Intervention Instruction in Structuring Numbers 1 to 20: The Case of Nate

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Nate was one of 200 participants in a research project aimed at developing pedagogical tools for use with low-attaining 3rd- and 4th-graders. This involved an intervention program of approximately thirty 25-minute lessons over 10 weeks. Focusing on the topic of structuring numbers to 20, the paper describes Nate's pre- and post-assessments, including major gains on tests of computational fluency. Relevant instructional procedures are described in detail and it is concluded that the procedures are viable for use in intervention.

The Numeracy Intervention Research Project (NIRP) aims to develop pedagogical tools for intervention in the number learning of low-attaining third- and fourth-graders (8- to 10-year-olds). Within the NIRP we have developed an experimental learning framework for whole number knowledge comprising five key domains: number words and numerals, structuring numbers 1 to 20, conceptual place value, addition and subtraction 1 to 100, and early multiplication and division (Wright, Ellemor-Collins, & Lewis, 2007). For each of these domains we are developing a set of instructional procedures. The case study reported here is part of the research program developing a detailed set of instructional procedures for the key domain Structuring Numbers 1 to 20. The main purpose of the case study is to document the instructional procedures that appeared to contribute to the learning of a student who made significant progress in knowledge of Structuring Numbers 1 to 20.

Background

Addition and Subtraction in the Range 1 to 20

In early number learning, children use strategies involving counting by ones, for example solving 8+7 by counting on seven from 8, using fingers to keep track. Children achieve facile additive thinking when they develop a rich knowledge of number combinations, and solve additive tasks with non-counting strategies, for example 8+7 as: 8+8 is 16, less 1 is 15 (Fuson, 1992; Steffe & Cobb, 1988; Wright, 1994). Facile strategies in the range 1 to 20 are foundational for further arithmetic, and for developing number sense (Bobis, 1996; Heirdsfield, 2001; McIntosh, Reys, & Reys, 1992; Treffers, 1991). Thus, the development from counting to facile non-counting strategies in the range 1 to 20 is a critical goal in early numeracy (Wright, 1994; Young-Loveridge, 2002).

Some children do not achieve this facility. Instead, they persist with strategies involving counting by ones for addition and subtraction in the range 1 to 20, and in turn use counting strategies in the higher decades. Persistent counting is characteristic of children who are low-attaining in number learning (Denvir & Brown, 1986; Gervasoni, Hadden, & Turkenburg, 2007; Gray & Tall, 1994; Treffers, 1991; Wright et al., 2007). Persistent counting can disable students' progress with numeracy. An extended review of this literature is available elsewhere (Ellemor-Collins & Wright, 2008).

Numeracy is a principle goal of mathematics education and there are calls for intervention in the learning of low-attaining students to bring success with numeracy (Bryant, Bryant, & Hammill, 2000). In developing numeracy intervention, there is a pressing need to design instructional procedures that are likely to progress students from counting strategies to non-counting strategies. Designing such procedures is a central goal of the present study.

Instructional Design of Structuring Numbers 1 to 20

Structuring Numbers 1 to 20 is an instructional topic designed to build rich knowledge of number combinations and develop facile non-counting calculation strategies in the range 1 to 20, developed from the work of Treffers (1991); Gravemeijer, Cobb and colleagues (Gravemeijer, Cobb, Bowers, & Whitenack, 2000); and Wright and colleagues (Wright et al., 2007). Drawing on the *emergent modelling* heuristic (Gravemeijer et al., 2000), instructional design involves devising instructional procedures which foster students' *progressive mathematisation* from informal, context-bound knowledge to more formal, generalised knowledge. We seek to devise instructional *settings* in which students can first, develop context-bound knowledge of combinations and non-counting strategies—such as identifying five red and three green dots as eight dots—and then, reflect on their activity, and generalise toward more formal reasoning about numbers—such as partitioning eight into five and three to solve the written task 8-3 without counting. Building knowledge of combining and partitioning numbers supports the development of facile calculation (Bobis, 1996; Fischer, 1990; Young-Loveridge, 2002). For addition and subtraction in the range 1 to 20, informal non-counting strategies commonly develop around doubles combinations, combinations with 5 and 10, and tens-complements (9+1, 8+2, 7+3, 6+4, 5+5) (Gravemeijer et al., 2000). Useful settings for these combinations are the *ten frame* for the range 1-10 (Bobis, 1996; Treffers, 1991; Young-Loveridge, 2002), and the *arithmetic rack* for the range 1-20 (Gravemeijer et al., 2000; Treffers, 1991) (see *Instructional Settings* below).

