Students' Attitude Towards Using Materials to Learn Algebra: A Year 7 Case Study

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This paper examines the affective responses of students to an algebra intervention in the last year of primary school. The intervention was based upon extensive use of materials, discussion activities and specifically designed algebra games. Data on student attitudes and beliefs were collected with an eight dimension Likert scale, student interviews, analysis of student sketches and classroom observations. While the overall results indicated student appreciation of the use of materials and educational games, some students perceived their value to be transitory.

Introduction

Students with limited capacity to think and operate algebraically are effectively limited from Intermediate and Advanced Mathematics study in the senior school, in effect competency and confidence in algebra acts as a critical filter for more advanced courses in mathematical thinking and problem solving (e.g., MacGregor, 2004; Stacey & Chick, 2004). There is considerable evidence that confidence and competence interact, and low levels in either or both predict reduced participation in more advanced study (Ethington, 1992; Wigfield & Eccles, 2000). TIMSS data (Thomson & Fleming, 2004) indicates that the transition from later primary into early secondary school is accompanied by a sizable reduction in student confidence about mathematics. Much of this decline in confidence can be attributed to the introduction of symbolic algebra in early secondary school (Kaput, 1987; MacGregor, 2004; Stacey & Chick, 2004). The challenges in teaching algebra have been well documented and include and can in part be attributed to the quality of standard texts which tend to direct the nature of algebra learning. In particular algebra resources found in standard texts frequently do not encourage teachers to enact appropriate pedagogy to foster algebraic thinking (Mousley, 2007; Stacey & MacGregor, 1999). It has been recommended that algebra teaching include (a) the explicit teaching of the nuances and processes of algebra a central feature of algebra learning (e.g., Kirshner & Awtry, 2004; Sleeman, 1986; Stacey & MacGregor, 1997; 1999), especially in transformational activities (e.g., Kieran & Yerushalmy, 2004); (b) that algebra study be embedded into contextual themes (National Council of Teachers of Mathematics, 1998) and (c) texts should use multiple representations of key concepts and incorporate the use of technology (e.g., Kieran & Yerushalmy, 2004; Van de Walle, 2006).

With this in mind, the authors implemented a Year 7 algebra intervention that focused on making the nuances and processes of algebra explicit using multiple representations at times within contextual settings, and examined the effect of the intervention upon students' perceptions about the value of the learning activities. The research questions in this paper were: (1) Did students believe the specific learning activities enhanced their understanding of algebra? (2) Did the intervention enhance students' confidence to undertake high school algebra?

Theoretical Framework

Kieran (2004) suggested that school algebra could be said to be of three types of activity: *generational, transformational* and *global/meta-level*. *Generational* activities involve the forming of expressions and equations that are the objects of algebra. This activity is the representation (and interpretation) of situations, properties, patterns and relations and much of the meaning making of algebra is situated in this activity. For example it may involve students developing a 'rule' that describes an expanding pattern and this can be converted into an algebraic expression. *Transformational* activities focus on symbolic manipulation and include activities such as collecting like terms, factorising, expanding, substituting, solving equations and simplifying expressions. *Global/meta-level* activities use algebra as a tool for problem solving and include modelling, noticing structure, generalising, justifying and proving. Clearly the latter is dependent upon the former two activities.

Method

The methodology was essentially a case study (Yin, 2003). The involvement of the researchers as active participants with the teachers in the trials also gave the methodology a participatory collaborative action-research element (Kemmis & McTaggart, 2000).

Participants

Twenty-four Year 7 students participated in this study. The study school was selected on the basis of convenience. The classroom teacher had recently completed a Masters in Education and had a particular interest in the use of material representations in developing students' understanding of number concepts. The classroom teacher and the author wished to extend this teaching approach to the learning of algebra. The school was located in a lower to middle class suburb in outer Brisbane. The students had done a very limited amount of work on patterns in earlier years, so this was their first formal introduction to algebra.

