Investigating a Phase Approach to Using Technology as a Teaching Tool

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Technology, such as dynamic geometry software (DGS), is available in many secondary mathematics classrooms. Whilst studies have highlighted the motivational aspects of DGS to explore geometric concepts, there exists a need to explore specific strategies for using technology in the mathematics classroom as a teaching tool, as opposed to a display tool. The project presented explored a phase approach to incorporate technology into the teaching/learning cycle to facilitate developmental progression in a secondary mathematics classroom. This paper presents the findings of a project which linked theory and practice within a technological environment.

Over the past thirty years there have been numerous studies that have characterised the nature of levels of understandings in Geometry (Burger & Shaughnessy, 1986; Currie & Pegg, 1998; Fuys, Geddes, & Tischler, 1985; van Hiele, 1986; Serow, 2002). In particular, the van Hiele Theory (van Hiele, 1986) is comprised of a five level framework of development in Geometry from which to view students' understandings. Some studies have extended this work and focused upon developmental pathways and identified hurdles leading to higher-order thinking (Serow, 2002, 2007a). There is an urgent need, however, to explore teaching/learning practices that facilitate student developmental progression. This exploratory study, is part of a larger study which explores the effectiveness of dynamic geometry software as a teaching tool. The activities are structured within the van Hiele teaching phases framework.

The project presented extends previous work which identified developmental pathways associated with class inclusion concepts (Serow, 2002). These pathways highlighted the difficulties associated with students' attempts to understand and utilise networks of relationships in geometry. Serow (2002) highlighted the reasons students find class inclusion concepts in Geometry difficult to grasp, and detailed the hurdles encountered by many students through the characterisation of the development of relationships among figures and relationships among properties. The philosophical stance taken by van Hiele in regards to teaching Geometry is grounded within the notion of insight. The opportunity to exhibit and develop insight is described by van Hiele (1986) as the aim of teaching mathematics. Thus, for the promotion of growth in understanding, learners require geometrical tasks that allow them to control their individual problem-solving environment (Hoffer, 1983, p. 205). Essentially, Dynamic Geometry Software (DGS) provides the potential for student-centred problem-solving tasks that remain in the control of the individual student. DGS allows the "continuous real-time transformation often called 'dragging'. This feature allows users, after a construction is made, to move certain elements of a drawing freely and to observe other elements respond dynamically to other altered conditions" (Goldenberg & Cuoco, 1998, p. 351).

The tools within The Geometer's Sketchpad, Version 4.0 (Jackiv, 2001) a form of DGS, provide teachers with the opportunity to explore the relationships of quadrilateral figures and properties both intuitively and inductively. Goldenberg and Cuoco (1998, p. 396) found the "dynamic nature of these tools makes them both exciting and accessible, even to elementary students" (p. 396). In addition, dynamic geometric investigations are possible when students have time to consider their mathematics ideas as opposed to concentrating on the technicalities of pen and paper constructions (Tikoo, 1998).

It has been contended by McGehee and Griffith (2004) and Coffland and Strickland (2004) that teachers need to focus on the ways that technology may enhance mathematical thinking and enhance conceptual understandings. Many teachers are comfortable using technology to display material but often lack confidence in sequencing technological tasks as an integral component of a teaching/learning sequence. A teaching framework that has the potential to address this need is the basis of the work of Dina van Hiele-Geldof (van Hiele, 1986). The five teaching phases represent a framework to facilitate the cognitive development of a student through the transition between one level and the next. The van Hiele phases are centred on the notions that progress is easier for students with careful teacher guidance, the opportunity to discuss relevant issues, and the gradual development of more technical language. The phases are organised in such a way that they acknowledge the assumptions underpinning the van Hiele levels, while providing students with the opportunity to exhibit insight. The van Hiele teaching phases address the concern that "teachers often feel reluctant or uncomfortable because their pedagogical knowledge perhaps does not include a framework for conducting technology-based activities in their lessons" (Chua & Wu, 2005, p. 387). A description of each of the five van Hiele teaching phases is summarised in Table 1.

