

# Deepening the Mathematical Knowledge of Secondary Mathematics Teachers who Lack Tertiary Mathematics Qualifications

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A professional learning program for unqualified practising secondary mathematics teachers regarding senior secondary mathematics teaching is described in this article. Professional learning episodes, artefacts and reflections of three teachers who participated in the program are analysed to identify the development of these teachers' mathematical and pedagogical content knowledge. The findings indicate that a program designed for senior secondary mathematics can enable practising teachers to deepen and broaden their understanding of junior secondary mathematical pedagogy.

In Australian secondary schools significant numbers of teachers of junior secondary mathematics (almost 50%) and even teachers of (usually less advanced) senior mathematics subjects (32%) do not hold the recommended tertiary mathematics qualifications for teaching secondary mathematics (McKenzie, Kos, Walker, & Hong, 2008). In Victoria, it is recommended that teachers of secondary mathematics complete two years of tertiary mathematics (elsewhere in Australia the minimum is three years) and an approved qualification in education (VIT, 2003). The subject of this paper is a professional learning project initiated by one school faced with the prospect of having too few teachers with the knowledge of senior secondary mathematics to be able to provide a full range of senior mathematics options for their students in the new future. This school decided to prepare some of its teachers of junior secondary mathematics to teach advanced senior mathematics. They approached our university to design a professional learning program for teachers from their school and other schools in their region who had not completed the recommended tertiary mathematics study or pre-service training in mathematics. In this paper we describe and analyse the mathematical and pedagogical content knowledge of three teachers who participated in the program.

## Theoretical Framework

### *Professional Development Models*

Pedagogical change and innovation has been the main goal of in-service professional development programs of mathematics teachers in Australasia and many practice-based models are structured around designing and/or implementing different teaching approaches and tasks (e.g., Goos, Dole, & Makar, 2007; Watson, Beswick, Caney, & Skalicky, 2005/2006). However it is not clear how teachers may develop their understanding of mathematics in such programs:

Advocates for practice-based professional development argue that learning experiences that are highly connected to and contextualised in professional practice can better enable mathematics teachers to make the kinds of complex, nuanced judgements required in teaching. Yet, evidence is generally lacking regarding if and how teachers might enhance their knowledge of mathematics through such professional development experiences. (Silver, Clark, Ghouseini, Charalambous, & Sealy, 2007, p. 261)

The importance of teachers' mathematical content knowledge is recognised as critical for improving students' mathematical learning in recently reported Australasian studies (e.g., White, Mitchelmore, Branc, & Maxon, 2004) and engaging teachers in mathematical thinking through working on mathematics-related tasks, and reflecting on these experiences is common to many in-service programs. For example, Biza, Nardi, and Zachariades (2007) used tasks that were situated in teaching practice that involved reflecting upon the learning objectives of a particular task, examining a flawed student solution and describing, in writing, feedback to the student. The professional learning tasks used by Silver et al. (2007) were also situated in teaching practice. These tasks began with an initial non-trivial mathematics problem. Teachers then analysed and discussed a teaching case and finally collaboratively planned and reflected on a mathematics lesson. On the other hand Leikin and Winicki-Landman (2001) used only mathematics tasks that focussed on definitions of mathematical concepts. Teachers were prompted to discuss the logical relationships between different mathematical statements related to a concept. These mathematics tasks entwined both mathematical knowledge and issues of pedagogy. Both kinds of tasks, that is mathematics tasks and pedagogical content knowledge

tasks imbedded within a practice-based model of professional development would seem to provide optimum conditions for enhancing teachers' mathematical and pedagogical content knowledge.

