as "allow students to work at their own level", "use strategies at their own level of understanding, and "access to a range of ability levels".

Various teachers also commented on the emphasis that might be placed on student responses such as "(a need to) focus on sharing strategies", "making generalisations and seeing patterns", and "translating insights into mathematical expressions".

In other words, many teachers were able to restate to us the purposes and operation of the tasks in the language and form that we had suggested.

### *Some Examples of the Tasks that Teachers Valued*

In the survey, the teachers were asked "of the tasks of this type that you have tried in your class this year, which worked best". They were also invited to describe the "next two best tasks". Not only did no particular tasks emerge as more popular, but the most striking feature of the responses was the diversity of tasks that were valued. Examples of tasks that were mentioned by more than one teacher were:

> A closed rectangular box is tied up with 1 metre of ribbon. If the bow takes 30 cm of ribbon, what might be the dimensions of the box?
>
> Using the map on *google earth*, plan a walk around the school that is 4 km long.
>
> What might be the missing numbers? $\_\ \_ \times 8\ \_ = \_\ \_\ 0$

These three tasks are appropriate exemplars of this type in that there is a variety of possible responses to each, the range of responses can be interrogated by students and teachers, the students have to make choices in finding one or more solutions, and the problems are not solved by the application of a procedure.

There were also examples such as the following suggested:

How much water is wasted by the school drinking taps over a year?

This has some characteristics of open-ended tasks in that the students have to make active decisions on what is important and how to collect data, and there would be sense of personal ownership. The task also has many characteristics of type 2 tasks (see D. Clarke, this volume) in that it addresses a practical context. The task is also similar to interdisciplinary tasks, which is our fourth type.

The teachers' responses indicate that their suggestions of open-ended tasks are compatible with the material they had been presented with in teacher learning sessions.

### *The Advantages of Open-ended Tasks as Seen by the Teachers*

In our teacher learning sessions we have emphasised the following potential advantages of using open-ended tasks: there is considerable choice in relation to strategies and solution types; generalised responses and patterns can be found; there are opportunities for class discussion about the range of approaches used; and the range of solutions found can lead to an appreciation of their variety and relative efficiencies. Teachers were asked:

What do you see as the advantages of using this task type in your teaching?"

The most common responses related to the choices that students make about their approach to tasks, such as "how various students go about solving maths problems", "every student has a chance to solve it in their own way".

Many responses related to the nature of the students' thinking such as "encourages students to broaden their thinking", "creativity", "opens up possibilities",  "students think more deeply", and on a slightly different note "encourages students to persist".

Teachers also commented on the ways the tasks can be accessed by all students such as "all achieve some success", "can cater for range of abilities", and "work at their own level". Having used such tasks in the classrooms, these responses suggest not only compatibility with the perspectives that we presented to them, but also further interpretations that were derived from practice, with emphasis on the idiosyncratic ways that students respond, and teachers' intention to support students individually.

### *The Constraints on the Use of Open-ended Tasks as Seen by the Teachers*

In the teacher learning sessions we discussed the potential constraints posed by such tasks, especially the resistance that some students have to taking the risks that such tasks present (see Desforges & Cockburn, 1987). In the survey, the teachers were asked

What makes teaching using this task type difficult? What are challenges in using this type of task?

The most common response related to the issue we had addressed, that is that some students prefer more closed tasks. Teachers comments included "some students are not risk takers", "challenge for the students who want to go straight to an answer", "requires thinking", and "the hard thinking and little direction can be confronting for some kids".

Other aspects of students' response that may be connected to their unfamiliarity of such tasks were "students who don't want to put in any effort", "some find difficulty finding an entry point", "their need for confidence" and "some students don't know where to start".

Some teachers clearly saw such tasks as more difficult noting that some students might experience difficulties such as “limited mathematical knowledge”, and “not all students have the right level of learning”.

There were pedagogical aspects mentioned such as “not always sure what maths will come out of it”, “correcting the different solutions”, “holding back on explanations”, and “being ready for what arises”.

There were also planning considerations mentioned such as “finding the tasks” and “needs additional resources”.

These responses clearly arise from reflection by teachers on the use of such tasks in their own classrooms. It is possible that the constraints might act as a deterrent to the use of such tasks. A significant aspect of our project is to explore the obstacles these constraints represent and to develop ways of working with our teachers to overcome them.

## Conclusion

After participating in teacher learning sessions on this task type, on a survey teachers gave adequate definitions and useful examples, could identify the advantages of the tasks, and articulated some constraints associated with their use. While it is possible that their responses were merely reproducing what had been said to them, their comments did seem to be derived from their practice. The hypothetical definitions and recommendations about implementation aligned with their experience, and it seems that teachers are both ready to take advantage of opportunities, and aware of the potential constraints they may experience.

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