The Growth of Early Mathematical Patterning: An Intervention Study

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A case study monitored the development of 53 preschoolers' mathematical patterning skills in two similar preschools, one of which implemented a 6-month Intervention promoting patterning concepts. Pre- and post-Intervention assessment data and follow-up data evaluated the impact of the Intervention on the growth of Repeating and Spatial Patterns. Intervention children outperformed Non-Intervention children across a range of patterning tasks and this trend was maintained 12 months after formal schooling. Intervention children readily identified the *unit of repeat* and the structure of spatial patterns. Without exposure to Growing Patterns, Intervention children identified, extended, represented and justified triangular and squared number patterns.

Background to the Study

Despite recent research interest in early algebra (Kieran, 2006), there is little known about the role of young children's mathematical patterning in the development of algebraic reasoning. Studies conducted before the 1990s contributed to the belief that algebra was best left for the later years of schooling. The 1990s saw a shift in research to children's mathematics reasoning and problem solving which included the development of combinatorial thinking. This was paralleled by studies with children aged 4 to 9 years supporting the idea that young children could learn more complex mathematics than previously thought. Further, much of the research in the 1980s and 1990s on early numeracy that focused on the development of arithmetic strategies influenced research on the relationship between arithmetic structure and algebraic thinking. However, there were few studies focused on underlying processes of patterning and abstraction with very young children.

Research on Patterning in Early Mathematics

Recently, mathematics education researchers have focused more seriously on the early development of patterning and its role in early mathematical thinking. Some studies have incorporated patterning as one component of investigation in early mathematical development. A series of studies have indicated that first and second graders' use of *pattern and structure* generalises across a wide range of mathematical content domains and this can be described as a general cognitive characteristic (Mulligan, Mitchelmore, & Prescott, 2006). Children's identification and representation of the structure of patterns was critical to successful task solution and the level of sophistication of structural awareness. Children's patterning knowledge has also been found to influence the development of analogical reasoning and the ability to identify, extend, and generalise patterns important to inductive reasoning (English, 2004).

Studies of preschoolers have found that they are capable of symbolic and abstract thought far beyond traditional expectations (Ginsburg, 2002). Young children have been observed developing skills in argumentation (Dockett & Perry, 2001) and algebraic reasoning (Blanton & Kaput, 2004). Some studies have included aspects of patterning such as simple repetition, part-whole thinking, spatial and geometric patterns, subitising, and counting patterns using calculators. However, few studies have focused explicitly on young children's development of patterning skills in early childcare settings. One recent observational study by Waters (2004) found that preschool children initiated and described their own patterns, ranging from simple repetition to geometric forms. Waters highlighted the limited pedagogical content knowledge of preschool teachers who needed to become more aware of the types, level and complexity of patterns. Her study suggests that more research is needed to support the inclusion of patterning in early childhood programs, and to develop a more coherent understanding of how early patterning skills develop. The study of patterning has also been explored through early childhood programs designed to enhance mathematical development generally (e.g., Ginsburg, 2002). Although it appears that patterning forms an integral part of these types of programs, the scope and complexity of patterning has not necessarily been informed by research that describes explicitly, the informal development of mathematical patterning. It is not yet clear how simple repeating patterns are extended to other mathematical contexts or how they are linked to growing patterns and functional thinking. Although contemporary studies of children's early algebraic thinking, such as exploring repeating and growing patterns, and functional thinking are mainly concerned with children in the 6-8 years age range, there remains unanswered questions about how and when early algebraic thinking develops in the years prior-to-formal schooling.

A case study was therefore designed to describe the development of patterning skills from preschool through to formal schooling and to investigate the role patterning plays in the development of early mathematical concepts and processes. Four key research questions were addressed: What are the characteristics of mathematical patterning young children develop naturally prior-to-school? In what ways does an intervention promoting mathematical patterning impact on the complexity of children's patterning concepts and skills and the development of other mathematical processes such as multiplicative thinking? Is the influence of such an intervention maintained after one year of formal schooling? If so, in what ways? What is the role of patterning in the development of early algebraic thinking?