Method

The NIRP adopted a methodology based on design research (Cobb, 2003), with three one-year design cycles. In each year, teachers and researchers implemented and further refined an experimental intervention program with students identified as low-attaining in their schools. The program year involved (a) in term 2, a range of pre-assessments of the students; (b) in term 3, a ten week instructional cycle; and (c) in term 4, post-assessments. In total, the project has involved professional development of 25 teachers, interview assessments of 300 low-attaining students, and intervention with 200 of those students.

Assessments

The primary assessment instrument was an individual task-based interview, approximately 40 minutes in length, videotaped for later analysis. Tasks addressed the five key domains of the learning framework. The interview assessment informed instruction, and enabled an assessment of progress sensitive to the intended instruction. A written screening test was used to help identify students low-attaining in their cohort. It included numeral sequence tasks, horizontal addition and subtraction tasks, and word problems. Among other shorter assessments, significant for the present case study were the *One Minute Tests of Basic Number Facts* (Westwood, 2003), for addition and subtraction. The addition test uses a page listing 33 additions of two single digit numbers written in horizontal format; the student is given one minute to write answers to as many as possible. The subtraction test is structured similarly. Each of these assessment instruments was administered as pre-assessments in term 2 and as post-assessments in term 4.

Instructional Cycle

In each school, two students were taught individually, six in trios. The instructional cycle consisted of approximately thirty 25-minute lessons across 10 weeks. Each lesson typically addressed three or four domains of the learning framework. Individual lessons were videotaped, providing an extensive empirical base. The analysis of the learning and instruction is informed by a teaching experiment methodology (Steffe & Thompson, 2000).

Instructional Settings

Ten frames 1-10. A 2x5 frame with a standard configuration of dots for a number in the range 1 to 10, either pair-wise (such as four dots on each row) or five-wise (five and three).

Tens-complements cards. Ten frames with ten dots of two colours in the combinations: 9 and 1, 8 and 2, 7 and 3, 6 and 4, 5 and 5, configured pair-wise and five-wise.

Ten frame addition cards. The 25 frames having 1-5 red dots on one row and 1-5 green dots on the other.

Arithmetic rack. Two rods, each with five red and five blue beads. Like on a counting frame, beads can be moved to one end of the rods to present certain configurations, such as 5-and-1 on upper and 5-and-1 on lower; or 10 on upper and 2 on lower.

Expression card. Two addends in the range 0 to 10, in horizontal format (such as 2+7). The set of expression cards includes all 121 such expressions.

Case Study Overview

Nate was eight years old and in the 3rd grade. His intervention teacher was Louise. Nate was selected as a case study of Structuring Numbers 1 to 20 for three reasons. (a) He participated in the third year of the study, when the instructional procedures in Structuring Numbers 1 to 20 were most developed. (b) He began the intervention with some knowledge of non-counting additive strategies in the range 1-10, so he had some scope for progress in the range 1-20 within the ten weeks of instruction. (c) He made major progress in non-counting strategies and timed computational fluency over the course of the intervention. The purpose of the case study is to document Nate's progress, and to describe the intervention instruction that seems to have been significant. The study informs our knowledge of the potential usefulness of the instructional settings and procedures.

The Case of Nate

Below, we compare Nate's knowledge of addition and subtraction in the range 1-20 in his pre- and post-assessments. We then describe relevant episodes of the instruction.

