Teaching Area and Perimeter: Mathematics-Pedagogical-Content Knowledge-in-Action

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This paper examines the influence of teacher's mathematics pedagogical content knowledge (MPCK) in the teaching of area and perimeter to Grade 4 pupils. Lessons of a beginning teacher were studied to determine the activities and teaching strategies used to bring out ideas associated with area and perimeter. Observable MPCK-in-action outcomes are also being studied through video-taping the beginning teacher. In addition, this paper also studies how concept of area and perimeter is developed in class. The complex interplay between area and perimeter concepts was sometimes handled well by the beginning teacher, whereas on other occasions, gaps in mathematics pedagogical content knowledge had the potential to cause misconceptions for pupils.

We know who the good mathematics teachers are and we can recognise good mathematics teaching when we see it; yet, it is not very easy to describe what comprises good teaching. This is often because mathematics teachers often have to apply knowledge from various domains of teacher knowledge to their classroom instructions. Among these domains are knowledge of pedagogy, knowledge of learners, subject matter knowledge, and pedagogical content knowledge (Shulman, 1986). Pedagogical content knowledge (PCK) is that unique knowledge domain of teaching that differentiates the expert teacher in a subject area from the subject expert. PCK is that special professional understanding that teachers have whereby they can incorporate, change and represent subject matter knowledge in ways that are comprehensible to pupils. At the same time, area and perimeter of plane figures have become more important components of the curriculum, particularly at the primary school level. This paper examines the approach of one beginning teacher who uses both content and pedagogical content knowledge to assist Grade 4 pupils to develop key concepts in area and perimeter of plane figure.

Background Issues

Mathematics Pedagogical Content Knowledge and Area and Perimeter

We may have encountered teachers who are very knowledgeable about their content area but are somehow unable to present the content in such a way that their pupils can comprehend. Shulman (1987) considered this to be that very critical part of teacher knowledge where one moves from personal comprehension to preparing for the comprehension of others. A teacher cannot hope to explain mathematical concept if he does not have full comprehension of that mathematical concept. Nevertheless, case study evidence suggests that the influence of teachers' mathematics pedagogical content knowledge (MPCK) has a strong influence on children's learning outcomes (Shulman, 1986). In the teaching of mathematics, Ball (2000) stressed how the depth of teachers' understanding of MPCK is a major determinant of teachers' choice of examples, explanations, exercises, items, and reactions to children's work. Mathematics educators have been constantly urging the mathematics teachers to teach mathematics with relational understanding (Skemp, 1978, Van de Walle, 1994). Only when the mathematics teacher understands something well enough is he able to teach others. He needs to overcome the various obstacles that might otherwise deny his pupils access to knowledge. Studies have shown that beginning teachers often struggle to represent concepts in an understandable manner to their students because they have little or no PCK at their disposal (Kagan, 1992; & Reynolds, 1992). Beginning teachers, for example, do not see much difference in telling and explaining because they have not developed their PCK.

For the area and perimeter content domain, the place of area and perimeter in the curriculum was well established. Many researchers have reported on lower primary pupils' ideas related to the measurement of area (Battista, Clements, Arnoff, Battista, & Van Auken Borrow, 1998; Dickson 1989; Heraud, 1987; Lehrer, Jenkins, & Osana, 1998; Outhred &Mitchelmore, 2000). Although their findings are insightful, their studies were about pupils in grade 4 or below. There were also studies which indicated that pupils in the middle years (Year 5 to 8) confuse the concepts of area and perimeter even though they may give correct answers to standard assessment questions requiring the use of formulae (Kidman & Cooper, 1997). Mulligan, Prescott, Mitchekmore and Outhred (2005) suggest that to avoid such confusion, teaching should focus on the recognition of the structure of an area grid and is likely to lay the foundation for a deeper understanding of area measurement. Other mathematics educators such as Martin and Strutchens (2000) and Van de Walle (1997) recommend the use of squares to make "length×width" understandable. It is very easy for pupils to cover an area empirically with squares and to use multiplication to show how many squares were used. Squares are easy to quantify because they are discrete quantities. Although area and perimeter of plane figures are characterized by procedural techniques, there are deep conceptual issues to address. Significant among these are the ideas that lengths and areas are continuous quantities. It will be extremely difficult for pupils to understand how two lines (the length and the width) can produce an area when they are multiplied. When presented with two lengths perpendicular to one another starting from the same point, the pupil must have a concept of the area as a matrix consisting of "an infinite set of lines infinitesimally close to one another" (Piaget et al., p. 350) to make sense of length x width which is the area of rectangle. In order to provide these, it may require the teachers to have significant content and pedagogical content knowledge of area and perimeter. Unfortunately, a study conducted by Menon (1998) on 54 preservice elementary teachers' understanding of area and perimeter, it was found that they have a procedural understanding of area and perimeter rather than a conceptual and relational understanding. Moreover, the 2003 NCTM Yearbook entitled Learning and Teaching Measurement (Clements & Bright, 2003) has six chapters that deal, at least in part, with the area of a rectangle. However, the authors of all six of these chapters assumed that a square was the unit for area for pupils and did not mention any issue related to the fact that length and area are continuous quantities. Establishing the idea of length and area as continuous quantities requires time. Mathematics teachers in Singapore may thus view it as a costly procedure given the crowded curriculum in Singapore.

