# Developing Positive Attitudes Towards Algebra

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This paper reports on one teacher's attempts to teach critical algebra understandings to a Year 9 class in ways that engage the students and help them to develop positive perceptions of their ability to learn algebra in a "rigorous and symbolic way". This paper describes a 6-week algebra intervention based upon connecting concrete representations with symbolic expressions and equations through the careful use of formal algebra language. The teacher had expressed her aspirations that interventions such as this would encourage more students to undertake intermediate and advanced mathematics courses in senior secondary years. The study collected data on student perceptions about their learning experiences including perceptions about mathematics as a subject domain, engagement with the activities, development of algebraic understanding, and the quality of discourse within the classroom. This study found that the students valued the classroom discourse much more than they did the normal mathematics learning experiences. These findings have implications for inservice and pre-service teacher education.

## Introduction and Background

This paper analyses one teacher's (Jane) attempts to teach critical algebra understandings, in particular, how to solve equations with variables on both sides. Jane is the mathematics subject head of department in a large suburban school situated in a middle to lower class outer Brisbane suburb. Historically very few students in the school opted to study Advanced Mathematics and Jane's goal was to increase the proportion of students enrolling in these subjects (Mathematics B - Intermediate Mathematics and Mathematics C - Advanced Mathematics). In recent years between 4% and 7% of senior students enrolled in Advanced Mathematics, much lower than the national average of 11.7% in 2004 Barrington, 2006). Approximately 25% enrolled in Intermediate Mathematics, which is on a par with the national average (Barrington, 2006). With this goal in mind, Jane devised an algebra intervention for Year 9 to be extended to Year 10 that she considered would foster student success in middle school algebra and, consequently, would encourage a higher proportion of the students to enrol in the more advanced senior mathematics subjects. That is, she hoped that if students had success and formed positive perceptions about mathematics they would be more inclined to tackle Mathematics B and C.

The research literature indicates that declining student perceptions and participation in mathematics study is of broad concern. For example, a decline in student perceptions of the worth of mathematics study has been reported from about Year 4 onward (e.g., Thompson, & Fleming, 2003), and Barrington (2006) has reported the declining participation in intermediate and advanced mathematics between 1995 and 2004. He concluded that:

There has been a significant nett loss of students taking the Year 12 mathematics options in which higher-level mathematical skills are taught. This has implications for the recruitment of students to undertake tertiary studies in the quantitative sciences, and for the national capacity for innovation in engineering and technology. The effects are much wider: fields such as finance and molecular biology are developing into quantitative and sophisticated areas. (p. 4)

Student attitudes to algebra are central to this process because early failure in algebra is likely to result in passive withdrawal from further study in the area or active rebellion (MacGregor, 2004). Hence, algebra study may act as a filter for further study in mathematics (e.g., MacGregor, 2004; Stacey & Chick, 2004), therefore, the development of positive attitudes to the subject are essential to increase student enrolments in advanced mathematics subjects.

Researchers have noted that early educational and socialisation processes are critical to children's learning and perceptions and subsequent participation in education (Khoon & Ainley, 2005). Student perceptions, which include their expectations of success and the value that they attribute to particular tasks, have been found to correlate strongly with later participation in study (Ethington, 1992; Wigfield & Eccles, 2000). The analysis of this relationship in the TIMSS data for Australian school students by Thomson and Fleming (2003) supports theorised connections between perceptions, participation and performance. Perceptions shape the information individuals attend to and how it is interpreted (De Bono, 2004). In summary, the decreasing participation of students in mathematics can be related to the interaction of three perceptions held by an increasing proportion of students:

- 1. Algebra is perceived as uninteresting and based upon symbolic manipulation with limited meaning and little relevance to every day life (e.g., Boaler, 2000; Kaput, 1995; MacGregor, 2004; Stacey & Chick, 2004).
- 2. Algebra is perceived as difficult (e.g., MacGregor, 2004).

Some critical aspects that have led students to see mathematics in these ways include an over-reliance on textbook work with a procedural focus, teacher dominated discourse, and closed learning activities that result in a lack of understanding and capacity to transfer knowledge (e.g., Hollingsworth, Lokan, & McRae, 2003). Gregg (1995) described this "school mathematics tradition", as a tradition that is well entrenched and resilient (Perry, Howard, & Tracey, 1999). Repeatedly, students report that they neither understand important mathematical concepts nor appreciate why they are worth the effort of learning (Watt, 2005). What is true for mathematics in general is especially true of algebra since its understanding assumes knowledge of the specialised processes and language nuances associated with symbolic representations (e.g., Stacey & Chick, 2004). The student perceptions that they cannot understand mathematics and that it is a hard subject is linked to an image students have of mathematics as an abstract collection of rules and processes (Boaler, 2000; Kaput, 1995). This is particularly in the case of algebra where resources found in standard texts frequently do not encourage teachers to enact appropriate pedagogy to foster algebraic thinking (Kaput, 1995; Stacey & MacGregor, 1999). Further, it has been reported that if students engage in "extensive symbolic manipulation before they have developed a solid conceptual foundation for their work, they will be unable to do more than mechanical manipulation" (Kirshner & Awtry, 2004, p. 39). That is, they did not think deeply about mathematical concepts and structures and were not challenged to think about solving problems, rather, classroom discourse was dominated by the practice of routine operations. In terms of the difficulty students have with algebra study, Stacey, and MacGregor (1999) found that only 8% of 116 Year 10 students (16 years old) could solve an equation with variables on both sides if it included fraction operations. Given this lack of success in algebra study it is hardly surprising that many students developed the perception that algebra was a hard subject and that they had little confidence in succeeding. In addition, MacGregor (2004, p. 315) noted:

Academic learning based upon reading and writing, such as the traditional school algebra of symbolic manipulation and word-problems, is not appropriate for students with weak literacy and numeracy skills.

