Speaking with Different Voices: Knowledge Legitimation Codes of Mathematicians and Mathematics Educators

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This paper uses a textual analysis of two documents prepared by the mathematics community and the mathematics education research community to the National Numeracy Review in 2007 to uncover and compare knowledge legitimation within these two fields. The paper shows that knowledge within these disciplines is based on different epistemic devices, and hence that debates surrounding mathematics education arise, at least in part, from differing ways of viewing knowledge.

Curriculum debates rage in the United States between proponents of a "reform" curriculum and those of a "mathematically correct" curriculum. Reformists accuse mathematically correct advocates of a reductionist, back to basics approach that subjugates the process of learning mathematics to a set of well-defined procedures. On the other hand those who claim to be mathematically correct accuse reformists of being "fuzzy", of valuing any method so long as it works, and of allowing students to work everything out for themselves (Klein, 2007).

Similar debates are rising to the surface in Australia. On the one hand mathematics educators, in particular university-based teacher educators and mathematics education researchers, call for a mathematics curriculum that is responsive to a changing society, that values and incorporates the use of technology and that recognises the hesitant way in which students construct knowledge. On the other hand, Donnelly (2007, p. 55) influenced by some mathematicians and mathematics teachers, calls for a more rigorous curriculum, arguing against constructivist approaches, against "outcomes-based and politically correct" education and against "fuzzy maths". This call foregrounds mathematics as a precise discipline, valuing clear definitions and standard procedures.

This paper uses a framework that looks at how knowledge is produced and legitimated within a discipline (Maton, 2000). It shows that knowledge within the disciplines of mathematics and mathematics education relies on different epistemic devices, and hence that debates surrounding mathematics education arise, at least in part, from differing ways of viewing knowledge. I use a textual analysis of two documents prepared by the mathematics community and the mathematics education research community to the National Numeracy Review in 2007 to uncover and compare the epistemic devices in these two fields. The purpose is not to privilege one view of knowledge over another, but rather to promote greater understanding, and hence to promote greater acceptance of a divergence of views and move the debate forward.

Locating the Issue

Mathematicians and mathematics educators in Australia naturally take a keen interest in the school mathematics curriculum. This interest was particularly evident in the early 1990s, with the development and introduction of *A National Statement on Mathematics for Australian Schools* (Australian Education Council, 1991) and its associated document *Mathematics – a Curriculum Profile for Australian schools* (Australian Education Council, 1994). The mathematics education community and the mathematics community were united in their concern over the process by which the documents were produced, citing lack of adequate consultation in their development and the apparent determination of the writing team to pursue a particular agenda. Both groups also expressed over the content of the documents, however these concerns had very different bases (Ellerton & Clements, 1994).

Mathematics educators were concerned that "reductionist behaviourist approaches to teaching and learning mathematics … give rise to atomistic approaches to curriculum development and encourage methods of teaching and learning that fail to assist the development of a holistic view of mathematics" (Ellerton & Clements, 1994, p. 10). A behaviourist approach, it was stated, was contrary to the view of leading national and international educators who, throughout the 1980s, had argued for a curriculum that promoted relational understanding (Skemp, 1976). While also being concerned about atomistic approaches to curriculum, mathematicians condemned the *Statement* and *Profile* for a lack of quality of mathematical thinking. "(I)f the documents do not faithfully reflect the history of mathematics and do not represent quality contemporary mathematical thinking, then the school mathematics programs engendered by these documents will inevitably be less than satisfactory" (Ellerton & Clements, 1994, p. 10). Mathematicians expressed concern at the omission of important topics in mathematics and at the lack of rigour expected of teachers and students in the pointers contained in the *Profile*.

Given the recent development of the national *Statements of Learning for Mathematics* (Curriculum Corporation, 2006) and the subsequent establishment by the Rudd labor government of a National Curriculum Board to develop national curricula in English, history, science and mathematics, it is opportune to examine the philosophical bases of the views of those with an interest in school mathematics.

Theoretical Framework

This paper argues that the debate over what counts in mathematics education and the school curriculum is, in effect, a battle for control of the epistemic device (Moore & Maton, 2001) arising from conflicting beliefs about the production and validation of knowledge. This epistemic device "regulates: who can produce legitimate knowledge; the ways in which antecedent knowledge is selected and transformed in the course of producing new knowledge; and the criteria for adjudicating claims to new knowledge" (Moore & Maton, 2001, p. 30). The epistemic device thus describes the relationship between knowledge and the knower, casting light on why people view the world as they do and in turn shaping the way they respond to new ideas.

