Using Task-Based Interviews to Assess Mathematical Thinking of Primary School Students

Chris Hurst
Curtin University of Technology
<c.hurst@curtin.edu.au>

This paper describes part of a larger study of an intervention program involving strategies designed to develop the mathematical thinking of upper primary students. It considers the effectiveness of task-based interviews in identifying the extent to which different modes or levels of thinking were used by eight students in a multiple case study following the implementation of an intervention program. The modes of thinking are based on mathematical, contextual and strategic knowledge.

Introduction

There is no universally accepted definition for numeracy though the myriad attempts to describe it have a number of common features. It has been equated with the idea of 'statistical literacy' (Watson, 1995, 2004), 'quantitative literacy' (Steen, 2001), 'mathematical literacy' (Steen, 2001), and 'realistic mathematics education' (van den Heuvel-Panhuizen, 2001). As well, it has been described in the *Numeracy Framework* (Morony, Hogan & Thornton, 2004; Willis, 1998) that outlined three modes of thinking, or components of numerate behaviour, and these were:

- Mathematical knowledge – the skills, techniques and concepts necessary to solve quantitative problems encountered in a real context.
- Contextual knowledge – an awareness and knowledge of how the context impacted on the mathematics being used.
- Strategic knowledge – the confidence, disposition and skills to find out what needs to be known in order to act numerately (Morony et al., 2004).

All of these descriptions of numeracy have in common the notion that students are able to understand mathematical ideas in various contexts and to apply those ideas to learn more about the context in which they are embedded.

Theoretical Framework

The study on which this paper is based (Hurst, 2007) interpreted the *Numeracy Framework's* description to develop indicators for each of the three modes of thinking that characterise numerate behaviour. Each mode of thinking is considered to be evident when a student exhibited some of the following respective sets of criteria.

Mathematical knowledge

- Recalls or identifies specific items of mathematical information.
- Recognises and reiterates examples of mathematical information.
- Uses statistical information to suggest or perform a mathematical operation.
- Poses questions that require the use of a mathematical operation.

Contextual knowledge

- Interprets specific data contained in the context and/or poses questions that require the interpretation of specific mathematical data.
- Describes, in one's own words, the main mathematically related ideas contained in the contextual information.
- Interpolates or extrapolates from aspects of the data.
- Poses questions that suggest connections between different aspects of the data and uses data to explain such connections.

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Strategic knowledge

- Predicts or suggests how the data could be used to develop a new idea.
- Develops a scheme or method for representing the data, other than that already presented.
- Evaluates aspects of the data for consistency and validity and/or identifies anomalies in the data or misleading information.
- Evaluates aspects of the data to clarify related issues and make decisions.
- Poses or responds to questions which require substantial evaluation of aspects of the data.

The original study was based on the notion that there exists a cyclical relationship between the three modes of thinking. That is, a student may approach a situation with embedded mathematical ideas by initially using mathematical knowledge in an attempt to primarily understand the nature of the context. Alternatively, the situation could be approached with contextual thinking where a student might have experience with the particular context and use some of the embedded mathematics to connect his/her understanding with related aspects of the context. Finally, an experienced user of mathematics might exercise strategic thinking and immediately identify an apparent anomaly in some embedded data, or a misuse of that data, and then use mathematical knowledge to prove how that was so. Figure 1 reflects this cyclical relationship.

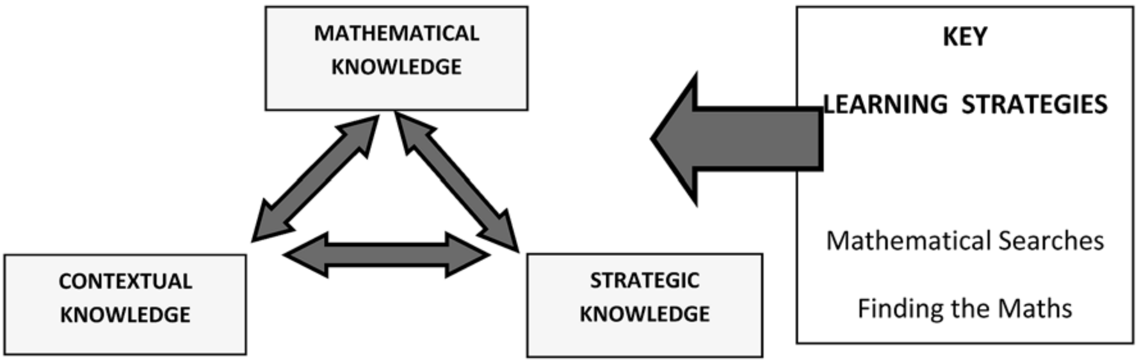

Figure 1: Cyclical relationship between modes of thinking.

