

Gaining Insight into Alice's Pedagogy with Respect to Five Dimensions of Numeracy

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Numeracy is a complex construct and has become an essential capability for the twenty-first century. This paper reports the case of one teacher, Alice, with respect to her positioning of numeracy in a reform curriculum. By analysing her conversations, Alice's numeracy pedagogy was described and visually represented according to five dimensions of practice. The case of Alice, as presented in this paper, demonstrates the potential for gaining insight into the numeracy beliefs and practices of individual teachers and for further understanding of the complex nature of numeracy teaching.

Curriculum reforms occurring in Tasmania over the past five years have brought a focus on pedagogy within the mathematics classroom. The importance of being numerate has been emphasised rather than solely knowing and doing mathematics. Tasmania's Essential Learnings curriculum framework (Department of Education, Tasmania [DoET], 2002) put thinking skills and strategies at the centre of a values-based curriculum where the connection of knowledge and concepts across the curriculum was encouraged. The reforms have matched those occurring nationally and internationally in mathematics education in terms of a shift from procedural teaching approaches to a conceptual approach focusing on student thinking and reasoning (Anderson & Bobis, 2005).

The pedagogical shift required of teachers in reform environments has required that teachers extend their knowledge of mathematics to include what Shulman (1987) described as pedagogical content knowledge (PCK). PCK goes beyond knowledge of the subject matter itself, to knowledge of how to teach the subject, knowledge of curriculum and resources, and knowledge of how students learn. Teachers' knowledge, beliefs, and practices significantly influence student learning (Hill, Rowan, & Ball, 2005) and it is important to examine the pedagogy of teachers with respect to the teaching of numeracy.

The aim of this study is to describe the numeracy pedagogy of one teacher, Alice, to gain insight into the idiosyncratic nature of numeracy pedagogy, and to consider the implications for the teaching of numeracy.

Theoretical Framework

Numeracy is accepted as having its foundations in mathematics (Australian Education Council, 1990). Beyond this there are many and varied definitions of numeracy, each with its particular theoretical underpinnings. Just as Green (2002) advocated a synthesis of the operational, cultural, and critical dimensions that play a role in the development of literacy, consideration of the different dimensions that are necessary for the development of competent and effective numeracy practice is important. Mathematical language, skills, and functions are required for students to make sense of, and critically evaluate, the contexts in which the mathematics is embedded. The socio-cultural and critical aspects of knowledge construction enable the selection of appropriate mathematical tools and informed critique of both mathematics and society. This study is informed by social constructivist theory (Prawat, 1996; Shepard, 2001), drawing from contemporary cognitive, constructivist, and socio-cultural theories and acknowledging the important contribution each element brings to a comprehensive definition of numeracy.

A comprehensive review of the literature has resulted in the development of a view of numeracy, incorporating five dimensions of practice, presented in Table 1 (Skalicky, 2007). In particular, the work of AAMT (1998), Steen (2001) and Queensland School Curriculum Council (1999), in presenting comprehensive balanced views of numeracy extending across foundational mathematical concepts and skills, strategic thinking, disposition, recognition of context, and critical practice, informed the description of the five dimensions. Essential to all these conceptions of numeracy is the view that mathematics is a vital tool in today's society, a tool that should be accessible to all members of society. The conceptions also acknowledge the complexity involved in numeracy and the many aspects, beyond mathematical skill, that contribute to a high level of numerate behaviour.

Table 1*Dimensions of Numeracy (Skalicky, 2007, p. 3)*

Aspects of knowledge construction	Dimensions of numeracy	Description
FOUNDATIONAL	Mathematics	The foundational understanding and use of the language, concepts, and skills of mathematics, as they relate to number, measurement, space, data and chance, and pattern and algebra.
PSYCHOLOGICAL	Reasoning	The use of (i) mathematical thinking strategies to question, identify, represent, explain, and justify mathematical approaches relevant to a given context, and (ii) general thinking strategies to support the problem solving process, from lower level cognitive processes, such as recall and application, to higher level critical thinking processes involved in evaluation, judgment, decision making, and creativity.
AFFECTIVE	Attitude	The confidence and disposition to choose and use mathematical understandings wherever required. Willingness to take risks and persevere in approaching new mathematics and new contexts.
SOCIO-CULTURAL	Context	The ability to select and apply the appropriate mathematical tools for sense-making in a given context and understanding how the context impacts on the mathematics. Contexts related to school and everyday life, public and social issues, and an awareness of mathematics connected to history and culture.
CRITICAL	Equity	Awareness that mathematics can be used inappropriately, can be represented to promote bias, and can therefore promote inequities in society. The ability to question assumptions and use mathematics in an analytical and critical manner to make decisions and resolve problems and investigations.

