The Case of Mathematical Proof in Lower Secondary School: Knowledge and Competencies of Pre-service Teachers

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A case study of the preparedness of pre-service mathematics teachers to teach proof at lower secondary is reported. Data collected by questionnaire and problem-centred interviews were subjected to in-depth qualitative content analysis. Knowledge and competencies demonstrated by two future teachers are examined. Both displayed high affinity with proving and a formalist image of mathematics but their inferred abilities to convey this affinity to students at this point in their careers differed. Implications for teacher education and university mathematics are outlined.

A renewed interest in proof and proving in school can be observed worldwide. This is reflected by the planning of an ICMI study focusing on proof. In the call for papers of this study Hanna and de Villiers (2008) state that this "renewed curricular emphasis on proof" (p. 1) in school mathematics is occurring at all grade levels in many parts of the world. However, others point out that "many school and university students and even teachers of mathematics have only superficial ideas on the nature of proof" (Jahnke, 2007, p. 80). Often secondary students prefer procedures of validation in mathematics that are empirically based even when they know they are expected to produce a deductive argument (Healy & Hoyles, 2007; Heinze & Kwak, 2002). The fact that this preference occurs for even high performing students leads Jahnke (2007) to conclude that "the usual teaching of mathematics is not successful in explaining the *epistemological meaning* of proof" (p. 80). Indeed, when the word "proof" is used in schools this might not mean a formal mathematical proof. As in mathematical research these days, proving in school "spans a broad range of formal and informal arguments" (Reiss & Renkl, 2002, p. 29).

Thus, as Hanna and de Villiers (2008) point out, "mathematics educators face a significant task in getting students to understand the roles of reasoning and proving in mathematics" (p. 1). This applies as much to tertiary mathematics educators as it does to teachers in schools. One of the questions raised by Hanna and de Villiers in the ICMI study discussion document is: "How can we design opportunities for student teachers to acquire the knowledge (skills, understandings and dispositions) necessary to provide effective instruction about proof and proving?" To be able to do this it is first of all necessary to be able to determine the competencies and the nature of the knowledge (mathematical or pedagogical content) pre-service teachers possess with regards to proof. These could have implications for both teacher education and tertiary mathematics courses that pre-service teachers take. Some studies involving pre-service teachers have been carried out (e.g., Selden & Selden, 2003; Simon & Blume, 1996; Stylianides, Stylianides, & Philippou, 2007). There is, however, a dearth of research literature in this area especially in the case of pre-service mathematics teachers for lower secondary school.

Selden and Selden (2003), for example, investigated the ability of 8 undergraduate students (including 4 secondary pre-service mathematics teachers) in a "transition to proofs" course (p. 18) to judge the correctness of four student "proofs" of a mathematical statement. The researchers concluded that the undergraduate students tended to focus on the surface features of arguments and their ability to determine whether an argument proved a theorem was limited. They point out that eventually most mathematics students learn to do this but cannot say when this happens. "Validation of proofs is part of the implicit curriculum, but it is a largely invisible mental process. Few university teachers try to teach it explicitly" (p. 28). This is somewhat of a concern for the preparation of secondary mathematics teachers who will be expected at some point in time to be able "to judge the correctness of their own students' proofs or novel solutions to problems" (p. 29). A further complication is that the reducing nature of mathematics requirements for entry into teacher preparation courses might mean that many of these future mathematics teachers have ceased their mathematics studies before they have learnt, or even experienced, how to carry out validation in a mathematical context.

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In the recent report from the international study, *Mathematics Teaching in the 21st Century* (MT21), where the goal was to examine how lower secondary mathematics school teachers were prepared to teach in six countries (Schmidt et al., 2007), there was reference to one of the objectives of mathematics being "student understanding of mathematics as a formal discipline" (p. 17) but no explicit mention of proof. There were several items focussing on proof in this study but these were not mentioned in the report. As a supplement to MT21, a collaborative study between researchers at universities in Germany, Hong Kong, and Australia has begun and one of the initial areas of interest is pre-service teachers' preparedness to teach argumentation and proof in lower secondary school.