Nate's Assessments in the Range 1-20

Pre-assessments. Nate's pre-assessments were in April and May. In the interview, he could say how many more to make ten for 5, 9, and 7 fluently, and for 2 and 4 with four seconds of thinking each. He found partitions of 7 using his fingers, but then found partitions of 6, 12, and 19 without fingers or counting. Asked the doubles 8 plus 8, 9 plus 9, and 7 plus 7, he thought for five seconds for each answer, but did not apparently use counting. For 9 plus 9 he answered "19". He added ten to each of 7, 5, and 1 successfully, without counting by ones. Five tasks were presented in written horizontal format. He solved 6+5 using a near-doubles strategy. 9+6, 8+7, and 11+8 were solved by counting-on by ones, using fingers to keep track of his counts. 17-15 was solved counting back 15 counts from 17. The written screening test included 12 addition and subtraction tasks in the range 1-31, in horizontal format. Nate incorrectly answered four subtractions: 15-9 (9), 14-11 (2), 9-4 (3), and 23-17 (7). In summary, in the range 1-20, Nate knew some useful combinations such as tens-complements, doubles, and tens-combinations, but still tended to use counting strategies for unknown calculations, and was error-prone to some extent.

Post-assessments. Nate's post-assessments were in October. In his interview, Nate was more fluent on the tens-complements, partitions, doubles, and tens-combinations tasks, and did not use counting on these. He made one error, initially stating 9 and 11 as two numbers to make 19 and then saying "18 and 1". Asked for another partition of 19 he answered "12 and 6". He answered the five tasks presented in written format quickly and successfully, using non-counting strategies. On the written screening test, he answered all 12 addition and subtraction tasks correctly. In summary, in the range 1-20, Nate showed increased fluency with combinations and partitions, a shift from counting to non-counting strategies, and increased facility and success with calculation.

Progress on One Minute Tests of Basic Facts. These tests were not intended as proximal measures of the intended learning, rather as potential benchmarks of computational fluency. Useful for comparison, Westwood's study (2003) provides mean scores and standard deviations for ages 6 to 11 from a large cohort (n=2297) of students in Adelaide in 1995. Nate's one minute tests make a striking comparison from pre- to post-assessment. In April, Nate scored 12 of 33 for addition, and 12 of 33 for subtraction. In October, he scored 30 and 31, respectively. Comparing Nate's progress with the results for Westwood's 1995 cohort (see table 1), Nate has progressed in addition from half a standard deviation below the mean to two standard deviations above, and in subtraction from a mean score to three standard deviations above the mean. In both cases he has almost attained the maximum score, and made a marked atypical leap in computational fluency.

Table 1

Nate's Scores Compared with Westwood's 1995 Mean Scores and Standard Deviations

		Addition			Subtraction		
Month	**Age**	**Score**	**1995 mean**	**Std. dev.**	**Score**	**1995 mean**	**Std. dev**
April	8.5	12	15.54	6.43	12	12.28	5.59
Oct	9.0	30	16.79	6.10	31	13.11	5.79

Instruction–Structuring Numbers 1 to 20

The intervention included 24 individual lessons over 10 weeks, from July to September. Instruction in Structuring Numbers 1 to 20 was mostly restricted to the 19 lessons of the first six weeks. Below we describe episodes from weeks 1, 2, 5, and 6, to show some key instructional activities that seemed to be productive, and to indicate the progression of the teaching and learning across these weeks of intervention.

Weeks 1 and 2: Ten frames. Louise used a set of ten frame cards to present a pattern identification activity. She would show each card, and Nate's task was to name the number. Later she changed to flashing each card. In each of lessons 1-9, Louise worked twice through the pair-wise 1-10 set, or the five-wise 1-10 set, or both. Nate was generally successful and facile on these tasks. In lesson 5, Louise introduced the tens-complements ten frame cards. Nate's task for each card was to say how many of each colour, for example "6 and 4; 2 and 8." Louise showed the set twice, then flashed the set twice. During the first turn showing the five-wise tens-complements set, Nate made one error, identifying the 8&2 card as "8 and 1". He made the same error when the set was flashed. He commented that the five-wise set was hardest.