### *Pedagogical content knowledge*

Shulman (1987) defined pedagogical content knowledge (PCK) as “the blending of content and pedagogy into an understanding of how particular topics, problems, or issues are organised, represented, and adapted to the diverse interests and abilities of learners, and presented for instruction” (p. 8). Ma (1999), who focussed on primary teachers' knowledge of fundamental mathematics, demonstrated the strong relationship between profound understanding of fundamental mathematics and pedagogical knowledge. For her, profound understanding is more than procedural and conceptual knowledge; it is “an understanding that is deep, broad, and thorough” (p. 120), where depth means being able to connect a topic with “more conceptually powerful ideas of the subject” (p.121), breadth being able to “connect it with those of similar or less conceptual power” (p.121), and “thoroughness is the capacity to connect all topics” (p. 124). Such understanding is evident in connected approaches to teaching mathematics that use flexible and multiple representations of mathematics and an attitude that promotes mathematical inquiry and justification.

Chick, Baker, Pham, and Cheng (2006) proposed a framework of analysing PCK. They grouped the elements of PCK into three categories. In the first category, “clearly PCK”, pedagogical knowledge of teaching mathematics is entwined with knowledge of mathematics and included knowledge of general and specific teaching strategies, student thinking and misconceptions, knowledge of cognitive demands of tasks, appropriate and detailed representations of concepts, knowledge of resources, curriculum knowledge, and knowledge of the purpose of content. The second category in their framework focuses on mathematics subject knowledge needed for teaching and includes Profound Understanding of Fundamental Mathematics (Ma, 1999), deconstructing content into key components, mathematical structure and connections, procedural knowledge, and methods of solution. The third category concerns general pedagogical knowledge applied to teaching mathematics and includes general and specific goals for learning, getting and maintaining student focus, and classroom techniques.

## **The Study**

### *The VCE Mathematics Professional Learning Program*

The VCE (Victorian Certificate of Education) mathematics professional learning program (PLP) for senior secondary mathematics was designed for practising secondary teachers of mathematics who had no experience of teaching advanced senior secondary mathematics and who had not completed the recommended qualifications. The teachers wanted a program that was situated and practice based and that would enable them to learn the mathematics of grades 11 and 12 (VCE Mathematical Methods and Further Mathematics) and the methods of teaching this mathematics. In this professional learning program, teachers were students of mathematics (Leiken & Winicki-Landman, 2001), and this mathematics included algebra, functions and graphs, rates of change and calculus, probability and data analysis, including statistical modelling.

The PLP involved seminars as well as self-directed practice-based inquiry and portfolio development. Teachers attended 21 three-hour seminars conducted fortnightly on an afternoon during school terms over a school year. We provided each of the teachers with a CAS (computer algebra system) calculator and at least two different textbooks for each of the VCE mathematics subjects, and we encouraged the teachers to purchase a mathematics dictionary of their choice.

We used both mathematics and professional learning tasks during the seminars. A typical session began with finding out what the teachers knew about the mathematics concepts and procedures concerning a mathematics task (e.g., Figure 1). One of us would scaffold discussion of conceptual or procedural ideas relevant to the task. Teachers would then work on mathematics tasks, including closed tasks such as the one in Figure 1, open-ended tasks, tasks designed to explore concepts, tasks with the purpose of deriving or proving rules, or mathematics-related professional learning tasks. These professional learning tasks (Silver et. al., 2007) included analysis of mathematics problems and activities, analysis of teaching materials and teaching strategies, design of mathematics problems and activities, analysis of students' solutions, and review of VCE subject examiners' reports. The session would end with a discussion to summarise the key mathematical

concepts and procedures. Teachers reflected on their experience of working on these tasks and the outcomes of this discussion were used to identify potential student misconceptions and key points for teaching.