Table 1

Descriptions of the van Hiele Teaching Phases

Phase	Description of Phase Focus
1. Information	For students to become familiar with the working domain through discussion and exploration. Discussions take place between teacher and students that stresses the content to be used.
2. Directed Orientation	For students to identify the focus of the topic through a series of teacher-guided tasks. At this stage, students are given the opportunity to exchange views. Through this discussion there is a gradual implicit introduction of more formal language.
3. Explicitation	For students to become conscious of the new ideas and express these in accepted mathematical language. The concepts now need to be made explicit using accepted language. Care is taken to develop the technical language with understanding through the exchange of ideas.
4. Free Orientation	For students to complete activities in which they are required to find their own way in the network of relations. The students are now familiar with the domain and are ready to explore it. Through their problem solving, the students' language develops further as they begin to identify cues to assist them.
5. Integration	For the students to build an overview of the material investigated. Summaries concern the new understandings of the concepts involved and incorporate language of the new level. While the purpose of the instruction is now clear to the students, it is still necessary for the teacher to assist during this phase.

(Serow, 2002, p. 10)

The five-phase teaching approach provides a structure on which to base a program of instruction. As can be seen, the phase approach begins with clear teacher direction involving exploration through simple tasks, and moves to activities that require student initiative in the form of problem solving.

Serow (2002) identified a generic developmental pathway leading to an understanding of class inclusion concepts, incorporating networks of relationships, in Geometry. The cognitive processes undertaken by learners, and hurdles met along that path have been articulated. Whilst it is essential to have a framework as a content analysis tool, the framework is not the focus of this project. Table 2 outlines the categories of responses, in ascending order of complexity, concerning relationships among quadrilateral figures which provided the basis for analysis of student responses in the reported project.

Table 2

Categories of Responses Concerning Relationships Among Quadrilateral Figures

Category	Characteristics of Responses
A	A single property or feature is identified to link the figures. The focus of the response is upon the identification of an observed single quantifiable aspect, which places figures into spontaneous groups. There is a strong reliance on visual cues.
B	Classes of figures are known by name and are characterised by a single property. The class represents an identifiable unit. Links do not exist between classes, unless supported by visual cues. Observed differences in traditional shapes, such as right angles, play a significant role.
C	Similar to the Category B response above, the Category C responses incorporate classes of figures, which are known by name. These classes are characterised by more than one property. Links are not made between classes where differences in properties are accentuated by visual differences.
D	Relationships exist between classes of figures, which are based upon similar properties. Inclusive language is used to describe the classes of figures; hence, property descriptions allow for similarities to be acknowledged.
E	When prompted, tentative statements are made concerning the possibility of subsets within a class of figures. There is no acceptance of this notion, however, it is able to be discussed tentatively.
F	There is an unprompted acceptance of a class of figures containing subsets. While this notion of class inclusion is accepted and utilised, it is not justified adequately.
G	The notion of class inclusion is an integrating feature of the response. A class of figures incorporates subsets, which are inclusive of generic categories identified by other names. Each class maintains a workable identity while the focus is upon the network of relationships based upon the properties of each class.
H	The notion of class inclusion acquires further development. Conditions are placed upon the classes of figures, which acknowledge more than one system of relationships. This requires an overview of the interrelationships among classes and their subsets, which utilises subsets within subsets, and precludes inappropriate examples of figures.

(Adapted from Serow, 2002, p. 214)

In consideration of the background provided, the natural progression presented in this paper is a focus on suitable teaching strategies to assist students in meeting and rising above the identified cognitive hurdles within a structured technological environment. The research questions for this study were:

1. Is the van Hiele teaching phases framework an effective structure for designing teaching sequences involving dynamic geometry software?
2. To what extent does the implementation of student-centred tasks, which utilise dynamic geometry software, facilitate student growth in understandings of relationships among quadrilateral figures?