Method

The research reported in this paper forms part of a larger PhD project, investigating the positioning of numeracy by teachers of middle grade classrooms (Grades 5-8) in Tasmania's reform environment and student experiences of numeracy in these classrooms. The larger research project adopted a collective case study approach (Yin, 2003), with four case studies involving five participant teachers and their students. All teachers planned and implemented units of work informed by Tasmania's Essential Learnings curriculum framework (DoET, 2002). These teachers were positive toward the broader curriculum reforms, and in particular had an interest in the teaching of numeracy. In addition, a representation of middle years grades was sought across a range of schools, both government and independent. The larger study used a combination of interview, observation, document, and photographic data to provide insight into the positioning of numeracy as enacted in the classroom by each teacher and the experiences of her students.

In this study the numeracy pedagogy of one teacher, Alice, is reported, based upon interview data. The teacher interviews were semi-structured and lasted approximately 40-50 minutes. They were designed to gain an insight into teacher beliefs and practices with respect to current curriculum reforms, views concerning the place of numeracy within these reforms, and the teachers' planning, teaching, and assessment practices.

Cluster analysis (Miles & Huberman, 1994, p. 248) was used to code the interviews. Segments were clustered according to the five dimensions of numeracy as detailed in Table 1 of the theoretical framework. Further analysis was undertaken, informed by Bloom's revised taxonomy (Anderson & Krathwohl, 2001), to categorise

each dimension of numeracy as being low, moderate, or high in terms of the level it was displayed by the teacher. Table 2 presents an overview of the levels of performance that informed the second level analysis of the teacher interviews.

Table 2

Exhibited Levels of Numeracy Pedagogy

Coding Level	Degree to which dimension is exhibited	Key aspects illustrating the levels of numeracy pedagogy
0	N/A	Not demonstrated.
1	Low	Awareness demonstrated by describing and explaining concepts and ideas.
2	Moderate	Application in the classroom is described, using specific examples. Includes evidence of examination of practice.
3	High	Multiple examples used and present consistent demonstration of the dimension, with justification supported by an underlying philosophy. Critical reflection and evaluation of practice are evident.

Radar charts are used as a means of visually representing the numeracy pedagogy of each case at the time of the study and in context of the unit of work being taught. The radar chart was chosen due to the opportunity it provides to observe deviations in the degree to which each dimension of numeracy was evidenced.

Context of the Study

Alice was a Grade 5/6 primary school teacher. In 2004, Alice had participated in a DoET targeted professional learning program for teachers of middle years students, Grades 5 – 8. The program, *Being numerate in the middle years*, occurred over six days and brought together 48 teachers from across Tasmania. It explored the role of numeracy, planning and structuring numeracy learning, thinking and working mathematically, mental computation, and the important middle years concepts of proportional reasoning; average; and fractions, decimals, and percents (Watson, Beswick, Caney, & Skalicky, 2005). Alice experienced a major shift in her beliefs and practices concerning numeracy teaching and learning as a result of participating in this program. Following this, Alice worked toward changing her classroom environment and her numeracy teaching.

Alice participated in the research project approximately six months after her involvement in the program. Alice's beliefs and practices concerning numeracy and its place in the curriculum were explored through the teacher interviews, documentation, and observational data. This study is concerned with the interview data only.

During the research, Alice taught a three-week numeracy unit on graphing. This learning took place during three of the four weekly numeracy sessions that were part of Alice's usual program. Alice usually ran her numeracy program alongside an integrated unit of work or learning sequence. It was Alice's numeracy unit that formed the focus of the study. The motivation for the numeracy unit came from a concern that arose about the students' difficulties in interpreting and representing information during an integrated unit of work designed to explore the unique features of the Australian continent. Alice wanted her students to gain the understanding that there are a variety of ways that information can be represented and to have the knowledge and skills to make and implement those choices from an informed perspective.

Results

Alice was working on her numeracy pedagogy and bringing those aspects of her teaching that came naturally to her in other curriculum areas, particularly literacy, into her numeracy teaching. Since the introduction of the Essential Learnings, Alice was aware of and described some significant changes that were occurring in her practice. Alice had begun to challenge her own ideas of mathematics teaching and was working with students to help them see the connections among concepts, for example, fractions, decimals, and percentages, rather

than teaching them separately. Although Alice still found numeracy to be an area that did not naturally sit in her integrated learning sequences, she had made a significant shift in her focused numeracy time.

I was teaching the stuff, or some stuff, but I wasn't teaching them for understanding ... my planning [now] reflects key concepts and ideas, not just well today we'd better do some addition but thinking about what are the big ideas that I want these kids to know.

