Primary Teachers' Perceptions of Their Knowledge and Understanding of Measurement

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This study focused on primary teachers' perceptions of their knowledge and understandings of length, area and volume. It also explored their understanding of how children's growth of measurement concepts and processes develops. Data gained from in-depth interviews revealed that teachers' knowledge was often implicit and that they struggled to articulate their knowledge of measurement concepts and children's trajectories of learning.

Over the past decade an increasing amount of research has been concerned with the key concepts and skills that students need to understand measurement–particularly, relating to length and area (e.g., Barret, Jones, Thornton, & Dickson, 2003; Outhred & McPhail, 2000). However, relatively little is known about teachers' understandings of these concepts and how their knowledge impacts on their teaching practices.

The research reported here explores primary teachers' perceptions of their mathematical content knowledge and of how children's understanding of length, area and volume develops. It is also concerned with teachers' perceptions of how this knowledge impacts on their pedagogy.

Background to the Study

Measurement is a central component of primary and secondary school curriculum documents around the world (e.g., National Council of Teachers of Mathematics, 2000). Consequently, there is an extensive body of research relating to the teaching of length, area and volume. The majority of this research centers on the key measurement concepts or principles that need to be understood by students and teachers such as unit iteration, attribute identification and the use of formal and informal units (e.g., Lehrer, 2003). However, a number of studies also explore effective teaching strategies for these content areas (e.g., Bragg & Outhred, 2000; Outhred & McPhail, 2000). Teaching strategies and concepts involving length, area and volume are similar due to the nature of knowledge development. Although some research exists on teacher knowledge of length, area and volume, this area of investigation is still quite limited (Ball, 1990). Current literature suggests that teachers' content knowledge effects student learning and is improved by professional development (e.g., Hill & Ball, 2004). The literature surrounding the development of key concepts in measurement is linked to that of teacher knowledge due to the commonly accepted view that teachers must understand these key concepts in order to be effective teachers of mathematics.

"There is a considerable body of evidence showing many secondary students do not have a thorough knowledge" (Outhred & McPhail, 2000, p. 488) of length, area and volume measurement. This evidence includes results of comparison tests such as TIMSS (Hollingsowrth, Lokan, & McCrae, 2003) and PISA (Lokan, Greenwood, & Creswell, 2001) that show Australian secondary students struggle to understand simple linear measurement (Outhred & McPhail, 2000; Outhred, Mitchelmore, McPhail, & Gould, 2003). In particular, studies have shown that students do not understand the attribute being measured or the units that are used for measurement (Outhred & McPhail, 2000; Outhred et al., 2003). Such evidence raises questions about the effectiveness of the instruction students are receiving within these content areas.

Research in the domain of length, area and volume has highlighted a number of general principles or concepts that underlie the understanding of measurement. These principles are important for both teachers and students. One of the more recent principles that research has emphasized is that of unit iteration (Barrett, Jones, Thornton, & Dickson, 2003; Lehrer, 2003). Unit iteration refers to the knowledge that when measuring, a single repeated unit needs to be used in a way that leaves no gaps and causes no overlap (New South Wales Department of Education and Training [NSWDET], 2003). Recent research has emphasized unit iteration due to the fact that many teachers do not highlight this principle within the classroom (Outhred & McPhail, 2000). Combined with the basic understanding of how to use units when measuring, unit iteration is important in ensuring that children gain more than a just procedural knowledge of length, area, and volume.

In addition to understanding how to use units in measurement, a number of other measurement principles have been identified. These key concepts generally involve:

- Conservation (NSWDET, 2003)
- Attribute identification (Clarke, Cheeseman, McDonough, Clarke, 2003; Lehrer, 2003; Outhred et al., 2003); and
- The use of formal and informal units (Clarke et al., 2003; Lehrer, 2003; Outhred et al., 2003).

Research has also identified a number of skills or measurement processes as being important to the process of measurement. These skills include the ability to:

- Compare measurements (Barrett, et al., 2003; Grant & Kline, 2003)
- Choose appropriate measuring tools (Clarke et al., 2003); and
- Measure from a fixed point (Lehrer, 2003).

In order for students to effectively learn measurement, these concepts and skills, along with unit iteration, usually need to be developed through explicit teaching. Therefore, it is crucial that teachers have a thorough understanding of these principles and skills.