Mathematics[12] and mathematics education[13] are horizontal discourses characterised by a set of "specialised languages with specialised modes of interrogation and criteria for the construction and circulation of texts" (Bernstein, 1999, p. 162). In the case of mathematics these specialised languages consists of fields of study such as geometry, number theory or algebra. In the case of mathematics education the languages may consist of different research paradigms or different lenses through which to view theory and practice in mathematics teaching and learning. Within each of these disciplines knowledge is produced by people working within a particular field and validated by others in the academic community within the discipline. However the process of this validation is based on different principles. In the case of mathematics new ideas are knowledge validated while in mathematics education they are knower validated.

Mathematics has a strong internal grammar (Bernstein, 1999) consisting of accepted principles of logic, internal and external consistency and lack of gaps in reasoning. Andrew Wiles' proof of Fermat's Last Theorem is a classic example of the strong internal grammar of mathematics. Although few could comprehend Wiles' proof in its entirety, the grammar of mathematics allowed a gap in the proof to be detected. Wiles was then able to work on this gap to complete a proof that would stand up to rigorous scrutiny according to the logic of mathematics. Although Wiles' proof was evaluated by his peers in the mathematics community, ultimately it was the product rather than the person that mattered.

Mathematics education, on the other hand, has a weak internal grammar (Bernstein, 1999). Journal and conference papers in the mathematics education research literature are reviewed according to relatively flexible criteria such as whether the paper builds on and interrogates published research, the open-endedness and thoughtfulness of the research questions, the clarity of description of methodology, the ethics of the research and the cohesion of the argument (Gordon, 2002). "In an interpretive paradigm individuals construct their own meanings and a researcher cannot persuade practitioners by logical arguments that his or her story about the world is better and should be used" (Gordon, 2002, p. 2). While reviewers make every attempt to be fair, ultimately it is the person rather than the product that matters.

These differences in knowledge legitimation are described by Maton (2000) as knowledge or knower codes respectively. Maton claims that languages of legitimation are more than mere rhetoric; rather, they "represent the basis for competing claims to limited status and material resources" (Maton, 2000, p. 149). Knowledge and knower legitimation codes are based on underlying principles concerning the epistemic relation, that is the relation between educational knowledge and its object of study, and the social relation, that is between educational knowledge and its author. These principles structure both what can be legitimately claimed as knowledge within a given field and who can legitimately claim or validate that knowledge. Maton (2000) uses

12 It is acknowledged that the term mathematics is contested. In this paper no attempt is made to look at ethnomathematics, as school curriculum is dominated by a Western view of mathematics characterised by relatively hierarchical knowledge structures.

13 It is equally acknowledged that the term mathematics education is contested. Again, within this paper mathematics education is used to refer to research into mathematics teaching and learning within the dominant culture of Australian society, and particularly schools.

Bernstein's (2000) concept of classification and framing to discuss the nature of these principles. Classification refers to the strength of boundaries between categories or contexts, while framing refers to the locus of control within a category or context. The epistemic and social relations that determine the knowledge legitimation mode vary according to the relative strength of the classification and framing on each dimension.

In the case of mathematics, the epistemic relation is both strongly classified and strongly framed. That is, it is clear what counts as legitimate mathematics and there is tight control over what is accepted as legitimate mathematics. On the other hand the social relation is relatively weakly classified and framed. Cultural differences and social disadvantage notwithstanding, in the end who develops mathematical knowledge is less important than the knowledge itself. When the Wolfskehl prize of 100,000 marks for a successful proof of Fermat's Last Theorem was announced, the University of Gottingen received a flood of entries. "Regardless of who had sent in a particular proof, every single one of them had to be scrupulously checked just in case an unknown amateur had stumbled upon the most sought after proof in mathematics" (Singh, 1998, p. 143). Mathematics, then, has a knowledge mode of legitimation.

In mathematics education research, the strength of classification and framing of the epistemic and social relation are reversed. Mathematics education is, by its very nature interdisciplinary. It draws upon knowledge from a wide variety of fields such as psychology, sociology and philosophy, as well as mathematics itself (Presmeg, 1998). The epistemic relation is therefore weakly classified in that it permits, and indeed encourages, a wide variety of knowledge paradigms as legitimate knowledge. Furthermore these different paradigms exist in "different cultural traditions in mathematics education, arising from different communities and sub-communities" (Sierpinska & Kilpatrick, 1998, p. 31). Thus the epistemic relation is also weakly framed in that the locus of control is not located within a particular group. However the social relation is strongly framed and classified. Mathematics education research, particularly of an interpretive nature, is frequently culture-dependent, thus the researcher "needs to be part of this world, interpreting its events for an extended period" (Presmeg, 1998, p. 59). Of his list of thirteen critical problems facing mathematics education Freudenthal's (1981), first and most urgent was "Why can Jennifer not do arithmetic?" He distinguished this from the more abstract questions "Why can Johnny not do arithmetic?" and "Why can Mary do arithmetic?" In making the distinction Freudenthal described Jennifer as a living child whom he could describe in detail. Jennifer's experience in mathematics at school was context-dependent, and being able to understand those experiences depended upon being in that context. Mathematics education is thus strongly framed with respect to the social relation - it matters who does the research. It also matters who reviews the research as the reviewer must be able to place herself within that context, which depends on having personally experienced similar situations. As noted by Southwell (2004, p. 540) in her discussion of the reviewing process for articles submitted to the *Mathematics Education Research Journal* "(t)he skill of the reviewers will, in the end, determine the quality of the journal."