In the various Australian state curricula, the emphasis on proof differs at the lower secondary level. In the Victorian Essential Learning Standards (VELS) (VCAA, 2005), for example, proof forms part of the working mathematically dimension, as well as being mentioned in the various content dimensions. In working mathematically, level 6 (usually completed by students in Years 9 and 10), students are expected to

> formulate and test conjectures, generalisations and arguments in natural language and symbolic form. … They follow formal mathematical arguments for the truth of propositions (for example, 'the sum of three consecutive natural numbers is divisible by 3'). (p. 37)

Within the structure dimension at level 6, students are expected to "form and test mathematical conjectures; for example, 'What relationship holds between the lengths of the three sides of a triangle?'" (p. 36) whereas Pythagoras' Theorem is mentioned in the space dimension learning focus (VCAA, 2005). The following case study examines the preparedness of pre-service teachers to teach such content.

The Case Study

Participants

In the case study to be described here the sample from one Australian site (a university located in Victoria) is the focus. Mathematics entry requirements for the secondary teaching course at this university exceed those required for teacher registration in Victoria. Eleven (out of a possible 14) pre-service secondary mathematics students, at the end of their year-long postgraduate diploma course, volunteered to participate in the study. Only 9 of these students completed the section of the data collection which dealt with argumentation and proof. From these 9, 6 were chosen for in-depth interviews. Interviewees were chosen on the basis of their mathematical competencies and pedagogical content knowledge being consistent with their espoused beliefs about mathematics and the teaching of proof or there being inconsistency between these. The responses and experiences of 2 of the interviewed students are the focus of this paper.

Instruments

The study used a questionnaire (approximately 90 minutes to complete) for all participants followed by a problem-centred interview (approximately 60 minutes) for respondents selected for in-depth study. Both instruments evaluate the professional knowledge of future mathematics teachers and were designed and evaluated by the third author for his ongoing doctoral project (cf. Kaiser, Schwarz, & Tiedemann, 2007). The items in the questionnaire are open requiring extended written answers. These items bridge the domains of mathematical knowledge, pedagogical content knowledge, general pedagogy, and mathematical beliefs. Items test several components of the professional knowledge domains as well as beliefs, linking different knowledge facets in an action-based design.

During the interviews, the pre-service teachers were asked about why they chose to be a teacher, their beliefs about mathematics and knowledge requirements in general for teaching mathematics in secondary school, and the adequacy of their university courses to address these needs for them. In addition, further questions probing aspects of the teaching and knowledge of the area of focus are included based around problem tasks presented to the interviewee. "The problem-centred interview (PCI) is a theory-generating method that tries to neutralise the alleged contradiction between being directed by theory or being open-minded so that the interplay of inductive and deductive thinking contributes to increasing the user's knowledge" (Witzel, 2000, p. 1).

The mathematical topics which are the focus of both instruments are the teaching of mathematical modelling and argumentation and proof in Years 8-10. This paper will focus only on argumentation and proof. The questionnaire contained one item consisting of several sub-items related to a proof of the proposition that doubling the length of the side of a square also doubled the length of its diagonal and another multi-part item about the proof of the sum of three consecutive natural numbers being divisible by 3. The interview included questions related to the proof of the Triangle Inequality Theorem. As the data collection is still in progress, the actual items have not been released at this point.

Analysis

The questionnaires were evaluated using qualitative content analysis methods (Mayring, 2000) which is a much more in-depth process than superficial quantitative content analysis (Silverman, 1993). Extensive coding manuals were developed for the questionnaire for Australia, Germany, and Hong Kong based on first trials. Joint coding with coders from all three countries was used to develop the coding manual, avoiding the reflection of cultural biases. The coding of data for the Australian sites was done independently by a team of German coders and one of the Australian authors. There were thus two codings in which any inconsistencies were then discussed and resolved. Variables of interest were the affinity variable of affinity to proving in mathematics lessons and the cognitive variables of subject-related adequacy of a formal proof, adequacy in construction of proofs, didactical reflection on proving, and diagnostic competencies with respect to proof.

The responses of the future teachers in the questionnaires and transcribed interviews were used to develop patterns of misconceptions of the future teachers, their strengths and weaknesses in terms of their ability to explain concepts in both deductive and inductive ways, to explore innovative teaching methods as well as to generalise and make use of knowledge to solve related problems. The relationship, if any, to beliefs about mathematics and mathematics teaching was also examined.

The Cases

Based on their responses to the questionnaire, the majority of the students at this site had a high affinity with proving in mathematics lessons and a formalist image of mathematics when it came to proof, “seeing mathematics as an abstract system that consists of axioms and relations” (Schmidt et al., 2007, p. 14). Two of the interviewed students, Ling and Gabby (pseudonyms), were selected as exemplars. They had formalist images of mathematics but showed differences in consistency between demonstrated mathematical competencies and their affinity with proving in mathematics teaching at lower secondary.