Method

This study uses a pre-experimental design with one group (23 students), involving pre-tests and post-tests (Cohen & Manion, 1994, p. 165). In addition, the design incorporated a delayed post-test to assess longitudinal retention of demonstrated understandings. The teaching sequence was designed with two main elements; the teaching phases as a design framework, and the embedding of dynamic geometry software, in conjunction with spreadsheets (Excel) and concept mapping software (Inspiration). The intervention involved a two-week teaching sequence (eight sessions of forty minute duration) and was delivered to a Year 9 (ages 14–15) secondary mathematics class. The focus content strand of the teaching sequence from the K-10 Mathematics Syllabus (Board of Studies, 2002) was Space and Geometry and the target outcomes addressed by the teaching sequence were "classify, construct, and determine properties of triangles and quadrilaterals" and "verify the properties of special quadrilaterals" (Board of Studies, 2002, p. 39). The teaching sequence aimed to integrate dynamic geometry software using the van Hiele teaching phases as a framework (van Hiele, 1986) for maintaining student ownership of their mathematical ideas. This teaching sequence included student-centred tasks that aimed to acknowledge the progression from informal to formal language use. The pre and post-tests were in the form of written tasks. These tasks comprised of open-ended items (Cohen & Manion, 1994, p. 277) to elicit qualitative student responses at the commencement and completion of the teaching sequence. A written delayed post-test, delivered to each student in the sample, was implemented to determine the retention rate of conceptual understanding. All responses to open-ended items were categorised via the identified developmental pathways associated with understandings of relationships among figures (Serow, 2002).

Teaching Intervention

The two-week teaching sequence (Serow, 2007b), which is detailed below in Table 3 is sequenced using the van Hiele teaching phases. Each activity described includes the target teaching phase.

Table 3

Teaching Sequence

Activity and Phase	Activity Description
Information Phase and Directed Orientation Phases Activities 1: Mechanics and Recall	1. Students work through simple constructions in Sketchpad and brainstorm known quadrilaterals. Constructions involve: a) Write your name using sketchpad. b) Create a person and reflect the person. Measure a selection of corresponding sides and angles. What do you notice when you drag one of your people? c) Create a house design using the six quadrilaterals, namely, kite, trapezium, square, rectangle, rhombus, and parallelogram. At this stage, the students, in most cases, will construct their figures using the line tool. This will be extended in later phases. When the students are asked to drag (drag test) the quadrilaterals they have formed this way, they will notice that the constructions are not robust (do not remain the intended figures).
Explicitation Phase Activities 2: Robust Templates and Recording	2. Students create robust templates for each of the six quadrilaterals on separate Sketchpad pages. If the drag test allows the figure to remain as intended, the construction will involve known properties of each figure. Discussions will begin to occur concerning relationships among figures. For example, comments such as 'this is really strange, when I drag the parallelogram it is sometimes a rectangle, square or rhombus'. This activity will involve constructions such parallel lines, perpendicular lines, and transformations. It is essential for the students at this phase to describe their construction within a text box on Sketchpad and to record the properties for each quadrilateral on a teacher-designed table.