I suggest that these claims regarding epistemological differences are at the heart of the debate about school mathematics curriculum. One mode of legitimation is not more acceptable or appropriate than another, yet they compete for credence within the broad mathematics educational community. Each, together with the mathematics teaching community has a legitimate claim to a voice in the debate, and each has something unique to offer. The different voices of mathematicians, mathematics education researchers and mathematics teachers will be examined in the remainder of this paper.

Methodology and Data Analysis

The paper uses text analysis to examine the knowledge legitimation codes in two documents. In selecting the documents I chose to use ones which purported to represent the views of the mathematics and mathematics education research communities regarding numeracy in Australian schools. The papers were prepared by the Australian Mathematical Sciences Institute (AMSI) (Australian Mathematical Sciences Institute, 2007) and the Mathematics Education Research Group of Australasia (MERGA) (Mathematics Education Research Group of Australasia, 2007) respectively in response to the Australian government's National Numeracy Review in 2007. The Review aimed to analyse research about teaching, learning and assessment practices in mathematics, examine mathematics pre-service and practising teachers' pedagogic content knowledge, identify the relationship between teachers' content knowledge, pedagogic content knowledge and practice, and identify effective assessment methods (Monash University, 2007).

The documents were analysed for conceptual content using Leximancer, which allows the researcher to examine large amounts of text using automatic recognition of the main concepts within the text together with their relative strength, relation to each other, and contextual similarity. The results of the analysis are presented as a visual map that enables the researcher to analyse the conceptual structure of the document and to refine the search for concepts and their relationships using further iterations through the text. Leximancer has been used for conceptual modelling of text in areas as diverse as risk management (Martin & Rice, 2007) and analysing the rules of baseball and cricket (Smith & Humphreys, 2006). Conceptual mapping of the responses to the National Numeracy Review enables a comparison to be made of the level of importance afforded to, and the relations between, various concepts in each of the submissions.

Results and Discussion

Each document was analysed, in turn, using Leximancer. The software was used to make an initial pass of 1000 iterations through each document to produce a visual map of the most common concepts. Although it is possible within the software to delete or combine concepts, or to add new ones, the decision was made to retain those identified by the software. Following the initial pass through the documents, a further 2000 iterations were performed, by which time the conceptual map of each document was relatively stable. It should be noted that the results reported are by no means an exhaustive analysis of the documents. Nor is the analysis a detailed text analysis using, for example, systemic functional linguistics or critical discourse analysis. This is potential further research.

The visual maps of the AMSI and MERGA documents are presented in Figure 1.[14] The relative frequencies of the most common concepts in the documents are presented in Table 1.

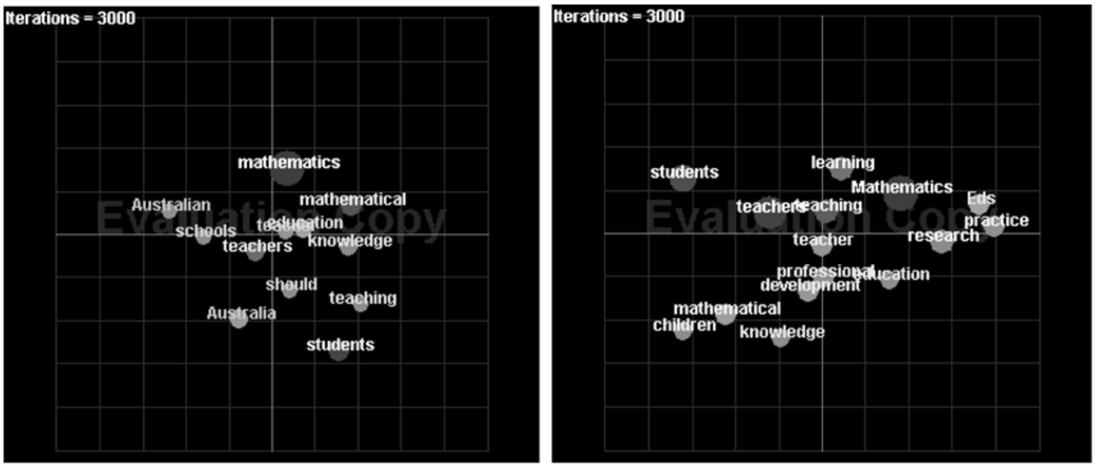

Figure 1. Conceptual maps of AMSI (left) and MERGA (right) responses to National Numeracy Review.