The following subsections detail how Alice's conversation about her teaching could be described according to the five dimensions of numeracy as stated earlier in the Theoretical Framework in Table 1.

Mathematics. Alice expressed a sense that numeracy was more than the mathematical skills that she had previously focused upon. Alice revealed some uncertainties in her own mind about the distinctions between mathematics and numeracy, and also whether such distinctions were important. At the same time, Alice described numeracy as "that whole notion of using mathematics and transference of that kind of knowledge".

When Alice was describing the structure of her numeracy time she referred to focused time at the beginning of each numeracy block, for the development of number sense, and the inclusion of "explicit teaching time" where the strands of mathematics, including measurement, chance and data, pattern and algebra, and space where explicitly taught, with her overall objective being to plan "for understanding of key mathematical concepts and ideas". In sharing her discontent with seeing her mathematics teaching as disconnected from her other work, Alice evidenced her belief that mathematics has a role to play in understanding ideas and concepts planned for in other areas of the curriculum. She wanted to do this in a "positive way that [was] not a contrived way".

Alice sought to equip her students with the "language of mathematics" so that they could share effectively their strategies and solutions. The emphasis she placed on student understanding of important mathematics concepts was evident in Alice's conversations. For example, she discussed the concept of "doubling", and wanting to support her students in making connections between mathematics concepts, by considering the meaning and application of doubling in terms not only of number, but also in considering measurement and pattern relationships.

For Alice, mathematical skills also remained important: "I see these skills that you have to teach, data and how do we read and how do we collect and all those sorts of things ... so many skills that we have to logically work through". With respect to the foundational mathematics in her thinking and program, Alice displayed a *moderate* level of absorption in terms of the criteria described in Table 2.

Reasoning. The development of mathematical thinking and reasoning played a very important role in Alice's classroom. She was very interested in seeing how the language of thinking applied to mathematics, with students often asked to "justify" their position or choice of a particular strategy or to "elaborate" on their thinking. Students in Alice's classroom were given the freedom to select and apply problem solving strategies. This had driven the shift in the culture of the classroom with students "beginning to see themselves as problem-solvers".

Alice had participated in and led many professional learning sessions in the field of Thinking, as a result of the curriculum reforms. She was very interested in the application of this within the numeracy classroom. Alice worked with her students to use tools to support their thinking within mathematical settings. For example, cooperative learning strategies, such as jigsaw techniques (Aronson & Patnoe, 1997) and think-pair-share activities (Kagan, 1994) were specifically taught to provide students with tools to share and describe their own thinking. Time was allocated at the end of every numeracy session for the whole class to share and to reflect upon and articulate their learning. Alice was clear about the outcomes that this shift in her teaching practice brought to student learning.

They are actually making connections and seeing a range of possibilities ... they are making those connections for themselves. ... They are moving away from that memorisation ... that rote learning being their only strategy.

In terms of the psychological reasoning dimension of numeracy, Alice displayed a *high* level of absorption and implementation.

Attitude. In sharing the changes in her numeracy pedagogy, Alice identified a distinct connection between these changes and the changing attitude of her students toward their mathematics learning. She reflected upon the previous general view of her students in “[not] seeing themselves as capable in the area of mathematics”. Alice had transformed not only the structure of her numeracy time and the classroom learning environment but she had also found that her own attitude to mathematics teaching had undergone a shift wherein numeracy was highly valued and seen as an important component of the curriculum. She was no longer accepting disruptions from outside during this time, “I had to change what was going on and let the kids see that this is really important to me and make it really important”.

Alice wanted to build a “community of learners” where students shared and reflected upon their learning and she described these times as “exciting” for both her and her students. Her efforts to “create a learning environment . . . so that the kids will take risks” were resulting in the students “taking chances in the classroom”. Alice believed that there was “real potential for these kids to be self-motivated, confident, articulate users of mathematics”.

Students are now free to manipulate numbers, they were frightened my kids, of numbers because I was going to ask them something really hard and they never put their hands up. Now they play around with numbers and they talk about numbers and the conversations in my classroom are really exciting.

The increasing development of a positive attitude toward mathematics for Alice and for her students was very entrenched in the changed culture of the classroom and the embedding of thinking within mathematical learning. With respect to the affective dimension of numeracy, Alice displayed a *high* level of absorption.

Context. In describing numeracy, Alice felt that “everything [had] to be in context to be meaningful”. She was “trying to make [mathematics] meaningful and connected with the real-world”. Her initial focus was on the transfer of number calculations to real-world situations. Alice also gave an example of using cooking to further develop students’ understanding of the concept of doubling, and discussed the relevance of doubling in terms of measurement and use of recipes.