Directed Orientation Phase Activity 3: Irregular Quadrilateral and Midpoint Construction	3. Students are instructed to: a) create any irregular quadrilateral using the line tool; b) construct the midpoints; c) join the midpoints to construct another quadrilateral; d) answer the question, What do you notice?; and, e) investigate the properties off this shape to justify what you have found, and record your justification in a textbox.
Explicitation Phase Activity 4: Further Exploration of Properties and Figures	4. Students design a spreadsheet where the six quadrilaterals are contained in the first column, and the first row contains all possible properties of quadrilaterals. Particular care needs to be taken to include diagonal properties such as 'diagonals meet at right angles'. The students record the properties of each figure by ticking the appropriate cell. There is an element of surprise in the classroom when the students notice that the square has the maximum number of ticks.
Free Orientation Phase Activity 5: Diagonal Starters Game Design	5. This activity is designed to reinforce diagonals as a property and not merely a feature of the quadrilaterals. Students are given the challenge to create the diagonal formation needed for each of the quadrilaterals. The aim is for the students to construct templates for younger students to complete the figure and explore the properties.
Free Orientation Phase Activity 6: Concept Maps and Flow Charts	6. Students create; a) a concept map b) a flow chart, to summarise their known relationships among quadrilateral figures.
Integration Phase Activity 7: Information Booklet Design	7. Students organise the constructions that they have made, justifications, tables, spreadsheets, concept maps, and flowcharts to produce an information booklet to explain what they know about the relationships among quadrilaterals and relationships among quadrilaterals figures. Students are instructed to include an overall summary of their findings.
Integration Phase Activity 8: Sharing and Routine Questions	8. Class sharing of booklet designs. Routine questions involving known properties and relationships.

The teaching sequence was presented to the class using a team-teaching approach involving their classroom teacher and researcher. Each student had individual access to a computer and relevant software.

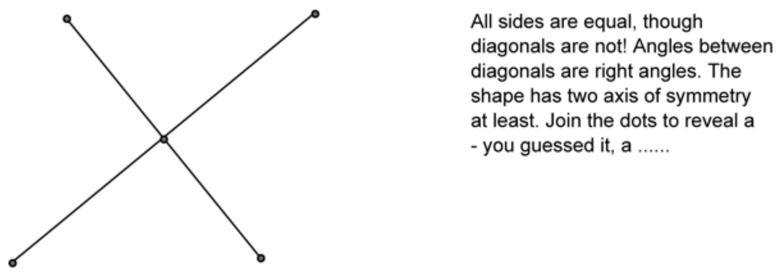
Instrument

The pre-test, post-test, and delayed post-test included the same set of items. The items included are summarised below in script form:

1. Int: Write a list of all the quadrilaterals that you know above the line.
2. Int: We will now check to see if we have them all. Did you include rectangle, square, parallelogram, rhombus, kite, and trapezium?
3. Int: Below the line, write the shapes that you may have missed.
4. Int: We are going to create a concept map with each of these quadrilaterals. You will have enough time to include all the information that you feel is important.
5. Int: Draw a diagram to illustrate each quadrilateral. Make sure your drawings clearly indicate each quadrilateral. Draw lines to indicate relationships among the quadrilaterals. Use circles if you would like to show groups. Write your reasons for the groups you have identified. Write one paragraph justifying the manner in which quadrilaterals are related to one another.
6. Students were asked to comment (in written form) on the following two scenarios.
   Scenario 1: John states to the class "The square is a rectangle". Do you agree or disagree? How could he justify this statement if he was asked to explain it?
   Scenario 2: Megan writes on her paper that "The rhombus is a parallelogram".

Results and Discussion

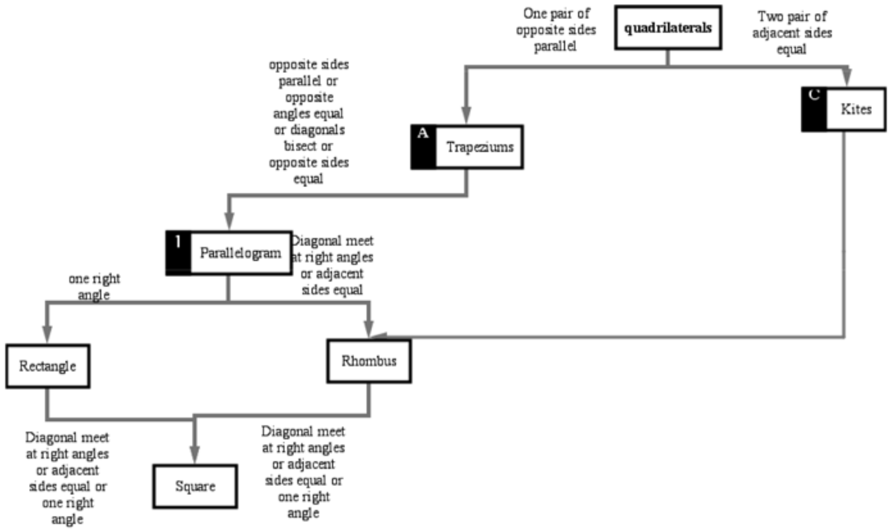
In relation to research question 2, the van Hiele teaching phases was an effective design framework for sequencing activities that involved dynamic geometry software. In addition to the overall increase in complexity of the student responses at the completion of the intervention, the students remained on-task through each of the activities and conversed with one another as they gradually moved from informal to formal language use. The following student samples typify the tasks completed by the students when immersed in the activities. Figure 1 below is a student sample of one section of the diagonal starter's game (free orientation phase) where the task focussed the students constructions on diagonal properties and the resultant figures and properties.