MacGregor (2004), citing Marks and Ainley (1997), indicated that only about 20% of 14-year-olds had the literacy and numeracy skills to cope with algebra study. Stacey and MacGregor (2004) have reported that the major reason for student difficulties with using algebraic methods for problem solving is that they do not understand its underpinning logic. Students tend to wish to calculate in the first instance, a behaviour that is consistent with their arithmetic learning. However, algebra requires an analysis of the problem and transforming it into algebraic equations. That is, students need to recognise, construct and manipulate algebraic expressions before applying their computational skills. Many students struggle with this change in operating and have had little support to make the transition.

Teachers are central to any model of effective educational reform and renewal (e.g., Doerr, 2004). Consequently, it is imperative to analyse systematically all aspects of teachers' classroom practices including the intended curriculum (curriculum guidelines, lesson plans), implemented/enacted curriculum (co-construction of classroom knowledge), and attained curriculum (what students actually learn) (Taylor, Muller, & Vinjevold, 2003). Thus the purposes of this paper are as follows:

- 1. To describe briefly an algebra intervention designed to immerse students in active learning through engagement with concrete materials and careful use of language.
- 2. To describe the students' perceptions about the algebra intervention including: whether they were engaged to think deeply and understand mathematical ideas, perceptions of fun, availability of teacher support, how hard they worked and how much they were challenged, the collaborative nature of tasks and perceptions about the nature of mathematics, in particular, whether they viewed algebra as essentially symbolic manipulation or about mathematical ideas.

The cognitive gains of students have been analysed and described in a companion paper (Norton & Irwin, 2007).

#### Method

The overall methodology is a case study that uses design based research, in so much as cycles of design, enactment, analysis and re-enactment, analysis and further design take place. As in all design-experiments, the specific research questions investigated in each cycle are conjured out of analysis of recent successes and/or failures of previous cycles of the research (Bereiter, 2002). This paper reports on Jane's third iteration of her teaching intervention commenced in 2006. Each iteration was essentially identical in terms of teaching approach. The third iteration was concurrent with the researcher's engagement with the school algebra project. Future iterations of the intervention will reflect what has been learnt from the analysis reported in this paper. The involvement of the researcher as an active participant in this process gave the approach a participatory collaborative action research element (Kemmis & McTaggard, 2000).

## **Participants**

The participants in this study were the classroom teacher, Jane, and the 18 students engaged in a 6-week algebra course. The students in the study were drawn from the 180 students in the Year 9 cohort. All 180 students were tested for general numeracy and more specifically to determine those who were "comfortable with the use of symbols to describe

patterns" (Jane, field notes). Students who scored in the top 1/3 on the pre-test were offered the algebra extension. There were three cohorts of about 20 students in each group. These students were drawn from the 7 mixed ability classes in Year 9. Although the students had done some patterning, backtracking and work with quazi variables, this was their first real exposure to algebraic symbolism. Since the selected students came from most of the 7 mixed ability mathematics classes, we can conclude that the students' descriptions of the mathematics classes that they were drawn from are representative of middle school teaching in that school.

## Description of Intervention

The intervention will be described in terms of the concept Instructional Discourse, which refers to the rules for selecting and organising instructional content (Bernstein, 2000). The instructional discourse was based on an underpinning theoretical framework put forward by Booker, Bond, Sparrow, and Swan (2004, p. 20).

While the role of materials and patterns they develop is fundamental, materials by themselves do not literally carry meaning ... it is language that communicated ideas, not only in describing concepts but also helping them take shape in each learner's mind.

Jane's selection of activity sources was, therefore, based on helping students make connections between materials, verbal language initially, and then symbolic language. The primary sources of activity were A Concrete Approach to Algebra (Quinlan, Low, Sawyer, White, & Llewellyn, 1987) and Access to Algebra Book 2 (Lowe, Johnston, Kissane, & Willis, 1993). Jane used match stick patterning to introduce variables and activities with cups, counters (blobs), and envelopes to explore writing expressions, equivalent expressions, simplifying expressions, expanding expressions, and writing equivalent equations. Activities from Lowe et al. (1993) were selected that emphasised the links between materials, patterns, and variables, and used the balance model for representing and solving equations including those with variables on both sides. The third source of student activities was based on the symbolic recognition and manipulations of algebra terms covered above embedded in algebra games that Jane had devised. The algebra games were constructed according to principles outlined in Booker (2000). The games were played by two or three students, and enabled them to consolidate and attain competency in the mathematics learnt in prior activities. A companion article describes the teaching approach in more detail (Norton & Irwin, 2007).