In an earlier report, Papic and Mulligan (2005) presented preliminary findings of initial assessment data from the study. This paper describes the assessment data focusing on changes in children's patterning skills at pre- and post- Intervention and following 12 months of formal schooling.

Method

The study was designed as an intervention employing a mixed-method approach: integrating a traditional constructivist-based teaching experiment with more contemporary aspects of a design study. Following pilot work, an interview-based assessment of children informed the development of an instructional framework implemented through the Intervention. The Intervention provided explicit opportunities for children to explore and develop their patterning skills through problem-based tasks. The researcher (as *participant observer*) collaborated with teachers to model opportunities for the development of *Repeating Patterns* and *Spatial Patterns*. Observations included data showing how children constructed and justified patterns in a variety of modes. Further, the Intervention included on-going professional development on the importance of pattern and structure in early mathematical learning, which assisted teachers in modifying the emergent curriculum to incorporate patterning skills.

Setting and Participants

A large long-day care centre in the South-Western area of Sydney that operated a preschool program was selected as a case study for the Intervention (for details see Papic & Mulligan, 2005). A similar long-day care centre was identified within the region as a "contrast" group (Non-intervention preschool). It was not intended to generalise the results from this case study but every attempt was made to select two similar preschools that were considered to be typical of centres in this region. The sample comprised 53 preschoolers, balanced for gender and broadly representative of the children in the final year of each preschool. Thirty-five of the initial sample were reassessed on completion of the preschool year and 32 of these on completion of the first year of formal schooling. Despite the substantial attrition, there was no indication that the final sample was biased. Analysis of the data collected at each assessment showed that, for both groups, the children who were not retained had given a fair distribution of responses at the first assessment.

Data Collection and Analysis

Data collection included three interview-based assessments on children's patterning skills and an additional numeracy assessment at the third assessment (Schedule for Early Number Assessment 1, NSW Department of Education & Training, 2001). A systematic interview protocol was employed to elicit each child's explanations and strategies used to solve each assessment task. A range of data sources collected throughout the Intervention included photographs, video recording and observations of children's patterning in structured and play situations. Work samples were compiled in individual portfolios. Figure 1 provides a summary of the data collection points. Preschool and Kindergarten teacher surveys were conducted at the conclusion of the study. The first researcher conducted all interview-based assessments and teacher surveys.

Intervention Preschool	June 2003 Interview 1 (n = 27)	→	July – Dec 03 Intervention Program	→	Dec 2003 Interview 2 (n = 19)	→	Dec 2004 Interview 3 (n = 20)	Dec 2004 Teacher Survey (n = 18)
Non-intervention Preschool	June 2003 Interview 1 (n = 26)	→	Regular Preschool Program	→	Dec 2003 Interview 2 (n = 16)	→	Dec 2004 Interview 3 (n = 12)	Dec 2004 Teacher Survey (n = 12)

Figure 1. Data collection points.

The classification of children's responses to assessment tasks was supported by other data: drawn representations, photographs of children's patterns and solution processes, interview transcripts, observation notes and digital recordings (20% of interviews). The analysis of assessment data involved initial coding of responses for accuracy, followed by classification of solution processes focused on the level of complexity of pattern recognition. Initial coding was verified by an independent coder (intercoder reliability calculated at 89%).

Three key aspects of patterning were identified from the research literature and initial analyses (Papic & Mulligan, 2005): *Repeating Patterns*, *Spatial Structure Patterns*, and *Growing Patterns*. Eleven task categories were derived from these key aspects (see Table

1). These tasks were devised to investigate children's ability to create, identify, extend, and copy from memory patterns, in a variety of modes. Tasks administered at Assessments 1 and 2 (for task descriptors see Papic & Mulligan, 2005) were identical, but tasks at Assessment 3 increased in complexity to accommodate Growing Patterns and children's growth in patterning concepts and skills.

