

One Secondary Teacher's Use of Problem-Solving Teaching Approaches

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This paper reports part of a larger study and examines one teacher's use of problem-solving teaching approaches in Years 7 and 9. Thirteen problem-solving lessons were observed over an 18 month period during which the teacher devised and used 7 different problem-solving tasks. Three tasks are described in detail and analysed in terms of task structure and implementation, and how the teacher managed whole-class discussions. Analysis highlights changes in the design and use of the tasks. They became more open-ended and the teacher improved the quality of whole-class discussions to promote student learning and reflection.

Anderson and White (2004) distinguish between *problem solving*, “the process of students exploring non-routine questions, using a range of strategies to solve unfamiliar tasks, as well as developing the processes of analysing, reasoning, generalising and abstracting” (p. 127), and *problem-solving teaching approaches*, “investigations, open-ended questions, and modelling tasks, as well as providing opportunities for students to pose questions and explore new ideas” (p. 127). Problem-solving teaching is therefore an approach in which “teachers see themselves as guides, listeners, and observers rather than authorities and answer givers” (Norton, McRobbie, & Cooper, 2002, p. 39).

Problem solving is an important part of what it means to do mathematics and students require frequent opportunities to solve complex problems and reflect on their thinking (National Council of Teachers of Mathematics, 2000). In New South Wales, the revised syllabus for Years 7 to 10 (Board of Studies NSW, 2002) highlights the importance of problem solving through its *Working Mathematically* strand which includes student processes of questioning, applying strategies, communicating, reasoning and reflecting as critical aspects of mathematics learning (Clarke, Goos, & Morony, 2007). Despite the prominence of problem solving in recent syllabuses and the professed support of teachers for it, research suggests that many teachers do not use problem-solving activities (Anderson, Sullivan, & White, 2004). The research reported here describes how one secondary teacher began to incorporate problem-solving teaching approaches into his lessons and documents the changing nature of the tasks he devised and how he used them.

Analysing Lessons and Tasks

A number of research studies have examined how to support student learning in reform classrooms. These studies broadly refer to three key elements: explicit teacher actions to communicate expectations and encourage student engagement, mathematical tasks and tools, and “building a learning community” (Sullivan, Mousley, & Zevenbergen, 2004) through the ways that teachers design and structure activities and interact with students to support their learning. Some illustrative studies are now described in more detail.

Artzt and Armour-Thomas (2002) developed a model for examining teachers' instructional practices and distinguished between *lesson phases* and *lesson dimensions*. Lesson phases refer to the ways in which teachers initiate, develop and conclude instructional stages of lessons. Phases include the introduction (establishing students' readiness to learn), investigation (helping students learn new concepts and construct new meanings), and summary (assisting students integrate what they have learned and extend it). Phases provide “a temporal sequence of teaching-learning experiences” (p. 11) occurring over one or more lessons. Lesson dimensions are aspects of instructional practice that foster student learning with understanding and include learning tasks (designed to capture students' attention and sequenced so that students can progress and connect new knowledge to what they already know), learning environments (the social and intellectual climate of the classroom designed to support student engagement) and discourse (including teacher-student and student-student communications, and especially the teacher's use of questioning and wait time to encourage student thinking).

Stein, Grover, and Henningsen (1996) examined the characteristics of classroom tasks used during reform-oriented instruction to enhance students' mathematical thinking. The researchers define a *mathematical task* as “a classroom activity, the purpose of which is to focus students' attention on a particular mathematical idea”

(p. 460). Tasks include students' productive output during a lesson, how they produce it, and the resources which they use. Stein, Grover, and Henningsen differentiate between the teacher's expectations of how the activity should be conducted, including the distribution of materials and the teacher's instructions (the *task set up*), and how students actually complete the task (the *task implementation*). They also define *task features* (aspects of the task which promote high-order thinking and permit the use of multiple representations or alternate solutions) and *cognitive demands* (thinking processes entailed in solving the task). They analysed 144 tasks ranging from 10 to 51 minutes duration and comprising an average 52% of total lesson time. Tasks found to support students' mathematical thinking and reasoning included those with high cognitive demands requiring students to explain or justify their reasoning. Teachers can assist students' mathematical development by designing task set up and features to maintain students' focus on mathematical processes and scaffold student learning, but not in ways that detract from the cognitive demands of the task.

Carpenter and Lehrer (1999) proposed three classroom instructional dimensions which may assist in developing students' mathematical thinking and reasoning: *tasks*, *tools* and *normative practices*. While noting the importance of appropriate task and tool selection, the researchers describe the crucial role of normative practices in fostering understanding. In particular, classroom practices should facilitate students' ability to relate their current understanding to new concepts and support the structuring of new knowledge by allowing students to apply and compare alternate solution strategies, make links between different representations of problems, and build on basic problem-solving skills. Teachers can provide extended opportunities for students to articulate and explain their thinking, both during problem-solving activities and in the whole-class discussions which follow.