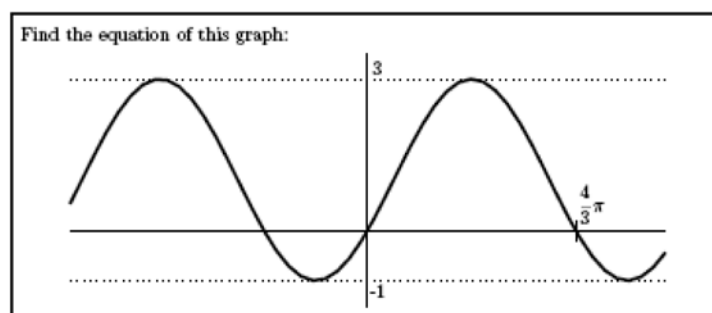


Figure 1. A task on transformation of trigonometric functions.

The sequence of topics for the seminars followed the sequence normally used by teachers of the grade 12 subjects. Hence we included seminars with discussions about the formal assessment tasks of the VCE subjects at roughly the same time that teachers were designing and assessing students with these tasks. Experienced senior secondary mathematics teachers also conducted a few sessions in the program. Their sessions focussed on curriculum knowledge, long-term planning for teaching and assessment, resources for teaching, and tasks used for teaching and assessment.

The practice-based component of the program occurred in the teacher's school between seminars. We encouraged participants to establish a mentor relationship with an experienced teacher of senior secondary mathematics to support their school-based self-directed inquiry. We recommended that they negotiate with their colleagues to observe and/or team-teach grade 11 or 12 mathematics lessons, observe students doing mathematics (in lessons or by tutoring students), reflect on observations, analyse student work, research and critique teaching and assessment resources and materials, and to participate in the moderation processes of student assessment for these subjects. The teachers documented their learning in an annotated portfolio that they presented to their peers in the final session of the program.

## Methods

The study used a qualitative design. Eleven teachers (six were women) from five government secondary schools located in a regional city and surrounding towns participated in the program. Ten teachers completed the program (the other teacher had to withdraw early in Term 2 due to family commitments and work responsibilities). Questionnaires, field notes and artefacts from the program and teachers' portfolios were the data used in this study. Data were analysed qualitatively using codes derived from the PCK framework (Chick et al., 2006) described above.

The pedagogical content knowledge with respect to algebra and functions of three teachers in the program from different schools are the focus of this paper. None of these teachers had completed the required tertiary mathematics study or mathematics education training for mathematics teaching. They were selected for this paper because their responses to the initial questionnaire indicated different levels of mathematics knowledge and teaching experience, varied professional learning experiences, and mentoring between sessions. These three teachers also illustrated the kinds of mathematical and pedagogical learning achieved by the teachers in the program.

## Teachers' Mathematical and Pedagogical Content Knowledge

### Gloria

Gloria, a second year teacher, with one year experience of teaching grade 12 chemistry and grade 10 mathematics, was teaching grade 11 General Mathematics in the year of the program. At the beginning of the course she described her mathematics teaching strengths as "recognising where students are having difficulties with understanding; and patient with explaining skills and formula's (sic) as well as providing

alternate explanations where needed” and rated herself as comfortable teaching basic algebra skills and using technology, but thought that she struggled to explain concepts and procedures concerning functions in the VCE mathematics curriculum. Gloria’s knowledge of algebraic procedures was demonstrated during one of the early seminars when she demonstrated to the group the procedure of completing the square to derive the quadratic formula for solving quadratic equations. She explained early in the program that she was “getting a deeper understanding of where the rules come from and why they work” indicating development toward thorough understanding of concepts.

When presenting the contents of her portfolio at the end of the program, Gloria described an investigation using Excel that she had used with her grade 10 students. She claimed that this task showed how she had developed during the program since she now understood the importance of investigations and how to implement them in the classroom:

I have included this resource in my portfolio because I think it is an excellent resource for introducing translations, dilations, reflections, etc in graphs. I have successfully used this at the year 10 level and observed students responding positively to it. While being involved in this course I have realised how useful this resource is for students continuing into VCE methods. This resource looks specifically at changes in quadratics graph, but would be easily developed to include other graph types; trigonometry graphs, logarithmic graphs etc, in a similar manner. It provides students with a quick and easy way to visually view the effects of translations in graphs. The questions at the end of each section help students to ensure that they understand what they have observed and consolidate the information.