Interview-Based Assessment Tasks

Table 1
Key Aspects of Patterning and Related Task Categories

Key Aspect	Task Category	Descriptor
Repeating Patterns	Tower Border Hopscotch Number	Repeating Patterns contain an element that continuously recurs. In these tasks patterns contained single or dual variable, simple and complex repetitions using coloured blocks, tiles or numerals.
Spatial Structure Patterns	Array Block Grid Subitising Triangular 1	Spatial Structure is the mental organisation of objects or groups of objects and their components. In these tasks the organisation of patterns was presented in the form of triangular patterns of dots and square and rectangular patterns of dots, arrays and grids.
Growing Patterns	Triangular 2 Square Tiles	Growing patterns increase (or decrease) systematically. Spatial Structure tasks were reformulated to explore the idea of more complex, growing patterns presented as the pattern of triangular numbers (triangular dots) and the pattern of squared numbers (square tiles).

The Intervention

The researcher, in collaboration with the preschool staff, developed, implemented, and monitored an intervention program. The Intervention was designed on the basis of children's existing patterning knowledge to: provide explicit opportunities to explore and develop patterning skills through problem-based tasks; develop children's mathematical reasoning in order to provide a foundation for later mathematical learning particularly in early algebraic thinking; provide a framework of assessment and learning experiences to guide emergent curriculum and scaffold individual children's learning; describe the development of patterning in both play situations as well as structured situations; and provide professional development for staff on the importance of pattern and structure in early mathematical learning to assist them in modifying their emergent curriculum to incorporate patterning.

The Intervention comprised three distinct components: structured individual and small group work on pattern-eliciting tasks, *Patternising* the regular preschool program, and observing children's patterning in free play. Structured pattern-eliciting tasks were based on the *Tower*, *Subitising* and *Hopscotch* tasks administered in the first assessment because they provided critical opportunities for developing patterning concepts. A *Framework of Assessment and Learning* that guided instruction and highlighted children's development was designed for both the *Tower* and *Subitising* tasks.

Discussion of Results

The following results compare Intervention (I) and Non-intervention (NI) children's responses across three assessment points. A discussion of the growth in children's acquisition of patterning skills is provided, supported by excerpts from interview transcripts and children's drawn and constructed representations. When interpreting data it must be noted that small differences in percentages, particularly NI at Assessment 3, are insignificant due to the size of each sample group.

Table 2 indicates the percentage of correct responses for the eleven task categories (data show the average score, as a percentage of correct responses on sub tasks within each category). The NI group was moderately more successful across most task categories at Assessment 1, but by Assessment 2, the I group was more successful across all task categories. This success was particularly evident in the task categories *Number*, *Grid*, *Subitising*, and *Triangular 1*. Number tasks were more challenging than other Repeating Pattern tasks because children were not provided with concrete materials and the tasks involved two variables, colour and number. NI children showed no improvement on Number tasks between Assessments 1 and 2, whereas I children improved substantially. Between the first two assessments, I children participated in various games and activities using dice and regular dot patterns as part of the 6-month Intervention. This may have impacted on I children's responses at the second assessment where their performance on *Subitising* tasks improved. Conversely NI children showed no improvement on *Subitising* tasks. It was observed that NI children were more focused on counting the individual dots or blocks in the patterns. For example, the simple three-dot pattern, which children immediately recognised at Assessment 1 was instead counted one-by-one at Assessment 2. This unitary counting strategy may have been attributed to the overemphasis on counting by ones in their preschool program.

Table 2
Percentage of Correct Responses for Task Categories at Three Assessment Points

Task Category	Assessment 1		Assessment 2		Assessment 3	
	I n = 27	NI n = 26	I n = 19	NI n = 16	I n = 20	NI n = 12
Repeating patterns						
Tower	34	47	85	73	93	47
Border	74	81	100	88	53	22
Hopscotch	16	28	55	45	65	8
Number	11	19	58	19	83	17
Spatial structure patterns						
Array	47	42	79	72		
Block	47	46				
Grid	33	27	79	25		
Subitising	15	20	58	16		
Triangular 1	7	8	50	13		
Growing patterns						
Triangular 2					38	0
Square Tiles					48	0

At Assessment 2, the *Array* proved to be the easiest of the Spatial Structure tasks. It was inferred that arrangements of dots in this task (e.g., 2 x 3 array of dots) made spatial structure explicit. In comparison, *Triangular 1* proved to be the most difficult of the Spatial Structure tasks. NI children found it difficult to identify the number, shape, size, orientation, spatial and numerical structure of the triangles when copying with counters and drawing triangular dot patterns. However, even without intervention, the NI group showed some progress at Assessment 2: *Tower*, *Border*, and *Array* tasks. However, there were marked differences between the two groups in terms of the patterning strategies employed to solve the tasks.