14 The maps were produced using an evaluation version of Leximancer, as the full version was in the process of being purchased at the time of writing the paper. Hence "evaluation copy" appears on the maps. The analysis process is, however, unaffected by whether or not the software is an evaluation copy.

Table 1

Relative Frequencies of Concepts in AMSI and MERGA Responses

Concept	Relative frequency of concept, compared to most common concept	
	AMSI	**MERGA**
mathematics	100%	100%
students	36%	60%
teachers	26%	82%
mathematical	28%	38%
knowledge	20%	24%
Australia	20%	<10%
teaching	19%	44%
education	19%	24%
time	17%	<10%
curriculum	16%	<10%
schools	15%	<10%
should	15%	<10%
learning	<10%	50%
Eds	<10%	49%
research	<10%	49%
practice	<10%	37%
children	<10%	35%
development	<10%	32%
professional	<10%	28%

Some striking similarities and differences emerge when examining the conceptual maps and the table of relative frequencies. The most obvious similarity is that in each of the documents the concept *mathematics* is the most common. In fact, each of the documents emphasises the centrality of mathematics, recommending the abandonment of the term *numeracy*, which is not prominent in either document.

The concepts *students* and *teachers* both appear prominently in each document, however with much greater relative frequency in the MERGA response than in the AMSI response. This may be seen as unsurprising given the nature of the two associations, however it is also suggestive of an emphasis on the person (a knower mode) rather than the content (a knowledge mode). The differences in emphases on concepts such as *teaching*, *development* and *professional* may reflect a similar emphasis on the person rather than the content. The conceptual map for the MERGA document clearly shows the conceptual proximity of the terms *professional* and *development*, suggesting that they could, in fact, be considered as one complex concept.

The concept *curriculum* appears with relative frequency greater than 15% in the AMSI document, but less than 10% in the MERGA document. Indeed, the AMSI document recommends as the first step "clearly defin(ing) the mathematics expectations for each year level in the compulsory years of schooling". It further states that "(w)e do not believe this would be very difficult but it must be done." This statement and the high relative frequency of the term *curriculum* suggest an emphasis on content (a knowledge mode) rather than the person (a knower mode).

The concept *Eds*, which is used in references at the conclusion of the document to papers in conference proceedings, and *research* appear frequently in the MERGA document, but not in the AMSI document. In fact the MERGA document contains 190 references to conference papers or journal articles in the mathematics education research literature. The AMSI submission contains 32 footnotes, of which one is a reference to a published conference paper. The emphasis on research suggests that the MERGA response sees its recommendations as much more dependent on evidence garnered from its members' and other researchers' contributions than does the AMSI document.

The concept *learning* appears frequently in the MERGA document, but not in the AMSI document. This again places an emphasis on people, as learning necessarily depends on interactions between teachers and students. This does not, of course, suggest that the AMSI document devalues concepts such as learning or the people involved in the learning process. It merely suggests that knowledge is seen to be a priority.

The above analysis does not purport to be a complete analysis of the documents. There are other concepts that could be discussed, and further work could be done in identifying other aspects of the text such as its ideational, interpersonal and textual function (Morgan, 2006). Nor does the analysis purport to represent the intentions of the authors of the documents, which requires further research such as interviews, or indeed the views of mathematicians and mathematics educators more generally. However the analysis is indicative that the groups do have different modes of knowledge legitimation, and that these differences are worthy of further investigation.

Conclusions

The analysis of these documents has shown marked differences in the construction of the epistemic device within the mathematics and mathematics education research communities. These differences have implications for the future of mathematics education in schools, in that each group has a legitimate claim for representation and input in the development of curriculum and in setting the agenda for school mathematics.

Debates over the introduction of national curriculum frameworks in Australia in the early 1990s have been well documented (Ellerton & Clements, 1994), as have the arguments promulgated in the so-called US Math Wars (Klein, 2007). The national curriculum frameworks in Australia spurned the formation of the Australian Mathematical Sciences Council (AMSC), which was an attempt to speak with one voice. However the AMSC was beset by internal divisions, resulting in the withdrawal of the Australian Association of Mathematics Teachers. Although the desire to speak with one voice is commendable, perhaps ultimately the knowledge legitimation codes in mathematics and mathematics education research make such a goal not only difficult but epistemologically impossible. Rather it may be more constructive, at a time when debate around national curriculum is set to increase, to see these as different but complementary voices.

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