Design and Methodology

The original qualitative study investigated the effect of using key learning and teaching strategies as shown in Figure 1 and sought to answer the following research question:

To what extent does the Mathematical Search enhance student capacity to recognise mathematical ideas embedded in a written context, and to display contextual and strategic thinking about mathematical ideas embedded in written contexts?

One of the data gathering tools used was the task-based interview. Task-based interviews were chosen as the main data gathering strategy for the original project because it was felt that the potentially 'data rich' environment they afforded would provide the best context for assessing and probing for the presence of the three modes of thinking both before and following the intervention phase of the project. As well, task-based interviews reflected the same type of contexts containing embedded mathematical ideas as did the key strategies used during the intervention phase. These strategies were:

- Mathematical Search – Students were given pieces of text and were asked to identify and describe the embedded mathematics, to say how the embedded mathematics helped them to understand aspects of the text, and to develop questions that could be asked using the embedded mathematical information.
- Finding the Maths – Students were asked to collect samples of published material (newspaper articles, brochures etc.) containing mathematical ideas and perform the same type of analysis as with the Mathematical Search.

During these interviews conducted at the beginning and end of the data gathering phase, students were shown a range of artefacts containing embedded mathematical concepts. These artefacts included maps, pictures of signs, shopping dockets, newspaper advertisements, and newspaper articles containing statistical information (see artefact samples in Figures 2 and 3). A program of questions and prompts was established to allow for as wide a range of student responses as possible and to enable the researcher to probe the thinking of students. Some of the questions asked were as follows:

- If we are shopping, coming to school, or watching TV, we see numbers around us. Do you take notice of them or wonder what they are telling you about?
- If someone gave you a page out of a book or newspaper, and asked you to describe the mathematics that you saw, what sort of things would you look for?
- Show the student the sample of the shopping docket/receipt and ask 'What mathematical ideas can you see and what do they tell you?'
- Show the student the map samples. What mathematical ideas can you see in the maps? How could you use those ideas to help you work out or learn something?
- Show the student the furniture advertisement. Ask 'What mathematical ideas can you see in that?' Ask 'How could you use that to help you work out something?' or 'What could you work out from that?' or 'How might that be useful to you?'

Figure 2. Artefact sample – computer advertisement.

Figure 3. Artefact sample – netball advertisement.

When shown the above artefacts, students were asked questions such as 'Is there a question you could ask about this?' or 'Does anything in there make you think of a question to ask?'

Eight female Year Six students from six different primary classes, aged eleven or twelve years, were selected as subjects for a multiple case study. Task-Based Interviews were conducted with these eight students at the beginning and end of the project to determine changes in their modes of thinking over the six month period of the intervention phase of the project. In addition, pre-project and post-project benchmark tasks based on interpretation of graphic and tabular information, and pre-project and post-project interviews with teachers of the case study students were conducted so that data triangulation could be achieved. As well, benchmark task samples were collected from the remaining 112 students in the six classes, hereafter termed the General Sample, to see whether or not results from them differed from those of the multiple case study.

Interviews were transcribed and student responses were categorised as mathematical, contextual or strategic, according to the criteria listed in the 'Theoretical Framework' section of this paper. Instances of each mode of thinking were compiled in table form to enable comparison of student thinking before and after the intervention phase of the project (see Table 1, following later).

Results and Discussion

The following excerpts from interview transcripts provide examples of how the student thinking was categorised. The excerpts are illustrative examples only and are not exhaustive representations of the responses of any of the students. Rather, they serve to indicate typical responses given and how those responses were interpreted by the researcher using the criteria previously listed in the theoretical framework section of this paper. The particular criterion used in each transcript sample is shown in parentheses in the second line of each sub-heading.