Teacher Knowledge of Children's Growth in Understanding

The key concepts of measurement, outlined above, form the basis for a number of 'learning frameworks' in measurement (e.g., NSWDET, 2003; van den Heuvel-Panhuizen & Buys, 2005). A framework sets out stages of development or conceptual growth points that students normally develop as their understanding of measurement processes and concepts progress to more sophisticated levels. These stages or growth points are a progressive list of mathematical attainments (Clarke et al., 2003). In 1999 the Count Me into Measurement Framework was introduced into NSW primary schools (NSWDET, 2003) and was used to inform the development of the new syllabus *Mathematics K-6* (Board of Studies NSW, 2002). The framework clearly highlights certain levels of thinking that children progress through when learning measurement. These levels of thinking reflect the key concepts that have been identified by research as important to understanding measurement. Research clearly indicates that when teachers have clear understanding of these frameworks or stages of growth in students' development of understanding measurement, it assists them to plan clear and appropriate learning activities (Clarke et al., 2003).

Teacher Knowledge and Classroom Practice

Students' lack of understanding in the content areas of length, area and volume has been intuitively linked to poor or ineffective teaching practices for some time, but growing evidence confirms these links. Research indicates that primary teachers often rely on worksheets, textbooks and inappropriate activities to teach measurement, resulting in an emphasis on procedure rather than process (Bragg & Outhred, 2000; Leinhardt & Smith, 1985; Outhred et al., 2003). Such reliance is considered to stem from a lack of confidence that primary teachers generally have when it comes to the content involved with measurement (Clarke et al., 2003; Sowder, Phillip, Armstrong, & Schappelle, 1998). The use of inappropriate activities may indicate a lack of understanding regarding the key concepts and effective teaching strategies for length, area, and volume. Outhred and McPhail (2000) found that teachers struggled with understanding basic concepts associated with unit iteration and effective teaching strategies. The researchers concluded that there was a reluctance to teach measurement concepts by primary teachers and that this may stem from contextual constraints including teacher knowledge. Such findings build support for the view that student misconceptions may simply reflect teachers' inadequate understandings of measurement concepts (Outhred & McPhail, 2000).

Given the context outlined above, this study sought to explore the following research questions:

1. What are teachers' perceptions of their knowledge and understanding of key mathematical concepts and processes associated with the measurement content areas of length, area and volume?
2. What do teachers know about children's developmental growth in understanding length, area and volume?
3. Do teachers' perceive their content knowledge and knowledge of how children develop understanding of length, areas and volume impacts on their pedagogy? If so, how?

Methodology

This research adopted a qualitative approach, utilising self-report data gained from in-depth teacher interviews. Interviews involved four primary teachers selected from three schools, ranging in teaching experience from one to 26 years. The inclusion of teachers from a range of career stages was based mostly on their willingness to participate in the study, but was also considered a strength of the study since such a range would most likely provide rich data to compare and contrast. However, caution is needed not to generalise findings to all teachers at a particular stage of their career without further study involving a larger sample of teachers. More specific information about each participant is presented in the results section.

The aim of the interviews was to gain information relating to: important biographical and contextual information about each teachers' background to teaching in general and more specifically to mathematics; teachers' perceptions about their understanding of mathematical content associated with length, area, and volume; teachers' perceptions about their knowledge of children's growth in understanding measurement concepts; and the impact this knowledge was perceived to have on their pedagogy.

Interview questions that were concerned with teachers' knowledge were presented in a way that let teachers display their perceptions about their knowledge of key concepts involved with length, area, and volume. Hence, participants were not explicitly asked what they did and did not know, rather they were invited to discuss what they considered to be important concepts, knowledge, and skills necessary for an understanding of length, area, and volume, what they thought were significant stages of development in children's understanding of these measurement concepts and how they perceived their own knowledge of these aspects impacted on their instructional decision-making.

Interviews were audio-recorded and later transcribed to assist analysis. Teachers were interviewed on one occasion for approximately one hour each. The interviews were semi-structured to allow teachers the opportunity to move the interview in any new direction as long as they appeared relevant to the research.

To analyse the data, participant responses were coded to assist in focusing on the essential themes emerging from the interviews that were specifically related to the research questions. Hence, interviews were coded based on the following categories:

1. Teacher knowledge of key concepts and measurement processes relating to length, area, and volume;
2. Teacher knowledge relating to students' growth in understanding about mathematical concepts concerned with length, area and volume; and
3. The types of strategies reported for teaching length, area and volume.

Information falling into each of these categories was then considered in light of their commonalities and differences and is reported in the next section.

Results and Discussion

This section presents and discusses the findings of the study. First, biographical and contextual information for each participant is briefly presented. Pseudonyms have been used for each participant. Following this, the research findings are discussed in terms of the commonalities and differences emerging from the interview data.