(Serow, 2007b, p.386)

Figure 1. Sample of student's diagonal starters diagram.

It was evident that the students were implicitly placed in a situation where they were required to use the properties of the figures if their constructions were to remain the intended figure when 'dragged'. The phase approach provided an avenue to design and implement activities which assisted in making the properties and relationships among the properties the explicit focus of the student activities. Hence, the use of spreadsheets for recording, text boxes for student recording of findings, and concept maps/flow charts played an important role in 'making the most' of the DGS activities. Whilst the concept mapping activity (free orientation phase) included typical venn diagrams, the flow chart design facilitated an environment where students considered the properties and figures that they had explored in the DGS environment and organised them into an hierarchical structure. A typical student response to this task is contained in Figure 2.



Serow (2007b, p.387)

Figure 2. Sample of student's flow chart diagram.

Table 4 details the coding for each of the students' responses to the pre, post, and delayed post-tests. When comparing the results of the pre and post-tests it is evident that the students' understandings of the relationships among quadrilateral figures did change after the teaching intervention. In reference to research question 2, using dynamic geometry software as an integral component of the student-centred activities did result in overall growth in understanding when considering the group as a whole.

Table 4

Relationships Among Figures

Category	A	B	C	D	E	F	G	H	Total
Pre-test	11 (48)	4 (17)	4 (17)	3 (13)			1 (4)		23 (100)
Post test		5 (22)	4 (17)	7 (30)	2 (9)		1 (4)	4 (17)	23 (100)
Delayed post test		5 (22)	4 (17)	7 (30)	2 (9)		1 (4)	4 (17)	23 (100)

Percentages of the sample for each category in each test are included in brackets.

In the pre-test, 48% of responses focussed on a single feature or property with a reliance on visual cue when attempting to describe relationships among quadrilateral figures In the post-tests, none of the responses were of this nature. The pre-test also indicated that 17% were characterising a class of quadrilaterals by a single property, and 17% were focussing on more than one property. Respectively, in the post tests, these figures were 17% and 30%. In the post-test a larger percentage of students were focussing on the relationships among classes of quadrilaterals based on similar properties. Overall, in the pre-test, only 4% of responses focused on the notion of class of inclusion (Categories G and H) and in the post-tests this has risen to 21%. Of this 21% of responses, 17% focussed on the placement of conditions upon the class of figures which enabled subsets within subsets. It is particularly interesting to note that the coding for the post-test and delayed post-test were consistent across each individual student.

Conclusion

This project aimed to undertake an exploratory teaching experiment to provide base line data on the effectiveness of dynamic geometry software to facilitate student growth in understandings of networks of relationships in geometry. A fundamental aspect of the project was the melding of cognitive frameworks, phases of teaching, and the embedding of Information and Communication Technology within a teaching sequence. This study highlights the importance of embedding technology within a pedagogical framework. In terms of mathematics education research as a whole, it raises interest in exploring the melding of existing theoretical frameworks with emerging technological tools that are currently available to secondary students.

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