Lesson 6, week 2: How many more to make ten? In lesson 6, after flashing the five-wise 1-10 set, Louise extended the task, asking Nate to say for each card both the number of dots and how many more to make ten. Nate rapidly answered the first three cards with "Five and five! Nine and one! Two and… two, eight." He then commented that the task was like the tens-complements cards. After this activity, Louise showed and flashed the pair-wise tens-complements set, and flashed the five-wise tens-complements set twice. Nate answered without errors, and with increasing fluency.

Lesson 6, week 2: Ten frame addition cards. Later in lesson 6, Louise introduced the ten frame addition cards. Nate's task was to say how many dots of each colour, how many altogether, and how many more to make ten. Thus, the activity involved identifying dot patterns, combinations less than ten, and complements to ten—a rich activity in structuring numbers 1 to 10. Louise posed 11 tasks, and Nate was successful with each task. Recognising the two numbers was easy for him. Some sums and tens-complements were apparently more difficult for him. In solving the sum of the first card (3&4), Nate answered "Three and four make nine…no (looks away from the card), three…three and four make…(looking up)…s-…seven". On the remaining tasks, Nate sometimes looked away from the card, and sometimes at the card, and may have used some counting by ones.

Lesson 13, week 5: Doubles and 10-plus. Louise used the arithmetic rack to present doubles and 10-plus tasks. Three sets of tasks followed a sequence: (a) not screening the rack, (b) screening and momentarily unscreening the rack, and (c) keeping the rack screened and posing the tasks verbally. Next, Louise posed doubles and 10-plus tasks verbally, without the rack. Nate was generally successful and fluent on these tasks.

Lesson 16, week 6: Structuring 1 to 10 consolidated. Lesson 16 included five sets of tasks involving structuring 1 to 10. Louise flashed each of the pair-wise and five-wise tens-complements cards. Nate was fluent with these despite that Louise had not used them since week 3. Next, Louise used the ten frame addition cards, as she had in lesson 6, but now only flashed them. Nate's task was to say the two numbers and the sum. By contrast with his effort in lesson 6, he was generally fluent on these tasks.

Following the addition card activity, Louise posed tens-complements tasks verbally: She would say a number in the range 1-9, and Nate's task was to say how many more to make ten. Nate answered fluently. Next, Louise introduced a set of expression cards with sums in the range 6 to 10. As she placed each card on the desk, Nate's task was to say the sum as quickly as possible. Nate was fluent and successful with all 31 of these tasks, and did not use counting by ones. The written setting (expression cards) is typically more challenging for students than structured settings; Nate's facility with number combinations to 10 in this activity was significant. Following the expression card activity, Louise presented tasks on jumping to the nearest decuple, which involved applying knowledge of tens-complements with numbers beyond 20. Nate was successful both in a screened ten frame setting, and with purely verbal tasks.

Lesson 19, week 6: 9-plus and 8-plus. Following an initial segment involving doubles and 10-plus tasks presented verbally, Louise presented a set of 9-plus tasks, using the arithmetic rack. First, with 9 on the rack, Louise asked "9 and 3", and moved 1&2 on the rack. Nate seemed to be confused. After four such tasks, they discussed why Louise was making the addend in a 1&X form, and she presented three more tasks. Next, Louise screened the rack. She called out the additions—"Nine and four more"— and after Nate answered, she lifted the screen for him to check. Nate was successful with these tasks and became more engaged, looking up from the screen and thinking hard. He was not using counting by ones, and it seems likely he was adding through ten and perhaps visualising the rack in doing so. Finally, Louise posed eight 9-plus tasks verbally. Nate was successful with these, generally answering within one second. Following these 9-plus activities, Louise presented similar tasks involving 8-plus, first on the arithmetic rack, and then verbally. Nate was successful on these tasks, apparently with less certitude than he showed with the 9-plus tasks. On the verbal task, 8 and 5 more, Nate nodded his head three times before answering "13", then stated that he had counted by ones. This suggests that his use of non-counting strategies was not yet fully routine. Rather, using a non-counting strategy was, to some extent, engendered by the setting, that is the arithmetic rack.