The Participants

A summary of the background information for the four teachers and their respective school contexts is presented in Table 1.

Table 1

Summary of Background Information for Teachers

Teacher	Grade currenlty teaching	Years of teaching experience	School context information
Dean	Kinder	1	Public school with approx. 530 students K-6. Forty percent NESB. Situated metropolitan region.
Vicky	Year 1	1	Public school with approx. 530 students K-6. Forty percent NESB. Metropolitan.
Pat	Year 3/4	6	Public school with approx. 200 students K-6. Predominately Anglo-Saxon population. Metropolitan.
Debbie	Year 1/2	26	Public school with approx. 600 students K-6. Students from diverse backgrounds. Metropolitan.

As part of the general background information, participants were asked to rate their general aptitude in mathematics. Both Dean and Vicky rated their mathematical abilities as quite "good", having done well in the discipline at school. Vicky recalled being accelerated in mathematics at High School and Dean doing mathematics as part of a Physics degree at university prior to undertaking his teacher education. Pat and Debbie, on the other hand, acknowledged that they had struggled in mathematics at school, but both considered that their knowledge had improved due to their teacher education and their experiences teaching it to children.

The rest of the interview asked teachers questions that specifically related to their perceived knowledge of length, area and volume—their knowledge of content, children's developmental growth and the impact of this knowledge on their teaching practices.

Perceived Knowledge of Measurement Content

Three of the four participants rated their knowledge of content associated with length, area and volume as "good." While Vicky, rated her knowledge of area as "good", she rated her knowledge of length and volume as, respectively, "adequate" and "minimal". She felt that her lack of knowledge of the latter two concepts would improve with more teaching experience. Despite rating his knowledge of mathematics, and measurement concepts in particular, as quite "good", Dean expressed frustration at "not always understanding" measurement concepts sufficiently himself, thus making teaching it to children more difficult. Thus, it was evident that both Dean and Vicky equated the adequacy of their own content knowledge with that needed to teach it.

When asked what they perceived to be the main concepts and processes involved with developing an understanding of length, area, and volume, all four of the participants provided limited responses. This could be due to the possibility that an interview might not be the most appropriate method to elicit this type of knowledge or that not having had to previously verbalise such knowledge, teachers found it difficult to coherently articulate it. Vicky's and Dean's answers possibly reflect their recent university studies, with them naming a number of common concepts and processes prevalent in research literature and syllabus documents such as the importance of comparison and the use of informal units. Dean added that "language" and "measuring from a fixed point" were also important, as was the progression to standard units. Pat also mentioned the need for a standard unit and referred to the "manipulation of units" as important, but was not able to elaborate. Debbie did not mention specific concepts or processes related to measurement. She focused her comments on the need for children to understand that "there is a purpose to it (measurement)". She confided that she considered her knowledge of "number" concepts to be more extensive. This was attributed to the fact that in her 26 years of teaching she had only ever experienced formal professional development that focused on number.

All four participants considered that they had gained their knowledge of content and student development from their teacher education programs, whether this was at university or at teachers college. Although all participants named their teacher education as important, they also thought that other factors had influenced their knowledge. Dean considered that he had learnt the mathematical processes at school, while university had allowed him to develop the skills to "articulate" these processes. Vicky believed that her knowledge was also developed from the Board of Studies units of work and from other teachers. Pat believed that teaching experience was the most important factor, stating that "at uni you get your general ideas but when you get out into the workforce" you learn a lot more.

It was evident from the interviews that teacher knowledge was both implicit and explicit. While the teachers often struggled to articulate their knowledge of key ideas involving length, area, and volume, it was apparent through their discussion of teaching methods and strategies that many key concepts were being treated within the classroom regardless of the teachers' abilities to explicitly identify key concepts. This may be reflective of the fact that in a classroom situation teachers are not required to articulate their own knowledge of key concepts, rather they are simply required to incorporate these into their teaching practice.

Knowledge of Student Developmental Growth in Measurement

Both Dean and Vicky described their knowledge of student development associated with length, area and volume as "adequate". Both teachers perceived their lack of teaching experience to be a major influence in this rating. Debbie believed that her knowledge of student development lay within the "adequate to good" range for the three sub-strands, stating that she understood more about number than measurement. Pat was the only participant who showed confidence in her knowledge of student development, rating herself as "good." She attributed her confidence to the fact that she had worked with Kindergarten through to Year 5 and was very aware of what measurement concepts children understood at the various grade levels.