Description of Intervention

Twelve one hour and half hour classes were conducted by the researcher over a 6 week period. In some of the other mathematics lessons during that period the classroom teacher supervised activities that consolidated algebra concepts introduced by the first author (usually through the medium of the algebra games) or on occasions unrelated mathematics work such as the revision of number computations was undertaken. The first researcher had developed an intervention, and taught the twelve critical lessons designed to introduce the following aspects of generative algebra: the concept of variable; connecting materials with tables of ordered pairs, word descriptions, symbolic equations and graphs of functions; understanding and transforming activities including construction algebraic expressions; the substitution of values into algebraic expressions; solving for an unknown in linear equations with the variable on one side only, and; solving linear equations where the unknown was on both sides of the equal sign. Meta-level activities included the construction of linear equations from word problems and solving for an unknown variable. The generative learning activities were drawn primarily from the text *Access to Algebra Book 2* (Lowe, Johnston, Kissane, & Willis, 2001). Transformational activities (algebraic manipulation – Kieran, 2004) associated with algebraic expressions, substitution and solving, used models similar to those described in *A Concrete Approach to Algebra* (Quinlan, Low, Sawyer, White, & Llewellyn, 1987). Materials were used to make the meaning and processes of solving explicit. The example below (Table 1) is taken from the first author's teaching notes. It attempts to link materials, natural language and symbols and emphasises an algebra structure as recommended by Stacey and Chick (2004). In the example below "ones" were represented by blue counters, "negative ones" by red counters and coloured blue and pink cups were used to represent "n" and "-n" respectively. Once familiar with the processes such as those illustrated in Figure 1, students played specifically designed board games (22 different games) to gain competency and solved word problems.

Let n =	Let – n =	Let 1 =	Let – 1 =
			

Say we try to solve 2n + 3 = 3n -1.

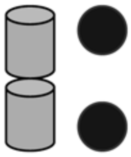
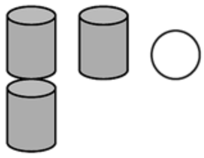
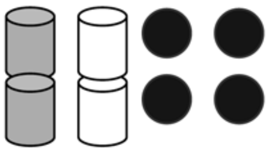
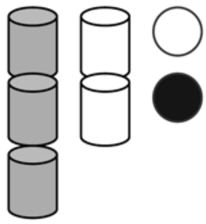
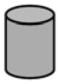
Symbols	Materials and language		
2n + 3 = 3n -1.		=	
It does not really matter what we work on first, the unknowns or the integers. We can add one positive to both sides, this will place all the ones on the LHS, then we could take two n's from both sides. In this instance we can use the zero principle by adding two negative n s to both sides. (I am doing this together simply do save space).			
2n – 2n +3 + 1 = 3n – 2n -1 +1+ 3	The + 2n and the – 2n will add to zero, and the ones will add to 4. 	=	The +3n and the -2n will result in one n, and the – 1 and the + 1 will add to zero. 
Thus  equals  4 = n. This can be checked with substitution.			

Figure 1. Excerpt from teaching notes on solving.

Data Collection

At the end of the intervention the students completed a Likert scale perceptions survey that was developed by the first author. The scale consisted of 32 questions related to eight attributes (with the following five response categories: strongly agree (5), agree (4), uncertain (3), disagree (2), strongly disagree (1). The scoring was reversed for negatively worded items. Each scale attribute consisted of four items, two of which were positively worded and two negatively worded. The first five attributes probed students' evaluation five key learning activities related to critical algebra concepts and processes, the remaining three attributes probed student attitudes towards algebra and mathematics study. The eight attributes and a relevant sample item are shown below. The associated citations indicate the importance of these processes and suggestions on how they might be developed.

1. The role of a balance activity in developing an understanding of the meaning of equals; e.g., *The balance activity helped me understand that you must treat each side of an equation in the same way* (Baroudi, 2006; Horne, 1999; Warren & Cooper, 2005).
2. The role of materials in helping students to develop an understanding of linear functions; e.g., *Using the match sticks to make patterns helped me to understand how to make up algebra rules or equations* (Lowe et al., 2001).
3. The role of materials in helping students to develop an understanding of algebraic expressions; e.g., *Using the cups and counters to make up expressions like 3x + 4 helped me to understand what they meant* (Lowe et al., 2001; Quinlan et al., 1987).