The critical role of the teacher in the reflective phase of an inquiry-based lesson was considered by Leiken and Rota (2006). They examined the frequency and duration of teacher behaviours such as listening and watching to begin, continue, or summarise a class discussion. They noted that teachers could support students' mathematical thinking during the discussion phase of lessons by asking open-ended questions and waiting for students to explain their answers fully before attempting to summarise them or move on. Whole-class discussions were found to be more fruitful when the teacher asked questions that related directly to student conjectures, stimulated replies from students, and built on students' explanations to develop new mathematical ideas.

Method

The research reported here is part of a larger study focussing on how secondary mathematics teachers in NSW responded to a new syllabus, particularly the extent to which they implemented its Working Mathematically strand (see Cavanagh, 2006). Data were sourced from questionnaires, classroom observations, student work samples and teacher interviews. One participant from the larger study expressed an interest in developing problem-solving teaching approaches and was invited to take part in a second round of interviews and classroom observations which are the subject of this paper.

Mr Richards

Mr Richards (not his real name) was the head mathematics teacher at an independent, co-educational school in a mid to high socio-economic suburb of Sydney. In 10 years teaching, he had taught all secondary grades, though he stated his preference for middle and lower ability classes in the junior secondary years since he felt there was less pressure from external examinations, allowing him greater freedom to experiment in his teaching. Mr Richards said his teaching was "not very conventional" because he tried to involve students actively in their learning, though at the start of the study he was not always sure about how to do this.

Research Design and Data Collection

An ethnographic case study design (Yin, 1994) was used to investigate how Mr Richards developed and used problem-solving tasks with two classes over an 18-month period. Data collection included lesson observations, teacher interviews, student work samples and researcher field notes. The observed lessons were digitally videotaped and the recordings made available during a stimulated recall interview with Mr Richards which lasted approximately 30 minutes and took place as soon as practicable after the lesson. Each interview was structured in three parts: First, Mr Richards discussed how he had developed the lesson materials and the intended purpose of the task; second, he viewed short segments of the lesson and the researcher encouraged

him to describe what he was doing and thinking during the lesson; third, he reflected on the entire lesson, discussing its effectiveness and describing how he might teach it differently if he were to repeat it. The interviews were audio-taped and transcribed.

In total, 13 problem-solving lessons were observed. Each lesson was 50 minutes long and based on a problem-solving task which Mr Richards had devised. Typically, each task required two or three lessons for students to complete in small groups and report their findings, and the lessons followed basically the same structure. After settling the class, Mr Richards introduced the problem and ensured students understood what they were required to do. Students completed the task for the remainder of the lesson while Mr Richards circulated among them to monitor their progress and deal with any difficulties they experienced. In the following lesson(s), students completed their investigations and sometimes Mr Richards chose individuals to present their results or he led a discussion on the mathematical ideas which had emerged from the students' work. This paper reports in detail on three of the tasks, selected because they show how Mr Richards' problem-solving teaching approaches developed over the course of the study.

Problem-Solving Tasks

Task 1. This task, shown in Figure 1, was the first to be observed and comprised three lessons. It was for Mr Richards' Year 9 class during a two-week unit on statistics. In the first lesson, students measured and recorded the length of eight of their body parts (e.g., height, foot length, arm span). That evening, Mr Richards combined all of the students' data in a spreadsheet so that they could work in pairs to analyse it in the next lesson using graphics calculators to display scatter plots and writing individual reports of their findings. In the third lesson, students concluded their reports and Mr Richards asked selected individuals to present their work to the class.

Body Parts Investigation Choose any two body-part measurements and do the following: • Enter the data into the List menu of your graphics calculator • Perform a list division Consider the results of the list division: • Does it look like there is a correlation? • Find the mean of the list division • Draw a scatter plot on your graphics calculator • Find the correlation coefficient Write a paragraph stating what you have discovered about the two measurements. Repeat the activity for any other two body-part measurements.
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Figure 1. Task 1.

Task 2. Mr Richards prepared this task, shown in Figure 2, for Year 9 to revise a unit they had just completed on volumes of solids. It was the second task observed and covered two consecutive lessons. In the first, students worked in pairs to draw designs of water tanks. In the next, they wrote individual summaries of what they had done and Mr Richards used the design created by one pair of students to demonstrate an algebraic solution to find the dimensions of a cylinder when its volume and radius were known.