Most primary school pupils have a good understanding of perimeter as a special application of length that measures the distance around a figure. Pupils are so accustomed to finding perimeters where the length of every part of a figure is given and they just had to add all the given numbers. Pupils who do not have an adequate understanding of perimeter will find it difficult to deduce the length of the side when it was not stated explicitly. On the other hand, a good relational understanding of perimeter includes reasoning based on relationships among the sides of a given figure. Another common finding from the literature is that teachers confuse the concepts of area and perimeter (Baturo & Nason, 1996; Fuller, 1997; Heaton, 1992), frequently assuming that there is a constant relationship between area and perimeter. Further, teachers often do not use appropriate units when computing area and perimeter, commonly failing to use square units when reporting measures of area (Baturo & Nason, 1996; Simon & Blume, 1994).

The Present Study

In light of the area and perimeter concepts discussed above, and mindful of the challenges inherent in teaching them, it seems timely to look at how a beginning teacher apply his content knowledge and pedagogical content knowledge in teaching area and perimeter. The present study was part of a larger project entitled Knowledge for Teaching Primary Mathematics (or MPCK Project) with the objective of studying the development of beginning primary school teachers' MPCK which involved observing and video-taping five lessons for a beginning teacher. An important part of the research was to determine what the researchers (from MPCK Project) term "MPCK-in-action" outcomes as observed being practised by teachers when teaching mathematics and to ascertain the relative importance of different practices in contributing towards effective pupil learning.

This paper reports on a study of a beginning teacher who presented five of his lessons on area and perimeter to his class of Grade 4 pupils. This paper also look at the area and perimeter concepts and the way in which CK and PCK impact on their development when the teacher delivered the lessons. It seeks to answer the following questions:

1. What key perimeter and area concepts were brought out in the lessons?
2. What are the observable MPCK-in-actions that are present in the teaching of area and perimeter?

Methodology

This paper focuses on this Grade 4 teacher, John (name is pseudonyms) who conducted five lessons on area and perimeter. The beginning teacher, John was from a typical government primary school and was teaching his own Grade 4 class. During each lesson, the video-camera followed the teacher and field notes were made by the author. The video data were analysed by the author to determine key points at which CK and/or PCK were evident in relation to understanding of area and perimeter. In particular, the way in which the chosen activities could be and were used to bring out key area and perimeter concepts were noted, along with other critical moments. The MPCK-in-action practices of this beginning teacher, John are identified through an analysis of video-tapes of five mathematics lessons.

Results

Despite the fact that only five lessons were observed for one beginning teacher, they were rich in their consideration of area and perimeter concepts. John planned the following seven specific instructional objectives for his five lessons:

1. Use the formula correctly to find the area and perimeter of rectangle and square;
2. Find all possible areas rectangle and square given a fixed perimeter;
3. Calculate one dimension of a rectangle and square given its area, perimeter and the other dimension;
4. Solve word problems involving area and perimeter of rectangles and squares;
5. Calculate the area and perimeter of the rectangle and square mentally;
6. Calculate the perimeter and area of the composite figure which made up of rectangles and/or squares.
7. Solve non-routine problems involving area and perimeter of a composite figure.

This beginning teacher, John, involved his pupils extensively in his lessons. Throughout the five lessons, John adopted teacher-centred approach. Only part of the first and fourth lesson, the pupils were involved in some group activities. He had a very effective and seemingly effortless questioning technique in which he was able to draw out from the pupils the ideas that he thought were important. Based on the syllabus, the teacher assumed that the pupils had learnt the concept of area and perimeter of rectangle and square in Grade 3 and proceeds to teach area and perimeter of composite plane figures.

CK, PCK and Area and Perimeter Plane Figures Concepts

In this section we look at the way area and perimeter of plane figures concepts were developed in the lessons. This discussion is not exhaustive, but is intended to highlight some of the critical events during the lessons where concepts were particularly affected by teachers' CK and/or PCK.