The increase in task complexity renders any comparison between Assessment 2 and Assessment 3 invalid. However, it is valid to compare performance between the I and NI children at Assessment 3. There were striking differences across all task categories in favour of the I children. Intervention children continued to show improvement across the more complex Repeating Pattern tasks at Assessment 3. However, the NI children found the tasks more challenging and performed well below the I group, particularly on *Hopscotch* and *Number* tasks.

Spatial Structure tasks were reformulated into more complex Growing Pattern tasks at Assessment 3. Neither I nor NI children had been exposed to Growing Patterns in the first year of schooling and these tasks had not comprised part of the Intervention. Nevertheless, many of the I children could construct, extend, represent, and justify these patterns. It appeared that about half these children depicted some underlying structure in the pattern. Forty-five percent could successfully continue a growing triangular number pattern “1, 3, 6”, presented as a triangular dot pattern and 55% could successfully continue a growing squared number pattern “1, 4, 9”, made with square tiles (see Figures 6 and 7 following). In comparison, Growing Patterns proved to be extremely difficult for all NI children, with no NI child giving a correct response.

Patterning Strategies

Repeating Patterns. By Assessment 2, I children developed a sound understanding of pattern as unit of repeat that appeared to lead to growth in the abstraction and complexity of patterning skills. Intervention children could successfully identify, construct and abstract the unit of repeat and calculate the number of repetitions. This was the dominant strategy used by I children at Assessment 2 and sustained at Assessment 3 (12 months later). Many I children were able to draw complex repetitions from memory, identify the pattern element, and number of repetitions as exemplified in the following excerpt.

Researcher: How do you know that you have finished making your tower?
Why didn’t you keep adding some more blocks?
Child I 19: I remembered red, blue, blue, black, three times.

In comparison, NI children relied on an alternating colours strategy to complete Repeating Pattern tasks. For example, when copying an ABABAB tower, NI children remembered the tower pattern as single alternating colours of “red, blue, red, blue, red, blue” rather than the element “red, blue” and the number of repetitions. For example, one NI child continued to add alternating colours of blocks, red then blue, and then after making a 9-block tower measured it against the tower that had been modelled to establish height. At Assessment 3, when the complexity of the tower was increased, (e.g., an ABBC repetition), and when asked to complete the task from memory, NI children’s alternating colours strategies became ineffective. Most NI children tried to remember the order of the coloured blocks and at times, the height of the tower. However, due to the complexity of the tower pattern they could not remember the sequence and thus made errors.

At Assessments 1 and 2 a simple repetition was presented in a vertical and horizontal hopscotch pattern with a unit of repeat created with four squares: Two vertical, two horizontal (see Figure 2). The *Hopscotch* category differed from other Repeating Patterns tasks in that it investigated changes in orientation of the pattern and children's transformation skills. At Assessment 1, both I and NI found it difficult to visualise the *Hopscotch* pattern when it had been rotated by 90º. At Assessment 2 both groups improved on the *Hopscotch* tasks. It could be assumed that exposure to a variety of concrete materials and viewing objects from different perspectives in the children's regular program assisted in developing these skills. For example, by the second assessment children had been exposed to a variety of activities such as block play and puzzles that encouraged transformation skills and this was critical to the completion of the *Hopscotch* rotation tasks. However, I children were more confident at drawing the rotated hopscotch from memory than the NI children. Figure 3 shows an I child's drawing of the hopscotch template rotated by 90° (on the left hand side) at Assessment 1 and her drawing 6 months later at Assessment 2 (on the right hand side).

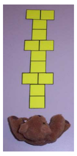

Figure 2. Hopscotch patterns.

Figure 3. Drawing of a Hopscotch pattern rotated at 90º, Child I 17.