Waggilendra's New Tank

On average, each person in the small country town of Waggilendra uses 30 litres of water per day. There are 2 500 people in the town. Waggilendra needs a water tank capable of holding 30 day's supply of water. You are a water tank consultant. You are to submit 2 designs for 2 possible water tanks. You are to:

- Find how many litres of water the tank should hold. Design 1 possible tank which will be sufficient for the town. The diagram does not have to be to scale. You must include the life-sized measurements.
- Design a second tank which uses a different type of shape. Include all life-sized measurements.

Figure 2. Task 2.

Task 3. This task, for Year 7, was observed near the end of the study. Students completed the task, in Figure 3, in two consecutive lessons. They began working in pairs to create open-top boxes by cutting out equally-sized squares from each corner of a piece of grid paper and folding up the sides. Mr Richards asked the students to calculate the volume of each box. In the following lesson, he led the class in analysing their results and used a data projector to display a spreadsheet containing values for the length of the cut-out squares (x) and the volume of the box (V). He also showed students a graph of V as a function of x , which he produced using a graphics calculator. The objective of the analysis was for students to find the dimensions which would maximise the volume of the box.

The Open-Top Box

It is possible to make an open top box by cutting squares from the corners of a rectangular card and then folding up the sides. If the card is 12 cm by 6 cm, what might be the volumes of some boxes you can make?

Figure 3. Task 3.

Data Analysis

Glaser and Strauss (1967) propose a constant comparison analysis of key concepts which gradually surface from the data analysis. Data for each task including lesson observations and post-lesson interviews were analysed prior to subsequent school visits. The lesson videotapes were viewed and sections which the researcher thought to be representative of Mr Richard's approach in the lesson were transcribed and read. A similar method was applied to the post-lesson interview transcripts so that general impressions of Mr Richard's statements could be made before a more detailed coding for recurring themes was undertaken. Emergent themes from the researcher's earlier field notes and observations were then used to interpret the subsequent lesson observations and interviews. Data from subsequent tasks were similarly analysed collectively before a final examination of all of the data from the different sources was then undertaken to cross-reference and confirm the main ideas which had been identified. The purpose of all of the analysis was to identify and track changes in the nature of the tasks which Mr Richards devised and how he used them in the classroom.

Analysis and Discussion

Tasks

There were significant changes in Mr Richard's design of the tasks, particularly in their set up and features (Stein, Grover, & Henningsen, 1996). Task 1 is typical of Mr Richard's early attempts to maintain tight control by carefully structuring student activities so they completed a number of steps in a specified order. Accompanying worksheets and Mr Richard's detailed instructions at the start of each lesson precisely described what students had to do. He also chose task features which provided a narrow focus for students' investigations. For instance, students were not encouraged to make any connections between the summary statistics and the shape of their scatter plots. Mr Richards repeatedly stated that his primary objective in the early tasks was for students to learn to persevere with the activity and produce detailed written summaries which explained their solutions clearly.

Tasks 2 and 3 were more open-ended and, instead of repeatedly exhorting students to follow the steps and set out their working carefully, Mr Richards now encouraged the class to “look for patterns in your answers”. Although Task 2 maintains a narrow focus, Mr Richards took advantage of the potential for multiple representations that were a feature of Task 3 and discussed them to a much greater extent than he did previously. Linking the numerical values for the side of the cut-out squares, x , and the volume of the box, V , to the graph produced on the calculator and then working with students to develop an algebraic formula for V in terms of x allowed Mr Richards to make connections between the different features of the problem and encourage students to relate them back to the original task of creating a box and maximising its volume (e.g., by interpreting the coordinate points on the graph with respect to the numerical values in the spreadsheet and the size of the cut-out squares). Although Tasks 1 and 2 could potentially have been used to demonstrate alternate representations, this did not occur. However, the coordination of multiple representations in Task 3 placed a significantly higher cognitive demand on students and ensured a more productive lesson.

Lesson Phases

The description of lesson phases (Artzt & Armour-Thomas, 2002) is used to report changes in the organisation of Mr Richards’ problem-solving lessons and Table 1 shows the lesson time allocated to each lesson phase in the tasks. While the introduction phase for each task decreased slightly over the three observed tasks, the most salient feature of the lesson phases is the extra time allocated to whole-class discussion over the three tasks.