4. The role of materials in helping students to understand the processes of solving linear equations; e.g., *By using the cups and counters I was more able to understand how to find the value of an unknown, like the value of x* (Lowe et al., 2001; Quinlan et al., 1987).
5. The role of algebra games in developing understanding of algebra concepts; e.g., *Playing the algebra games was a good way of learning algebra* (Booker, 2000).
6. The role of the course in developing confidence to do algebra; e.g., *This course has given me confidence to do algebra in secondary school* (Thomson & Fleming, 2004).
7. The role of the course in developing confidence to do high school mathematics; e.g., *My confidence in being able to do secondary mathematics has been improved by doing this algebra course* (Thomson & Fleming, 2004).
8. Student perceptions about the value of learning algebra; e.g., *This course has helped me to appreciate that algebra is important in understanding how variables (amounts of things) relate* (Wigfield, & Eccles, 2000).

Student sketches provided an additional data source on student perceptions. At the end of the study students were asked to draw a picture that summarised their activity and feelings during a normal maths lesson and a picture of an algebra lesson. Sketches were analysed for themes such as those that might emerge from the pictures. The final data set came from student interviews, the eight attributes associated with the survey formed the basis of the interviews conducted with pairs of students at the end of the study.

Results

Survey Results

The means, standard deviations for the survey instrument were calculated. The small numbers in the sample and the limited number of items in each scale limit inferences based on these estimates. That aside, the data provided information about trends and served as a starting point for probing student perceptions via qualitative methods. These descriptive outcomes are presented in Table 1 below. The mean values and standard deviations suggest that on the whole student responses fell somewhere between uncertain and agreement with statements affirming the value of the activities in helping them to understand algebra concepts. For example for the balance activity the mean is almost 4 with a relatively small standard deviation of 0.57 indicating that student responses aggregated about 4 or *agree* to the statements probing students perceptions of the value of this activity. In a small sample such as this a few students can substantially alter the mean and variance. It was found that six students accounted for almost all the 1 and 2 responses indicating a negative view of the value of the materials, almost all the remaining students responding with 4 or 5. The survey data indicate that the class fell into two groups; students who responded on the survey that they found the approaches useful and those who did not. Student interviews shed light on the reasons why these students responded as they did.

Table 1

Means, Standard Deviations, (n = 24)

Scale	Mean	Standard deviation
Balance activity	3.92	0.57
Linear functions	3.66	0.85
Materials and expressions	3.79	1.03
Materials and solving	3.68	1.11
Value of games	3.59	1.11
Algebra confidence	4.17	0.70
Maths confidence	4.30	0.70
Value of algebra	4.17	0.53

Interview Results

Two girls, recent African immigrants, had strongly negative views about the value of materials in algebra learning. Nicola and Marion had similar views (all names are pseudonyms). Nicola summed up the consensus between this pair:

> When you use counters I found it hard, when you write it down or do it in your head I found it easier.

Of the games this student simply said:

> The games did not help me learn much, but I liked the spinners.

Nicola could not articulate why she did not like to use materials, just that they confused her. A second trend emerged among three the boys and two girls, in this instance Chris was typical.

> It was good the first time, but once you understand it is easy, like now I do not need cups and counters and it just slows me down.

This comment shows recognition of the value of materials in establishing understanding, but once the mathematical structure was understood a number of students (e.g., Chris, Ash, Ben, Sally and Sasha) disliked being encumbered with material use. Sally added an additional condition to the use of materials:

> For people who are challenged with algebra you should use cups and counters, but for people who are not, you do not have to.

Each of these students was able to use the materials in problem solving contexts when asked. During the interview process eight students made comments that indicated that the materials were useful for adding meaning to the processes initially, but were not required beyond that initial phase of the learning progression. In four instances this was translated to an overall negative response on the survey items. The following comments are typical of explanations as to how the use of materials helped their learning of algebra:

> Corrine: The matches helped me see the pattern a lot.
>
> Connie: It helped a lot, you can see what you were doing instead of having to keep it in our mind (match stick patterns).
>
> Julia: You can see and feel everything what you are doing.
>
> Joelle: It made it much clearer. If you go step by step it is much clearer, rather than just say "this is how it is done."
>
> Sarah: The cups and counters help you get the picture in your head.