Ling has a Bachelor of Science (Honours) from an Asian university, where she majored in Chemistry but her degree included substantial mathematics studies in both first and second year. After graduating she worked for seven years as an industrial chemist in Asia before coming to Australia five years ago. She is now returning to the workforce but as a teacher because teaching is a “good job for a young family”. Ling’s questionnaire responses showed she had the highest possible level of affinity with use of proof in mathematics teaching. She believed that proof in mathematics lessons in secondary school was “important for training students to think clearly and logically and be able to argue the case but it could be too difficult for the general population. However, if it was taught early, perhaps in elementary school, it could become easier in secondary school.” However, the length of time between her formal mathematical studies and her going to teach, and her previous experiences with proof as a mathematics student at university meant she was restricted in her abilities to convey this effectively in her teaching at the point in time when she was about to begin her transition into the teaching workforce.

Gabby has a four-year undergraduate degree in pure mathematics from an eastern European university. The final year not only included mathematics subjects but also included mathematics teaching method, educational psychology and professional teaching practice. After graduation she worked for three years as a secondary school teacher before emigrating to Australia. However, before she could teach in Victoria, she was required to complete a graduate diploma in education, the course she had just completed at the time of data collection. She did not believe she had developed her teaching knowledge during her overseas undergraduate degree at all as “it is not like here so, umm, it’s not as intense as here in Dip Ed .… It was not as useful as this year here.” She also stated her first two years of teaching in a private school only taught her: “What I don’t want to do”. Her third year, this time in a state school, “was absolutely everything” and “nothing can surprise me anymore". Gabby's responses on the questionnaire showed she had a high level of affinity with proof in mathematics teaching. However, in the problem-centred interview (PCI), despite saying "formal proof, that is the proof I like and prefer", she revealed that

> when I was in school I didn't like it and then when I started teaching about it I was insisting on that. I hated it because I was learning all the proof and all congruencies and similarities off by heart because it was so hard to come up with formal proofs. … But proving using rules or axioms or some proved prior axiom, we can use it like working out from this and this. Yeah that's formal proof.

Gabby's most recent mathematical studies, her teaching experiences in Europe, and her previous experiences as a student with proof both in secondary school and university meant she had more resources immediately available to convey this notion effectively in her future teaching but some of her responses were enigmatic. She believed lower secondary students should be exposed to proofs done by teachers but not construct these themselves or even take much from the experience as the following PCI excerpt shows:

> I definitely believe they should be exposed to formal proofs all the time, so they get a feeling of it and being familiar with it. They will say, "Oh it is too hard" and "Why are we learning this?" and "What is this?" And probably, very likely, they are going to copy it only to copy it from the board but I think as long as something stays in their mind anyway. So I think that it is very important.

Competencies in proof construction. Ling exhibited very high competencies in formal proving when using Pythagoras' theorem for proving the proposition about doubling the sides of a square and its generalisation to rectangles but as she revealed in her interview the only theorem she could recall at all was Pythagoras' Theorem. On all other occasions when required to construct proofs whether geometric or algebraic her attempts were categorised as demonstrating low competency. She was unable to show algebraically the divisibility proposition for the sum of three consecutive natural numbers could not be generalised for k numbers. Her response showed she believed it was able to be generalised. When working with the Triangle Inequality Theorem she clearly believed, demonstrating with reference to a triangle, that it was possible to construct a triangle where the sum of the lengths of two sides was equal to the length of the third side. She then used an example of such a non-constructible triangle (with side lengths of 1, 9, and 10 units) to "disprove" the Half Perimeter Proposition (the half perimeter of a constructible triangle is always longer than each side of the triangle). Gabby, on the other hand, consistently demonstrated she had high competency in this area in all contexts. For instance, she was able to use her formal representation of the statement of the Triangle Inequality Theorem using algebraic notation to prove that the Half Perimeter Proposition for a constructible triangle was indeed true.

Mathematical content knowledge about different kinds of proof. Whilst Ling has a very high affinity for the use of proof in mathematics teaching, her own mathematical experiences have in some cases presented a negative view of proof and have not given her the basis to always successfully translate this affinity into her teaching practice as the following excerpt from the interview shows.