Table 1

Proportion of Time in Each Lesson Phase

Task	Introduction phase	Investigation phase	Discussion phase
1	20%	74%	6%
2	19%	68%	13%
3	15%	59%	26%

There were qualitative differences, too, in the teacher-student interactions in the discussion phase across the three tasks. In Task 1, seven students presented their findings to the class but each spoke for less than a minute and simply recounted their summary statistics and described the shape of their scatter plot. Occasionally Mr Richards asked a direct question such as “What did you get for the mean?” but otherwise he was silent. He did not further probe the students’ thinking, even when they provided fairly simplistic accounts of what they had done. In Task 2, Mr Richards spent about nine minutes leading the class through the design made by one pair of students in order to demonstrate an algebraic solution to the problem—a strategy which few had employed in their investigations. During that time, Mr Richards asked 22 questions of the class, but only two of these were questions which required more than a straightforward response such as a numerical result or calculation. His questioning adhered closely to the traditional IRE recitation pattern (Mehan, 1979) of teacher initiation followed by student response and teacher evaluation as the following excerpt shows:

- Mr R: I want to show you another way because most of you just picked a number and then used trial and error, which is perfectly fine, but I want to show you another way that you could have done this. [He uses the student example of a cylinder of radius 5mm]. What is the formula for the volume of a cylinder?
- S1: πr^2 times height.
- Mr R: So πr^2 is the circle and h is the height. Now, how much does the volume have to be?
- S 2: 2 250 [m^3]
- Mr R: Good. So 2 250 equals π times, and what’s π ?
- S 3: 3.14

Mr R: Okay, their r was 5 millimetres so that's $(0.005)^2$, times by h . Now if you were able to manipulate the formula you wouldn't have had to guess and check. So let's have a look. [He writes $2\,250 = \pi (0.005)^2 h$ on the board] What can we do to this so we can just plug numbers in and find out what h is?

Students: [no response]

Mr R: This is algebra. What do we do to both sides?

S 4: Divide it.

Mr R: By ...?

S 4: π .

Mr R: So if we divide by π we get $2\,250$ over π equals what?

S 5: $(0.005)2h$

Mr R: [He writes $2\,250/\pi = (0.005)2h$ on the board] Alright, the last step is to do what?

Students: [no response]

Mr R: What do we need to do to both sides to get h on its own?

S 4: Divide.

Mr R: Divide by what?

S 4: $(0.005)^2$

The conversation continued briefly until the class found the height of the cylinder but Mr Richards made no attempt to compare the solution strategies, nor did he ask students to apply the algebraic method to one of their own designs. The classroom discourse at the conclusion of Task 3 was different in a number of significant ways. Not only were multiple representations compared, but the style of Mr Richards' questioning changed: he asked fewer questions overall, but there were more open-ended questions that called for opinion and conjecture from students; he allowed wait-time for students to respond; and he sanctioned incorrect responses without immediately discarding them. For example, the following exchange took place when the students were discussing the spreadsheet showing values of x , the length of the cut-out squares, and V , the volume of the box:

Mr R: Okay, let's look for a pattern. Someone describe what's happening with those numbers there [values of V and x]:

S 1: Except for 0.3 they are all going up and then down.

S 2: They all increase up to 1.3 and then they drop.

Mr R: Yes ...

S 3: The 6.8 plus the 12.9 is almost 18.5

S 4: Just with the decreasing theory, what about 2.1, 2.2 and 2.3? Oh, sorry I thought it was 28.

S 5: If you plus 18.5 with 6.8 it's almost 23.3

Mr R: Yes, ok let's just realise what's going on here. The x -values are what?

Students: [No response].

Mr R: You need to know what is actually happening here. Hands up: what is x ? You've just done a whole lot of calculations and if you don't know what x is you're missing the point of the exercise. What are these x numbers? Have a think. Have a think about what you've just done ...

Conclusion and Implications

This article describes how one secondary mathematics teacher sought to change his classroom practice by incorporating problem-solving learning activities into his lessons. There were a number of factors which assisted Mr Richards in this process. His personal philosophy of teaching mathematics was conducive to allowing time for student investigations since he was not inclined to rush through the syllabus in order to cover the entire content. He also attended two professional development sessions which he described as very influential in alerting him to reform teaching approaches. His accounts of the workshops echo some of Guskey's (1986) guiding principles for professional development in that Mr Richards regarded the presenters as credible practitioners who offered realistic and achievable ideas, and the feedback on the effects of the changes he was making on student learning was positive; student engagement increased and their reports of what they had done became more sophisticated. Mr Richards also adopted a measured approach in reforming his practice. He quickly realised that he could not change everything at once and was judicious in choosing one or two specific areas on which to focus. In doing so, he gradually learned some of the new skills he needed to shift from a transmitter of knowledge to a facilitator of learning and he assisted his students in progressively becoming more active participants in the classroom. Mr Richards also acknowledged that the work he undertook with the researcher as part of this study had been helpful in focussing his attention and providing a mechanism for reflecting on his practice. There could be value in replicating and documenting a similar model of "researcher as sounding-board" in future studies that cover the professional growth of individual teachers.

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