As previously noted, the educational board games were based on material representations such as those illustrated in Figure 1 that link various algebraic representations. Students who responded negatively about games did so for reasons such as those articulated below:

> Ash: The games were not very useful, I would prefer doing sums, the games were too easy, I saw the answer too fast. Otherwise it gets boring.

One conclusion based on these data is that these students sought greater challenges. More generally, the survey data suggests that most students responded positively to the games and the interview data gives explanation:

> Corrie: The games were good; they helped you see the answer. And when I did not know the answer I could ask my friends.

Billy: They were fun to play, and you had to think about it. So it was practicing what you had learnt as well as having fun at the same time.

Themes arising from the survey data indicating approval for the use of games and included "fun", staying on task and an appreciation for the social aspect of learning through playing algebra games.

Sketch Drawing Results

Twenty-three students completed pairs of sketches representing "a normal maths lesson" and "an algebra lesson". Themes emerging from the drawings were categorised by the authors, and also reviewed by peers among the tertiary mathematics education community. Some sketches had a single identifiable theme, others had several. What is clear from the sketches is that most students portrayed a normal mathematics lesson as one dominated by procedural computations and simple representations (15 sketches), none showing the use of materials, a finding consistent with descriptors of the dominant discourse in primary mathematics classrooms (e.g., Vincent & Stacey, 2007). Students reported greater complexity in tasks associated with algebra activities (11 sketches). This complexity was indicated by multiple processes which required a number of different operations to be performed. Representations of materials and algebraic symbols were common in the sketches of the algebra class (14 sketches). Engagement (7 sketches) and understanding (4 sketches) was expressed more frequently in the intervention class as compared to the normal class (engagement 2, understanding 0).

Discussion and Conclusions

The authors are well aware of the inherent limitations in using concrete materials to represent algebraic statements because the inherent particularity of such models can run counter to the generality and abstractness of algebraic statements. However, in this paper we are interested in student perceptions of their learning, including the effectiveness of materials and material based games in helping them to understand the processes and structures of algebra at this introductory level. Student responses to the survey indicated that the class fell into two groups, those that found the materials helpful and a smaller number who did not. The interview data helped to expand upon this finding. With the exception of two girls, those who evaluated the activities based upon materials lowly (1 and 2 on survey items) said in the interview that the activities were useful initially to see the "pattern", and see the process "if you go step by step (with materials) it is much clearer," "see what you are doing…saw the structure…get the picture in your mind" but once they understood the concept or processes they preferred not to use materials since it slowed them down. That is the students recognised that symbolic processes were more efficient than processes tied to materials or material representations. This was especially the case for predominantly generative (Kieran, 2004) activities (such as creating and describing representations of linear functions, e.g., match stick patterns, tables of co-ordinates, graphs and word descriptions of relationships and equations) but also the transformational (Kieran, 2004) activities such as processes that solve for unknowns with the aid of cups and counters. Once students understood the structure they preferred more abstract methods such as mental computations and written algorithms. This implies that while students might gain from material based forms of algebraic activity, for a significant portion of students it is not necessary to labour their use. In the case of the use of mental computations, it is possible that some students were using numerical methods that need not necessarily parallel the formal processes of algebra. The use of more complex numbers (larger numbers and fractions) could have reduced these students' capacity to rely on mental and numerical methods and this will be taken into account in future algebra interventions.

With respect to the first research question, with a few exceptions and some conditions (some students saw initial value in material based activities but did not see the merit in persevering with their use) most students believed the learning activities enhanced their understanding of algebra. Overall student perceptions of their increased competency in algebra were reflected in increased confidence about approaching Year 8 algebra and Year 8 mathematics in general. Given the relationship between self confidence and mathematics achievement (Thomson & Fleming, 2004; Wigfield & Eccles, 2000) this is an encouraging finding.

The merits of learning mathematics in social and collaborative settings has been well documented (e.g., Grootenboer & Zevenbergen, 2007) and the algebra games afforded this learning setting. The comments of students about games in facilitating collaborative social settings support earlier findings (e.g., Booker, 2000; Norton & Irvin, 2007). Insomuch as these activities help to foster student confidence about their capacity to understand algebra, the use of materials and educational algebra games based upon material models warrants further research. The current outcomes indicate that there might well be a case for more widespread use of such approaches before secondary years.

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