Gloria’s investigation of quadratic functions is an example of a general strategy that is student-centred and engages students in thinking mathematically by exploring cases and paying attention to patterns; in this case, in the relationship between graphic and symbolic representation of functions. It also illustrates her use of multiple representations of concepts facilitated by digital technology. Gloria also indicates through this task that she understands the connections between topics; in this case, transformations of different functions. By generalising the teaching strategy in this context, Gloria believes that students will consolidate their understanding and, by implication, generalise the findings of transforming functions.

Gloria’s progress toward deepening and broadening her conceptual understanding was also evident in her analysis of the cognitive demands of tasks and the deconstruction of content to key components. Toward the end of the program, the teachers reviewed the examiners’ reports and student results for particular questions on the Mathematics Methods written examination 2 (VCAA, 2006). The two most difficult multiple-choice questions proved to be Q7, a question about the domain of a logarithmic function, and Q19, a question that required students to solve for an unknown in a probability density function, for which only 45% of students had been correct. Gloria argued that students had generally performed worse on the multiple-choice questions in which functions and equations were expressed in what she described as “general form” (Q7:  $g(x) = \log_e |x - b|$ , where  $b$  is constant; Q19:  $f(x) = \begin{cases} 1 + e^{\frac{x}{k}} & 0 \leq x \leq k \\ 0 & \text{otherwise} \end{cases}$ ). She also commented that perhaps students were

also having difficulty with questions about domains. She included these observations among her goals for teaching when documenting what she had learned during the program.

### *Helen*

Helen was also in her second year of teaching. She was a qualified science teacher, who had taught grade 7 mathematics for one year. She had also tutored grade 10 and 11 students from a local private school in mathematics for three years and had established strong professional relationships with mathematics teachers at this school who continued to mentor her. At the beginning of the program Helen wrote that she considered herself to be “a mathematical and scientific thinker” who enjoyed working with structure and “skilled in working with students of different abilities ... and good at finding problems and work that challenges all students.” While comfortable with teaching some algebra topics, Helen thought that she struggled to explain concepts and procedures concerned with solving exponential and logarithmic equations and graphing and transforming functions.



Early in the year Helen expressed that she was “beginning to see how concepts link up and ‘fit in’. I also feel as though I am generating a better understanding of ‘how’ and ‘why’, although this still needs further work”. At the end of the course Helen reflected on her own experience of learning mathematics, recalling learning the rule and applying it, but not “getting” the conceptual ideas. She observed this in the grade 10 and 11 girls that she tutored throughout the year. Helen admitted that she still looked for rules but that the program had enabled her to focus on understanding and concepts more. In her presentation Helen focussed on the need for students to develop a mathematical attitude toward the discipline of mathematics. She gave particular emphasis to students developing ways of thinking mathematically and developing persistence in problem solving:

I place a lot of importance and emphasis on students’ success in VCE Mathematics from their engagement and problem solving abilities in middle years mathematics. This VCE Mathematics program has therefore assisted me in my teaching of middle years mathematics (as well as VCE Mathematics) by inspiring me to work towards students being engaged, challenged and encouraged when developing understandings of new concepts.

Helen included a curriculum development map (or learning trajectory) on polynomial and modulus functions in her portfolio. She explained that she had based in on one about exponential functions distributed during the program. Helen’s map shows the sequence of connected topics from grade 9 and 10 to grade 11 Mathematical Methods and grade 12 Mathematical Methods. It includes symbolic and graphic representations of various functions and lists key skills and concepts to be developed. One thread of ideas through this map is set notation, first included for grades 9 and 10 as an introduction (#4). She listed symbols and examples including representation on a number line. At #7, as part of the grade 11 curriculum she included “develop deeper understanding of set notation, domain & range, and symbols to represent these.” Thereafter the map includes set notation when defining functions and their domain. For #13 in grade 12, Helen records that students need to develop “understanding/reading accurately all methods of expressing a function”. From this artefact we can see that Helen is developing depth in her mathematical understanding, being able to connect a topic with more conceptually demanding ideas. She thought curriculum maps were really helpful because they showed when the students learned particular concepts and skills and why some students might miss learning some concepts or skills. Helen’s curriculum map illustrates the way in which the teachers came to understand the connection between content in junior and senior secondary mathematics and hence the purpose of specific content in junior secondary mathematics for a range of topics.