At Assessment 2 children were also given an extension task where they were asked to design their own hopscotch pattern. Sixty-three percent of I children successfully designed their own hopscotch that showed repetition of elements. Many I children could additionally integrate a second variable, colour, in their hopscotch pattern and could extend the number of tiles that formed the pattern element. For example, in Figure 4 the child created a complex pattern element, "two horizontal, one vertical, two horizontal, two vertical, four horizontal" using a systematic arrangement of colours, and replicated it once. In Figure 5 the child created a pattern element of "three, two, one", creating a descending row of steps. In contrast, only 25% of NI children designed a hopscotch pattern that showed a single variable repetition and there were no examples of complex patterns; rather they were restricted to AB repetitions. All NI children attempted to make their own hopscotch but they seemed unaware of the need to create and replicate a pattern element.

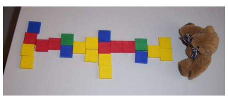

Figure 4. Design own Hopscotch pattern,
I 25, 5.1 yrs.

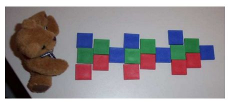

Figure 5. Design own Hopscotch pattern,
I 18, 5.0 yrs.

At Assessment 3, the Hopscotch task required the children to complete a cyclic pattern where they needed to identify the pattern as a sequence of 90° turns. Sixty-five percent of I children successfully modelled and predicted the pattern as a sequence of 90° turns. In contrast, the NI children did not identify the pattern as a sequence of 90° turns but saw the three hopscotch templates as an ABC pattern element to repeat.

At Assessment 3, children's ability to identify a pattern beyond a linear form was also explored. One of the *Border* tasks required children to identify an ABC repetition (3 x 5 border pattern of red, blue, green tiles) from multiple starting points. The task proved very difficult for both groups, with only a small number of children from each group accurately completing this task. The majority of children identified the pattern with a starting point in the top left hand corner. It may be inferred that this was due to the children's limited exposure to patterns presented as different spatial arrangements. This response could also be explained by the children's classroom experience of making patterns that were limited to horizontal and vertical linear forms that begin in a designated position, using left-to-right or bottom-to-top directions.

In another *Border* task, children were asked to identify the number of green tiles required to complete the ABC pattern. Structuring the task in this way allowed the researcher to observe whether children determined the number of times the pattern element could fit into the remaining spaces. Intervention children outperformed NI children on this task. This may have occurred because the children were more aware that the pattern element contained three colours and they needed only to count every third tile. Such a strategy would suggest a sophisticated understanding of pattern as repetition and reflect early multiplicative thinking. Many of the I children immediately identified every third position in the border by placing their fingers on the square where the missing green tile needed to be placed. It appeared that these children visualised the pattern element accurately; some *skip counted* every third position in the pattern, translating the repetition of colours into a number pattern of multiples. In contrast, most NI children attempted to complete the pattern by verbalising alternating colours to determine how many greens were required.

Spatial Structure Patterns. Intervention children outperformed NI children on all Spatial Structure tasks at the second assessment where almost all I children represented the structure of the patterns. For example, one *Grid* task required children to copy a grid of three connected squares. Most I children were able to draw the correct number of equal-sized squares in correct formation. Those who made errors, made counting errors rather than those related to the spatial arrangement. In another example, when presented with an *array* of dots (e.g., 2 x 3) a number of children clearly represented the structure of two rows of counters forming a rectangular shape however, there were two rows of four counters, rather than two rows of three counters presented. It seemed that the I children focused their attention on the spatial structure of the patterns. This is not surprising since teachers encouraged children to look for similarity and difference in the structure of patterns throughout the Intervention. In comparison, many NI children's incorrect responses lacked any structural features. For example, in *Array* tasks, children's responses did not represent the shape of the array and frequently included an incorrect number of counters. It was inferred that the children did not "see" the structure of the array or the rows of dots in alignment.

Growing Patterns. A number of I children, although not exposed to Growing Patterns throughout the Intervention or in the first year of schooling, were able to extend a growing triangular number pattern (see Figure 6) and a growing square number pattern (see Figure 7). Most of the I children who made errors in constructing the Growing Patterns were still able to observe holistically the increasing size of the triangles or squares, and attempted to make the pattern larger.