Area.

Although the pupils were able to articulate the area of rectangle and square well, it seemed that the pupils were not able to define area. In the first lesson, pupils were asked to define area, promptly answer, "length times breadth!" They understand area as a formula rather than as a concept – the amount of space covered by the boundaries of a two-dimensional figure. From the first lesson, no pupil was able to state that area is the amount of space in an enclosed figure and the teacher did not attempt to assist the pupils to define the term, area. Instead, the teacher showed examples on how the area of rectangle (flag) and area of square (chessboard) were computed using the formula. At the last 30 minutes of the first lesson, pupils were given twenty match sticks to investigate different areas having the same perimeter. He guided the class to work out the area of different rectangles and squares that were found in the worksheet. Finally, the pupils could show that although rectangles and square might have the same perimeter their areas were not equal.

Area of composite figures.

Due to emphasis in the Grade 4 syllabus, the teacher only provides examples of composite figures that made up of rectangles and squares. Opportunities to form composite figures involving rectangle and squares were also implemented in the fourth lesson. The purpose of the activity was to let the pupils experience how composite figures are formed. Pupils used the cut-outs rectangles and squares to form composite figures. The pupils actually measured the dimension of composite figures and made an attempt to form composite figures that were shown on the worksheet. Most of the pupils were able to find the area of the composite figures with ease. The teacher discussed the solutions with the whole class after most of the pupils complete finding the area of composite figures. This discussion included dividing each composite figure into squares and rectangles. He also discussed ways of deducing the length and breadth which were not given explicitly. Unfortunately, perhaps because of time constraints, during the fourth lessons John could not discuss or explore further how to find the area of composite figures in different ways. The examples that were given in the fifth lesson were appropriate but the teacher did not make an attempt to subdivide the composite figure out of the boundary. Pupils were always trying to find and divide the figure within the shape. In the fifth lesson, although there was some discussion of finding the area of composite figures which are more complicated, this received only limited investigation.

Perimeter.

There were two significant occasions at which the idea of perimeter was apparent. The first was at the beginning of the first lesson, in which he devoted five minutes of the lesson time to get pupils to define perimeter. This resulted in the pupils trying to recall the formula for the perimeter of rectangle or square. Although the pupils tend to focus on remembering the correct formula for the perimeter of rectangle and square, teacher emphasized the meaning of a perimeter for a plane figure. Even though the pupils had learnt the concepts of perimeter in Grade 3, at the beginning of the first lesson, John ensured that his pupils were able to define perimeter explicitly.

The second occasion associated with perimeter occurred because of the way in which examples that were showed in the class. A full analysis of the L-shaped figures example which John actually did in his fourth lesson showed that not all the lengths of the L-shaped figure were given. Pupils could not just simply add up all lengths to get perimeter of the L-shaped figure. In the fourth lesson hand-on activity, pupils were given rectangles and squares to form composite figures. Pupils have to match the rectangles and squares with composite figures showed in the worksheet. John made an attempt to lead the pupils to find the perimeters where the lengths of every part of a figure were not given and they just could not add all the given numbers. In the class discussion, pupils were also given the opportunity to reason and based on the relationships among the sides of a given composite figure to deduce the length of a side which was not given explicitly. This had assisted the teacher to tease out any misconception the pupils may have on the understanding of perimeter. In addition, the teacher also gave homework involving composite figures of lengths not stated explicitly. This provided an added experience for the pupils to find perimeter of such composite figures.

Observable MPCK-In-Actions

Sequencing of activities.

Throughout the five lessons, John was able to structure the examples from simple to complex. This was reflected in his fifth lesson where it was intended to solve only non-routine problems after the pupils had enough experience in solving simple problems from the previous lessons. In carrying out the match sticks activity and forming composite figures, teacher adopted the concrete-pictorial-abstract approach. Pupils were given the concrete experience of finding the perimeter before they worked out their computation in their worksheets.

Choice of activities.

John had a good understanding of his pupils' mathematical abilities. He had identified and selected activities that matched his pupils' learning needs. For example, in the fourth lesson pupils worked in pair to form composite figures. This gave the pupils a good concept of composite figures involving rectangles and squares as expected in the Grade 4 syllabus. The pupils responded extremely well to this activity.

Connections between concepts.

At the introduction of each lesson, John always elicited pupils' previous knowledge. He highlighted to his pupils the relevance of learning and applied the area of rectangle concept to a real life problem. For example, John tried to relate area concepts through an example in finding the area of floor plan in a new apartment. In addition, John was able to link the concepts of area and perimeter well. For instance, given a composite figure, pupils need to find the area and perimeter of the composite figure so as to help the pupils to "see" the difference.