Helen’s interest in student thinking and their misconceptions is also evident in this curriculum map. She included a section on common misconceptions, identifying students’ likely “confusion with symbols, & reading set and function notation, difficulties with asymptotes & domain/range of these graphs, translating in wrong direction, what to do when not in TP (turning point) form, modulus functions.”

### *Donna*

Donna was the most experienced teacher among these three teachers. She had taught junior secondary mathematics for three years. She was in the fifth year of teaching, having been a health-care professional in her previous career. Donna was one of two people in the program who rated their mathematics knowledge as quite low, claiming that she could only solve simple problems related to VCE mathematics content. She believed her strengths were in her “ability to explain concepts well once I have revised them, enjoy maths myself and succeed in building confidence in my students.” At her school the VCE mathematics coordinator mentored the two teachers in the program.

Midway through the program Donna commented on the importance of using the correct terms and their meaning and how this aspect of the professional learning program was impacting on her teaching of junior secondary mathematics. So it was not surprising that definitions and mathematical language figured prominently in her portfolio and presentation of artefacts at the end of the program. Donna included two packs of cards from Barnes (1991) that are resources for a small group activity that she learned about from her mentor. One pack depicts the equation, graph or description of the transformation from the standard form for various quadratic functions on the cards. Donna explained that when students worked in a small group to sort these cards into matching sets, they needed to explain their thinking and present an argument to support their claim of matching cards. She reflected:

I found these cards very useful when I was revising this work and correcting my own language when describing curves... This activity helps students become familiar with the both the idea and language of transformations.

This task illustrates a general teaching strategy of group problem solving that can be used purposefully for teaching and learning specific content. The task also illustrates the use of multiple representations since using symbolic and graphic representations of concepts, and moving fluidly between them, was important for the topics in the VCE mathematics subjects that we focussed on. At various times during the program examples of appropriate representations that these teachers used in their teaching in junior secondary mathematics were discussed and Donna was usually prominent in these discussions. For example, during the analysis of examiners reports, the teachers identified students' weakness with fraction knowledge and as part of the extended discussion on teaching and learning fractions Donna described using strip models of fractions to develop grade 7 students' understanding of equivalence.

## Conclusion

The PCK framework developed by Chick et al. (2006) was useful for analysing teachers' knowledge and the cases of teachers' knowledge presented in this paper illustrate the entwining of knowledge of mathematics and knowledge of teaching and learning (Ball, 2001; Davis & Simmt, 2006; Ma, 1999). While we set out to develop teachers' knowledge of mathematics needed for teaching senior secondary mathematics we discovered that taking this approach deepened and broadened teachers' understanding of junior secondary mathematics content and pedagogy. This was particularly evident through the connections that they made between mathematical concepts, the use and understanding of multiple representations, deconstruction of content into key components, their understanding of students' misconceptions, and their appreciation of the inadequacy of procedural and instructional thinking.

The model of in-service professional development analysed in this article shows promise as an effective means for preparing senior secondary mathematics teachers in the current context of mathematics teacher shortages. Important for deepening teachers' mathematical knowledge and developing their PCK was the "teachers as learners of mathematics" model used along with the professional leadership provided by mentors. However these findings need to be tested in a follow-up study that analyses the professional paths of these teachers, their pedagogical practices and their students' learning outcomes in both senior and junior secondary mathematics.

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