> Don't know much proof, didn't learn much about proof during my studies, and I remember I did proof only a very short experience in the first year of … maths in university, undergraduate study, and couldn't understand much of it. The lecturer just says, "Given this statement, this is how you prove it", and then [we] go to tutorial where we're given a set of questions. The tutor just walks around, and he would call any one of us to show the proof. If any can't show, they will just embarrass [laugh] and everyone will go to the classroom with a lot of fear [laugh]. "Have I found the answer or not?" And, um, yes, when anyone mentions proof, it's a fear [laugh] we don't know how to do it. It is too hard. We were shown how to do a proof to a certain statement, but how to get there, we don't understand at all, we just look at it. … No proof at all in high school…so the only experience was in first year of uni, and after that I dropped maths!

Gabby showed a high affinity with using proof in mathematics teaching and her previous experiences in secondary school and university where "we put lots of emphasis on proof" supported this. However, her pre-service diploma course experiences and exposure to pre-formal proofs in the questionnaire and interview problem tasks caused her to reconsider what she considered as "proof". When discussing the use of a rubber band on a board with three non-collinear nails to demonstrate the Triangle Inequality Theorem, she said,

> If I let them do it by themselves, it is not formal proof ... but it is like a certain degree of proving. At the junior level, it is more likely that's the proof for them which is not a proof of course, but this is empirical evidence. And what I have seen here, I have never seen before … It's like when they are proving things with that dynamic computer software, stretching or whatever ... I don't know if that can be considered as proved. I don't consider it as proof but if we do like, I think we can consider it as proof. Presently not, but I should leave a space for not so absolute 'only this is a proof', it might be accepted as a proof as well, the same thing with nails, and with that computer software.

However, she was fully aware from her teaching experiences that an over-reliance on empirical methods was misleading and allowed students to indulge in their preference for validation procedures that were empirical over formal methods (Jahnke, 2007).

> As long as something is measurable, they have no problem, or obviously visible or whatever. So the advantage is that you can actually show how it works. … it's misleading in the sense that students start thinking this is an actual proof, that it's not like evidence how this theorem works or an application. … So if you do that, then "Let's do the formal proof now", well they will start saying, "We already proved it! That's the proof!" So it's very potentially misleading that they got the impression that this is the proof and they carry out that impression like later on, that that's the proof.

Declarative knowledge about the different structures of proving. As a teacher it can be expected that in addition to being able to construct proofs, teachers will need to draw on their mathematical knowledge about the different structures of proving such as special cases/experimental "proofs", pre-formal proofs, and formal proofs (Blum & Kirsch, 1991) and pedagogical content knowledge when planning teaching experiences and when judging the adequacy or correctness of their, and their students', proofs in various mathematical content domains. The term "pre-formal proofs" means "substantial argumentation on a non-formal basis" (p. 184) where "the conclusions must be capable of being generalized directly from the concrete case" (p. 187). An example could be a geometric diagram used as a visual proof, such as the early Chinese proofs of the Gou Gu problem (cf. Pythagorean theorem) (Joseph, 1991, p. 180). Little attention had been given to pre-formal proving in the pre-service course at this university and the term "pre-formal proof" was met by the pre-service teachers for the first time in the questionnaire. A clear example was given but not all students realised it was able to be generalised and thus different from a special case.

Ling's questionnaire responses appeared to show she understood formal proof and possibly special cases, but she did not use these terms explicitly. In the PCI she thought pre-formal proof "would be using trial and error". She then continued: "informal proof is considering specific examples to show your statement is true or not true". With respect to formal proof, she stated that the prover "really [has] to go through these writing steps: If this, then that. Uhm, if this, therefore." Thus, Ling's responses showed only low levels of declarative knowledge about proving structures. Gabby showed evidence of high declarative knowledge about the structures of proving clearly distinguishing between empirical evidence, formal proof, and verbal argumentation that fell short of proof. However, she was unable to see the difference between pre-formal proof and the mere use of special cases. The pre-formal proof presented for doubling the diagonal of a square when its side is doubled, for example, was not sufficient proof for her "because it refers to a single case, in other words, this is proof for the case that is pictured in the associated diagram above. [It] does not cover all cases (squares) in general."