When asked about where they thought students encountered the most difficulty in developing an understanding of these concepts and measuring processes; responses reflected the varying experience of teachers. Dean identified that his Kindergarten class had difficulties understanding the concept of starting from a fixed point when dealing with length. Vicky identified that her Year 1 class had struggled with the concept of leaving no gaps when measuring area. The two more experienced teachers, Pat and Debbie, both perceived volume as being more difficult for children to understand than either length or area. They considered that this was due to students' inability to "visualize volume" with Pat claiming that the concept of volume is "hard to imagine". All participants thought that practice and time were the solutions to these difficulties, believing that students would progress to the next stage of understanding when they were "developmentally ready".

Vicky and Dean claimed to be familiar with the NSW DET Learning Framework in Measurement (NSWDET, 2003) having been introduced to it in their initial teacher education programs. Vicky reported not having used it to plan her unit on area although she believed that she would use it for her unit on length in the near future. Dean said that he had "looked at it a bit" but admitted to not being very familiar with it. Dean explained that his mathematics units had been planned in collaboration with other early stage one teachers at the school and they had "started looking" at the Framework in Measurement for future programming, but because the other teachers were not familiar with the Framework, it was perceived as an obstacle to whole-stage planning. Vicky and Dean were only able to vaguely recall some of the Framework's content. Debbie, on the other hand, had no knowledge of any formalised framework for describing children's growth in understanding of measurement, and Pat knew that "frameworks about the way kids learn" existed but could not "specifically think of one".

Teaching Practices

When asked about specific teaching and learning strategies that they employed to teach length, area and volume, all the participants named general teaching strategies such as "hands on" activities using "concrete" materials, rather than provide specific examples of experiences or strategies to address children's misconceptions or to introduce them to more sophisticated concepts of measurement on the basis of developmentally appropriate activities. There was a perception that using a variety of concrete materials allowed students to practice using measurement in real-life settings. This was particularly the thinking of Vicky and Dean who both described class experiences of going "outside into the environment and … giving them (the students) problems they can relate to".

In relation to the third research question about the perceived impact of teacher knowledge on classroom practices, there was a distinct perception that such knowledge had little or no impact. However, the very process of questioning the teachers about their understanding of developmental frameworks seemed to act as a stimulus (or reminder) that such knowledge was available and could inform their teaching.

The Learning Framework in Measurement was introduced into NSW Department of Education and Training primary schools in 1999 and extensively influenced the development of the measurement strands in *Mathematics K-6* (Board of Studies NSW, 2002). There was a distinct lack of explicit knowledge regarding this Framework or any other research-based framework describing children's development of measurement concepts. While the early career teachers in the study were aware of the Framework, their limited teaching experience, combined with the fact that their more experienced colleagues were unfamiliar with such a framework, meant that it was difficult for them to utilise this knowledge in their planning. This raises a number of interesting questions about how and when teachers are introduced to developmental frameworks of this nature. Is there a need for such knowledge to be learnt in a context in which it can be simultaneously applied for this type of knowledge to have more impact on teachers' planning and teaching. In particular, this has implications for the way such information is presented in initial teacher education programs.

Conclusion

The aim of this investigation was to explore teachers' perceptions of their content knowledge and knowledge of how children learn length, area, and volume and how they perceive this knowledge impacts on their teaching. Hence, the data presented does not document actual impact on instruction; such claims would need to be validated through additional data collection methods like observations of teaching.

While based on a small sample of teachers, the interviews revealed a number of issues that point to a need for further exploration of teacher knowledge relating to the measurement strand. The study revealed that the teachers in this study perceived their knowledge of measurement concepts and processes to be generally "good" but that this varied according to the different sub-strands; volume was perceived to cause more difficulties for teachers and learners than length or area. All the teachers struggled to explicitly identify key concepts and processes involving length, area, and volume. This inability may reflect the fact that teachers are not normally required to articulate their own knowledge of key concepts; they are simply required to incorporate these into their teaching practice.

Relying on teachers' self-reported perceptions to identify the impact of knowledge on teaching practices through interviews is less resource-intensive than conducting a large number of case studies involving classroom observations. Nonetheless, such case studies are a necessary component of further study in this area as they will allow exploration of how varying levels of experience and teacher understanding of research-based frameworks of children's mathematical thinking impacts on teachers' practices and ultimately on children's understanding of measurement.

Overall, the data highlights the inability of teachers in this study to clearly articulate their knowledge of important concepts and processes relating to length, area, and volume. It also points to the importance of professional development that focuses on the measurement strands of primary curriculum, including content knowledge and knowledge of how children learn measurement.

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