She still felt that “things [were] a bit contrived” and although she valued the transference of students’ mathematical knowledge to new contexts she expressed a desire to work on the implementation of this area in more meaningful ways. Alice described her biggest challenge as incorporating numeracy into her planning of transdisciplinary units of work informed by the Essential Learnings framework and her desire to “drag mathematics out there into my learning sequences in a more “positive way that is not a contrived way”. Alice displayed a *moderate* level of the context dimension of numeracy.

Equity. With Alice focusing on the development of understanding of mathematical concepts and building a classroom community that would enable students to begin to explore mathematical strategies this higher level element of critical engagement was not yet a part of Alice’s pedagogy.

Alice’s overall numeracy pedagogy at the time of the study is represented for the five dimensions, in the radar chart in Figure 1.

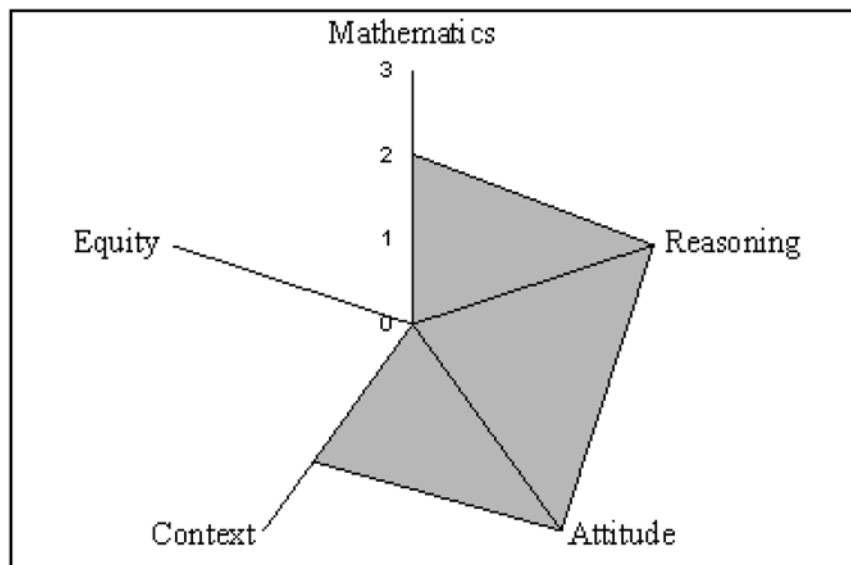


Figure 1. Alice's numeracy pedagogy.

Discussion and Conclusion

Alice was aware of and described some significant changes that were occurring in her practice. She was equipping her students with the language of both mathematics and thinking to support their participation in the learning environment. Alice was encouraging students to use and share their own strategies for problem solving and to develop understanding of mathematical concepts.

Alice was exploring ways that she could naturally connect mathematics to her integrated learning sequences, but found this challenging. She was therefore focusing on her dedicated numeracy block and the learning experiences she was providing for her students during this time. Alice felt that explicitly teaching thinking strategies and related language was important for students as it enabled them to approach tasks in a purposeful and meaningful way. The classroom learning environment was important to Alice, as she worked hard to establish a community of learners. The opportunity to work continually on this aspect of her teaching was a feature of Alice's practice.

The high levels exhibited by Alice in the dimensions of *Attitude* and *Reasoning* were reflections of her focus in these areas as she worked hard to develop her numeracy teaching within the reform environment. Alice's numeracy pedagogy, as presented in this study, provides a snapshot of her pedagogy at a point in time and in the context of a particular unit of work.

This study has examined the numeracy pedagogy of one teacher; distinguishing levels for each of the five dimensions, and when reported graphically, giving a visual representation of numeracy pedagogy at a point in time. The example case of Alice demonstrates the possibilities for considering a holistic view of numeracy teaching and the potential it provides to gain an in-depth understanding of the complex nature of numeracy teaching.

Alice's case is one example of the four case studies that formed part of the larger research project. All the teachers who participated in the larger project were recognised by their respective schools as being effective teachers of numeracy and all presented with distinctively different numeracy pedagogies from that of Alice. The four cases will be reported together at a later date.

Researchers in both Australia and the United Kingdom have identified key practices that are indicative of highly effective teachers of numeracy (Askew, Brown, Rhodes, Johnson, & Wiliam, 1997; Clarke & Clarke, 2002). This study seeks to acknowledge that, within the breadth of the many effective teachers of numeracy in today's classrooms, each teacher has their own idiosyncratic numeracy pedagogy with different strengths and areas of focus.

Recognising that teacher pedagogy is a changing construct, the results presented enable the distinction to be made within and across the different dimensions of numeracy both descriptively and visually. It is hoped that in describing numeracy pedagogy in this way it may help mathematics educators and teachers to continue to evaluate, reflect upon, and improve the teaching and learning of numeracy.

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