Feedback and diagnostic competencies. For the proof of the sum of three consecutive natural numbers being divisible by 3, a statement describing the proposition was presented along with several Year 9 students' "proofs" of this. The pre-service teachers were asked how they would respond as a teacher to each student in such a way as to both assess the "proof" presented and to provide motivation for further exploration or correction. For a student who had presented special cases to prove the given statement, Ling was able to evaluate the student's ideas: "You have proven for three sets of numbers. Very good." In addition, she suggested a path for the student to consider in moving more towards the notion of a generalisable proof: "Have you thought about other natural numbers that exist? Do you think this will always hold true? Why?" For a second student who provided an adequate algebraic proof, Ling assessed the response and provided meaningful feedback, "Excellent! You have used a generalisation and this is exactly what we do for proofs. It has to hold in general, for all situations." In this case she made no suggestion to help the student progress further. However, for an unfamiliar adequate diagrammatic proof she was unable to evaluate it or provide feedback as she could not understand it. It appears she was not expecting a visual proof to this proposition and hence could not make sense of one when presented. Clearly, when Ling was able to make sense of the student's work, she was able to provide both an evaluative response and suggestions for future progress. Gabby on the other hand was able to judge the responses correctly and provide meaningful feedback to these three students, not at all being perturbed by the unexpected format of the diagrammatic proof: "Very interesting solution. Correct, as well. Really, we can go on and on like that and there would be no remainder. What do you think, how can you prove correctness of the statement for any three consecutive numbers?" However, neither Ling nor Gabby provided meaningful feedback for a fourth student who had indicated that "you can't say" whether or not the statement was true and then attempted to argue why this was so, clearly not showing an understanding of the general nature of proof.

Didactical reflections about proving. To evaluate pre-service teachers' ability to reflect in a meaningful way about proof, they were asked firstly to judge if a pre-formal proof could be sufficient as the only kind of proof in mathematics teaching and explain their position. Ling firmly stated it could not be as "maths is like science, a hypothesis has to go through rigorous tests before it can be accepted as a proof. It has to be systematic." She was clearly interested in conveying an image of mathematics as formal and rigorous. Gabby's answer was coloured by her equating pre-formal proofs with empirical evidence which she clearly thought was not sufficient. Neither Ling not Gabby reflected on the variation in cognitive abilities of students or the complexity of theorems likely to be presented at this level of schooling. As further opportunities to reflect in a didactical manner about proving, the pre-service students were asked in the questionnaire and in the PCI the advantages and disadvantages of a formal and informal proof. Ling was unable to provide any meaningful examples in the questionnaire, whereas in the PCI she mentioned that a pre-formal proof "gives you the confidence that you are right". Gabby, on the other hand, showed high competencies in this area in both the questionnaire and the PCI, noting that pre-formal proofs were "easier to master" and could "show how it [e.g., a proposition] works" but could be "very misleading" if adopted as "actual proof" whereas "deductive (formal) proof is [much] more difficult for students to master, but once they understood it; it is conceptual understanding that lasts and can be retrieved and applied in future easily."

Discussion and Conclusion

Having a high affinity with proving in mathematics teaching at the lower secondary level and possessing an apparently adequate mathematical background well beyond that required for teacher registration are clearly not sufficient preparation for teaching the rather small amount of proof explicitly identified in the VELS. Whilst short pre-service courses must prioritise some topics at the expense of others, clearly time needs to be devoted to ensuring that future teachers are aware of their needs to revisit critical topics such as proof in order to activate dormant knowledge. Although both future teachers in this study demonstrated at least average competencies in the area of diagnosis of misconceptions about the nature of proof and providing meaningful feedback to enable students to develop their understanding of proof from their current status, the authors concur with Selden and Selden's (2003) suggestion that pre-service courses include opportunities for students to reflect on, evaluate, and provide feedback on a variety of student generated "proofs" to propositions at the lower secondary level. We also concur with Blum and Kirsch's (1991) "plea for doing mathematics on a preformal level" providing all students with opportunities to engage deeply with "preformal proofs that are as obvious and natural as possible especially for the mathematically less experienced learner" (p. 186). The present study also suggests that students even with strong mathematical backgrounds from tertiary studies are not necessarily experiencing proof in such a manner that they can convey a complete image of proving at the lower secondary level. Although one of the future teachers realised that students at this level had a propensity to accept empirical evidence and be dismissive of a necessity for "further" proof, her expectations did not include the necessity for students to be able to construct such proofs for themselves or "follow formal mathematical arguments" (VCAA, 2005, p. 37). Both future teachers in this study have a significantly difficult task ahead of them if they are going to be able to convey to their future students that "proof undoubtedly lies at the heart of mathematics" (Hanna & de Villiers, 2008, p. 2) and a school mathematics without proof is no mathematics at all.

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