Discussion

Over the course of the intervention, Nate made significant progress with addition and subtraction in the range 1-20, changing from using counting to using non-counting strategies, and attaining high computational fluency. The instructional procedures in structuring numbers 1 to 20 appeared to be significant for his learning. His progress is evident within single lessons, for example in lesson 19 establishing a non-counting strategy for 9-plus tasks. Progress is also evident in activities repeated over a series of lessons, for example in fluency with the tens-complements cards from lesson 5 through to lesson 16.

As part of the design research, the case study informs the design of the instructional procedures for Structuring Numbers 1 to 20. We describe four features of the instruction which seemed to facilitate Nate's learning. First, Louise generally focused each lesson segment on a particular aspect of structuring numbers. Aspects in the range 1-10 were: five-wise and pair-wise patterns for numbers 1-10, tens-complements, and combinations less than ten. Aspects in the range 11-20 were: doubles and 10-plus patterns for 11-20, 9-plus and 8-plus combinations, near-doubles combinations. These aspects are roughly identifiable as subsets of the combinations in the range 1-20, some are also identified with particular sets of ten frame cards or expression cards. In organising the instruction by aspects, the range of any one activity did not become unmanageable for Nate, and Louise could target the cutting edge of his knowledge of that aspect.

A second feature of instruction is the use of the ten frames and arithmetic rack. Over the first five weeks working in the ten frame setting, Nate became facile with combining and partitioning numbers in the range 1-10. In the 9-plus episode in lesson 19, the setting of the arithmetic rack seemed critical in supporting Nate's attention to reasoning without counting by ones. The case study affirms the usefulness of these settings.

Table 2

Settings Used in Each Week, for Each Aspect of Structuring Numbers 1 to 20

Week	Patterns 1-10	Tens-complements	Sums ≤ 10	Doubles, 10-plus	Sums > 10
1	Show TF →Flash TF				
2	→Flash TF	Show TF →Flash TF	Show TF		
3	→Flash TF →Flash AR	Show TF →Flash TF		View AR →Flash AR	View AR →Flash AR
4			→→→Exp cards		
5		→→Screen TF →→→Verbal		→Flash AR →→Screen AR →→→Verbal	
6		→Flash TF →→Screen TF →→→Verbal	→Flash TF →→→Exp cards	→→→Verbal	View AR →→Screen AR →→→Verbal

Note: TF=A ten frame set. AR=arithmetic rack. Exp cards=a set of expression cards. → =setting distanced.

A third feature of instruction is the progressive distancing of the settings, through the use of flashing and screening. Progressive distancing is evident at micro- and macro-levels of the instruction. Within a lesson segment with ten frames, Louise would move from showing to flashing the cards. Extended lesson segments, such as the 9-plus episode in lesson 19, consisted of a progression: Showing the setting, then flashing, then screening, and finally removing the setting and posing tasks verbally. This progressive distancing enabled Nate to apply his knowledge of patterning and partitioning to reasoning without counting by ones, on verbal tasks. On the macro-level, over the weeks, Louise moved from showing the structured settings towards verbal and written settings. Table 2 shows the progression of settings Louise used for tasks on five different aspects of Structuring Numbers 1 to 20. Progressive distancing of the setting is evident down each column.

A fourth feature of instruction is that the aspects were not treated as discrete or disconnected. Aspects were connected within lessons, for example, in the lesson 6 episode "How many more to make ten?", an activity identifying patterns 1-10 became an activity of reasoning about tens-complements, and in lesson 19, Louise reviewed 10-plus tasks before a set of 9-plus tasks. Further, the instruction in these aspects was not in lock-step fashion. As seen in Table 2, the introduction of each aspect is staggered across the weeks, and attention to the aspects overlaps, with each aspect at different stages of progress.

This case study provides a detailed overview of instruction that resulted in significant learning in the domain of structuring numbers to 20, and major progress on two timed tests of computational fluency. We conclude that the instructional procedures described here are viable as a basis for intensive intervention and we will continue to refine them.

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