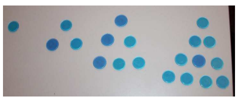

Figure 6. Triangular Growing Pattern, I 11, 6.3 yrs.

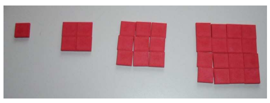

Figure 7. Square Tile Growing Pattern, I 21, 5.5 yrs.

Of particular importance was the I children's use of spatial structure to explain the pattern as an extension of the previous pattern element. This showed early signs of co-variational thinking where children were required to deal with a change in the structure of the pattern. This result supports the findings of Blanton and Kaput (2002), which highlight the importance of quantitative relationships in developing algebraic thinking.

Researcher: Can you tell me what is happening each time we make the triangle bigger.
Child I 18: It gets bigger.
Researcher: Can you tell me how it is getting bigger?
Child I 18: It's going one, two, three, four.
Researcher: What's going one, two, three, four?
Child I 18: See the bottom of the triangle, here it is one, then here it is two, then three, here it's four (*outlines each successive triangle when explaining it).*

Most I children who could successfully extend Growing Patterns could also justify the pattern. The following excerpt demonstrates one I child's justification of the pattern as growing systematically in two dimensions.

Researcher: Can you tell me what is happening each time we make the square bigger.
Child I 4: Yeh, here it has one, then it has 2 and 2 lines and it's bigger. Then this one has three and three lines and then four and four lines.
Researcher: What do you mean four and four lines.
Child I 4 See there's four in each line.
Researcher: So what would the next one in my pattern be?
Child I 4: Umm … five and five lines.

In contrast, NI children were unable to identify or extend Growing Patterns. Many saw the triangles and squares exclusively as *items* in simple repetitions in the same way as the simple repetitions that they were familiar with. Many successfully created an ABC repetition however, they did not see the pattern as a growing pattern.

Conclusions and Implications

Interview-based assessment of children's patterning skills identified that young children can develop complex patterning concepts prior-to-formal schooling. It appears that the Intervention experiences encouraged children to see the structure of simple repetition using a unit of repeat, and to represent patterns in different spatial forms such as borders, grids, arrays, subitising patterns, and numerical sequences. It was also apparent that the development of pattern as a unit of repeat promoted other mathematical processes such as multiplicative thinking and transformation skills.

Warren (2005), in her study with 9-year-olds, questioned whether growing patterns were cognitively more difficult, or whether the real difficulty could be traced to over-emphasis on repeating patterns in early mathematics curricula. The findings of this present study showed that the difficulty with growing patterns was not necessarily the absence, or predominance of repeating patterns in early mathematics curricula. Rather, the inadequate or inappropriate development of repeating patterns without a sound understanding of the unit of repeat, limited and possibly impeded the development of growing patterns. Commonly, when teachers are dealing with repeating patterns, the structure of the pattern is ignored or misinterpreted. Therefore, expecting children to observe other pattern structures such as growing patterns is unreasonable.

Algebra has at times been considered developmentally inappropriate for young children, lying well beyond their developmental capabilities. However, the findings of this study suggest that this is not the case. It can be inferred that older students' difficulties may not be a result of developmental constraints after all, but rather, traced to the limited opportunities and/or limited or inaccurate approaches experienced in the early years. These include a lack of awareness of unit of repeat and inadequate attention to structure. The results indicated that the predominant strategy used by NI children to solve patterning problems was an alternating colours strategy. In comparison, I children were able to identify the unit of repeat and use this to solve various complex patterning tasks. Therefore it might be questioned whether the approach to teaching patterns and algebra used in mathematics curricula encourages an alternating colours strategy rather than the identification of pattern elements and number of repetitions. Could teachers' lack of understanding and their approach to teaching repeating patterns limit children's development of patterning? Further research is needed to explore the impact on children's mathematical development if changes to curriculum and teacher pedagogy were to occur that explicitly encourage representation, abstraction, and generalisation of repeating and growing patterns in the early years.

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