Balance between concept development and mathematical procedures.

John consciously emphasised the underlying reasons and explained the mathematical procedures of finding the lengths of a composite figures which are not stated explicitly. When the pupils presented their solutions on the board, he focussed on the essential steps and necessary conditions in the procedures. In fact, he valued computational speed and accuracy so much that he allocated one lesson where the pupils just only worked out the area, perimeter and length of rectangles and squares mentally. Allowing the pupils to explore alternative procedures in solutions was only evident in the fourth lesson. Pupils were asked to find the perimeter of an L-shape figure. A particular pupil was able to find the perimeter of the L-shape in two ways even though length of certain side was not given explicitly.

Questioning techniques.

In most situations, John used structured questioning to establish mathematical procedures. He also exhibited effective use of questioning to elicit understanding from the pupil. John seemed to have a solid knowledge of procedures of finding the area and perimeter of plane figures, of its place in the curriculum and in his own pupils' understanding and of how to teach it. He made no effort to take appropriate actions to rectify errors or correct misconceptions as he circulated in the class to monitor the pupils' class work. Understanding pupils' common misconceptions and strategies for challenging such misconceptions are examples of pedagogical content knowledge which John had shown in most of his lessons.

Discussion and Conclusion

The results depict the complex interplay among concepts (as seen in the fact that some teaching episodes involved two or more area and perimeter concepts), the challenges associated with teaching these concepts and the relationships among them, and the importance of content knowledge and pedagogical content knowledge. The beginning teacher had chosen activities that were rich mathematically, considering the language of area and perimeter, the meaning of composite plane figure, the relationship of area and perimeter and the various ways of finding the area and perimeter of composite figures. The beginning teachers' PCK was evident not just in the choice of activities, but in the ways that he was able to link concepts to pupils' experience. For instance, he gave examples of finding the area of a flag, chessboard and area and perimeter of floor plan of an apartment. The teacher's approaches varied, giving pupils greater freedom to think about concepts and holding rich discussions with groups and individuals. At the initial stage of each lesson, the teacher tended to provide more direct teaching and guidance to pupils about what to do and, perhaps because his belief was that pupils must have a strong basic foundation of the skills and concepts first. From the teacher's direct teaching, class discussions and questioning, it appears that the teacher's CK was strong. He has no much difficulty exploring pupils' ideas when they deviated from his own. Except for one occasion when incorrect content knowledge was evident when the pupils stated that area is equal to length times breadth and he just accepted this definition.. John did not further define the term "area" explicitly for the pupils.

The complexity of pedagogical content knowledge was particularly apparent in the use of the match sticks activity. The match sticks activity provided opportunities to explore areas of different rectangles and squares if the perimeter is constant. The teacher allowed the pupils to do in pair and encouraged his pupils to go through this hand-on activity. In fact, the teacher's decision to do this was made during the first lesson where he planned it in advance. This may well have been the time during which the teacher's pedagogical content knowledge actually developed. However, the teacher did not plan any activity for the same area of different rectangles and squares and deduced if the perimeter of those rectangles and squares could be the same. Teacher was more careful in the presentation of the solutions, using words that would help his pupils understand its meaning. The important role of content knowledge in PCK was also evident in the lessons. On many occasions this was positive, as when the beginning teacher discussed the various ways of finding the area and perimeter of composite figures. The teacher's discussion about finding the area of composite figures showed his content knowledge, but also revealed the complexity of deciding how much to explain to pupils and the difficulty of actually explaining it in a way suitable for Grade 4 pupils. These lessons show that quite deep area and perimeter concepts can be considered in Grade 4. They also highlight the complexity of the concepts, and the importance of having teachers with appropriate content and pedagogical content knowledge. The key area and perimeter concepts identified above need to be understood by teacher himself, plus he must be able to recognise which activities would foster such understanding in his pupils and how to bring this to the fore in his lessons. While a number of primary teachers with weak content knowledge were predisposed to telling pupils rules and explaining algorithmic procedures, John with strong content knowledge appeared to provide conceptual explanation for each example and activity. However, some beginning teachers found it a challenge to provide conceptual explanations for the procedural tasks they performed.

Everything that the beginning teacher John had carried out involved planning lessons, implementing them, responding to what arose in the classroom, interacting with pupils—involved one or more aspects of PCK. Good teaching requires both the mastery of CK as well as PCK. PCK is developed over time. John is a teacher with rich PCK who can devise examples that illustrate a range of concepts, can highlight connections among topics, and identify which are the central ideas and which are peripheral. This was evident in John's teaching of area and perimeter of plane figures.

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