1. Pre-project interview with student 'Kerryn' showing *mathematical thinking*.
   [i.e., Uses statistical information to suggest or perform a mathematical operation.]

Interviewer: [referring to the furniture advertisement] OK, good. I'm going to show you one more thing; this is an advertisement about furniture. Have a look at that [showed Kerryn the furniture advertisement]. What mathematical things can you see there?

Kerryn: You can see the prices; you can see how much you've saved, and you can get the prices and compare the differences between them. It tells you when the day was, and what year it is [pause] when it's open from Monday to Friday, Saturday, Sunday [pause] then it tells you how tall the actual furniture is.

Interviewer: How could you use something like that?

Kerryn: To do length times width to see how big it is [pause] 100% leather means that it's all leather. There's like, five pieces so there's one whole and five parts [pause] and 1.65 metres long and 3700 metre squared for the store.

Interviewer: How would you use something like that information, that the table is that long?

Kerryn: In your house, you'd have to measure 1.65 metres to see if it would fit. Also, it has a map here to tell you where to get it; that's also mathematical.

2. Pre-project interview with student 'Tania' showing *contextual thinking*.
   [i.e., Interprets specific data contained in the context]

Interviewer: [referring to statistical charts taken from a newspaper]. I took these charts out of the newspaper. If you saw those charts, would there be any of them that you would look at and say "I wonder what that's telling me about"?

Tania: I'd look at … probably… this one (pointed to the fuel prices)

Interviewer: Why would you look at that one?

Tana: To look at the prices for petrol and to tell Mum where to go.

3. Pre-project interview with student 'Louise' showing *contextual thinking.*
[i.e., Poses questions that require the interpretation of specific data]

Interviewer: [referring to the furniture advertisement] And what's that bit telling us?

Julia: It's telling you the trade hours that they're open for.

Interviewer: How would you use that to help?

Julia: I would probably look at the trade hours and look at the time now and think "How long do I have in the shop?"

Interviewer: OK, if you wanted to buy some furniture there, which one of the shops would you go to?

Julia: I would probably go to that one.

Interviewer: Why that one?

Julia: Because it doesn't look as busy.

Interviewer: How do you know it's not as busy?

Julia: Because it doesn't have as many streets around it.

4. Post-project interview with student 'Mary' showing *strategic thinking*.
[i.e., Poses questions that require substantial evaluation of aspects of the data]

Interviewer: [referring to the netball advertisement – see Figure 3] Finally this is a piece about a netball team, the Perth Porcupines. Have a read of that (paused while student read the sample). Is there anything that makes you think about asking something?

Christy: Umm, say if I wanted to go join it, there's no website or address or phone number to ring. Where would I go, and because they've been playing netball for fifty years, would they be a really good team, because they'd been together and it said they had great success. Can adults play it and would they still be free? Is there a training date?

5. Post-project interview with student 'Sara' showing *strategic thinking*.
[i.e., Evaluates aspects of data to clarify related issues and make decisions]

Interviewer: [referring to the computer advertisement – see Figure 3]. Read this advertisement about a shop selling you computers (paused while student read sample). Is there something that you want to ask about.

Sara: Umm, sometimes when they do the 60 months, they might think why they are doing it when sometimes it's more expensive to do it that way. But, if you didn't have the money at the time, it would be better.

Interviewer: Is there anything else there?

Sara: Would you still get the huge monitor [with the 40 Gb]? In a way, I'd get that one [the 40 Gb] possibly because, if it wasn't for school and just for home, we don't really use 80Gb.

Interviewer: So you ask yourself if you needed all those gigabytes eh?

Sara: Yeah, that's what I do when I'm buying discs… when I'm buying discs for it, whether I need a big disc or a small disc.

The five transcript samples provided indicate that task-based interviews can be used to assess the mathematical thinking of students, particularly when artefacts such as advertisements and maps are used. In each of the transcript samples, only one particular mode or level of thinking has been identified though it could be considered that degrees of mathematical and contextual knowledge are shown when a student displays strategic knowledge such as in Sample 5. As well, not all of the indicators for identifying the various modes or levels of thinking, as listed in the theoretical framework, are evident in the transcript samples, as those indicators are representative of the types of criteria used to categorise students' thinking, rather than being a finite list.

The interviews were also used to assess the success of the intervention program conducted during the project at large and to see whether the strategies used in the intervention assisted in the development of the students' thinking. On the basis of information gathered from the pre-project interviews, all eight students in the multiple case study displayed both mathematical and contextual knowledge and during the post-project interviews all students displayed all three modes of thinking – mathematical, contextual and strategic.

This suggests that the intervention program using the *Mathematical Search* strategy was successful in helping to develop student thinking, and hence, the research question has been addressed. To reiterate, this question asked: *To what extent does the Mathematical Search enhance student capacity to recognise mathematical ideas embedded in a written context, and to display contextual and strategic thinking about mathematical ideas embedded in written contexts?*

The *Mathematical Search* strategy had not been encountered by students previously as was verified by responses during post-project interviews conducted with both teachers and students. Hence, it is reasonable to claim that it had at least some effect in enhancing student capacity to display different modes of thinking. Further evidence to support this claim is contained in Table 1 which illustrates the gains in thinking shown by the eight case study students over the course of the project. The abbreviations 'M', 'C' and 'S' correspond to 'mathematical', 'contextual' and 'strategic' thinking respectively.

Table 1

Gains in Modes of Thinking for Students from Pre-project to Post-project interviews

Student	Pre-Project	Post-Project
Mary	M/C	M/C/S
Sara	M/C	M/C/S
Jenny	M/C	M/C/S
Tania	M/C	M/C/S
Kerryn	M	M/C/S
Louise	M/C	M/C/S
Lexie	M/C	M/C/S
Sonia	M/C	M/C/S

Other aspects of students' thinking and attitudes about mathematics were also explored during the task-based interviews. For example, students were asked about whether or not they identified mathematical ideas in everyday situations and what sorts of things they looked for. The following two responses from students Tania and Sonia during pre-project interviews are typical in that the emphasis is on number aspects.

Interviewer: If somebody gave you a page out of a newspaper, or a book, that wasn't a maths book, and asked you to describe the mathematics you could see, what sorts of things would you look for?

Tania: Numbers and, you know, place value, like 300 000 type things.

Sonia: Umm,…I'd look maybe for some numbers, if there were any on there, and maybe some maths related words…. Words and symbols.

However, the following excerpt from a post-project interview with student 'Kerryn' indicates some different thinking.

Interviewer: So where would you be likely to find mathematical ideas?

Kerryn: Umm.. everywhere, yeah basically everywhere … The other day I was looking at a magazine called "Just Kidding" and I saw something that just grabbed my eye and I thought about maths so much and I related it back to the tasks that we were doing.

Conclusions and Implications

Task-based interviews can be useful tools for helping teachers assess the mathematical thinking of their students, particularly when mathematical concepts are embedded in everyday or 'real life' contexts. Given the time constraints of individual contact with students, it may be difficult for teachers to use task-based interviews on a large scale but the apparent benefits of their use suggest that they should at least be used with students identified as being 'at risk' or with students identified as being 'gifted and talented'.

In order to overcome the issue of time constraints, it should be possible to train students in interview techniques so that pairs of students could participate in informal interview situations. Interviews could also be conducted as small group activities with a teacher interviewing say three children at a time. This technique would provide opportunities for students to learn from one another and to see how other students interpret mathematical ideas embedded in a range of contexts.

The task-based interview format could also be modified to become a teaching tool. Having identified a student as being possibly 'at risk', a teacher might use the process with purposeful questioning to lead the student to recognise the embedded mathematical ideas and to begin to develop contextual and strategic thinking in applying and questioning the use of those mathematical ideas. Data from the project at large on which this paper is based suggest that the eight students in the multiple case study showed greater gains in their thinking than did their classmates who formed part of the General Sample. The only tasks in which the case study students participated that were not experienced by the General Sample students were the pre-project and post-project task-based interviews. This might indicate that the interviews were partly instrumental in the case study students showing greater development of their mathematical thinking, particularly with regards to contextual and strategic thinking.

The use of task-based interviewing would be particularly relevant where teachers use integrated programming. Specific mathematical concepts could be purposively embedded in artefacts and text samples based on content and themes related to associated learning areas such as Society and Environment, Science and Health. Hence, students would be more likely to appreciate the application of mathematical ideas to other areas, an essential element of numerate behaviour.

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