Typically the 1-hour lessons were divided into three sessions. In the introductory session Jane used the white board and an activity selected from Quinlan et al. (1987) or Lowe et al. (1993) as the basis to conduct a class discussion on the key concepts. In the second session students worked in pairs or threes on activities selected from Quinlan et al. (1987) or Lowe et al. (1993) and Jane scaffolded the learning of individuals or groups of students. Sometimes this activity continued to the end of the class. Generally, the third session was spent by the students playing the algebra games.

### Data Collection

Eighteen one-hour classes were observed and recorded on video. At the end of the intervention the students completed a 5-point Likert perceptions survey, developed by the first author, consisting of 40 questions related to eight attributes. The selection of eight attributes was informed by the literature on student attitudes towards mathematics and, in particular, to learning of algebra (e.g., Boaler, 2000; Kaput, 1995; MacGregor, 2004;

Stacey & Chick, 2004). These attributes of mathematics learning are thought to be important to the formation of student perceptions and subsequent participation in mathematics study. The items were positive or negatively worded and each question started with the phrase, *Compared to the way I usually study (or study) maths* .... The eight attributes and a relevant sample question are shown below,

- 1. Student perceptions on the depth of their mathematical thinking. E.g., ... the activities in this class help me to think deeply about mathematical ideas.
- 2. Student perceptions of fun and interest associated with the algebra learning. E.g., ... the learning in this class is more fun.
- 3. Student perceptions of their confidence to develop mathematical understanding underpinning mathematical processes. ... the learning in this class has encouraged me to believe I can <u>understand mathematics</u> better.
- 4. Student perceptions of support for learning provided by the teacher. E.g., ... the <u>teacher</u> in this class <u>helps</u> me more.
- 5. Student perceptions of how hard they worked in class. ... *I work harder* in this class.
- 6. Student perceptions about how challenging the activities were. *In this class I am <u>challenged</u>* to figure out how to solve problems.
- 7. Student perceptions of collaborative learning. ... *I and the people I sit with help each other more*.
- 8. Student perceptions about the nature of mathematics ... Before I thought maths was mostly about operations and symbols, this class has helped me see it is about ideas.

Subsequently, each student was interviewed about her/his responses, a process designed to increase the validity of the perceptions survey. For example, students were asked explain and expand upon their responses to the survey and, in addition, compare these responses with their perceptions of mathematics in their normal classrooms. These interviews were video recorded. Finally, the students were asked to draw two pictures, one to represent their perceptions with regard to their normal mathematics classroom experiences and the second to represent their perceptions with respect to their algebra learning experiences in the intervention class. The students then briefly interpreted their diagrams for the author and these were audio recorded.

#### Results

The results of the analysis of students' perceptions of the intervention are presented in three parts. The first part describes student responses to the perceptions survey. Second is the description and analysis of student interview data. Finally, an analysis of the pictures students were asked to draw about activities and feelings in the different classes is presented. The results of the survey are summarised in Table 1.

The responses indicate that students responded either strongly agree (5) or agree (4) on each attribute gauging their perceptions of Jane's intervention. In short, the high mean values for each attribute imply that on completion of their engagement in Jane's intervention all students perceived that they thought more deeply about mathematical ideas, perceived that the activities were more interesting and fun; that they had developed greater confidence in their capacity to understand mathematics; that they had the perception that the teacher had a greater role in helping them learn; that they worked harder and spent more time on task and perceived that the tasks were more challenging; that there was more collaborative learning; and that they had developed the perception that their concept of mathematics had shifted from one predominately associated with computations and symbolism towards one aligned with problem solving and mathematical ideas.

Table 1
Summary of Student Responses to the Perceptions Survey

Learning attribute	Mean n=18	Standard Deviation
1 Depth of thinking	4.48	0.44
2 Fun and interest	4.48	0.63
3 Confidence to understand	4.57	0.43
4 Teacher support	4.62	0.53
5 Hard working	4.57	0.46
6 Task challenge	4.22	0.47
7 Collaborative learning	4.39	0.37
8 Nature of mathematics	4.22	0.54

Interview data supported the key findings from the survey analysis described above. The interview data did reveal that, in stark contrast to positive outcomes from the teaching intervention, all 18 student participants reported that in their normal mathematics class they did little mathematics work. Several indicated that in one class a small group of boys was disruptive and diverted the teacher from teaching students who wanted to learn. It was also reported that classroom activities were usually boring and consisted mostly of repetitious work from a textbook or worksheets that the students found confusing. The students reported that – frequently – they did not understand the mathematics work and received little assistance from the teacher. The following comment can be described as a typical example of students' perceptions of their normal classrooms: "Text books are a bad thing, because, you are just doing the same thing from the same maths book over and over again. It gets really boring."

Explanations for the behaviour of peers who disrupted the normal classes included that these students had "given up trying to learn mathematics" and it was unlikely that any form of mathematics teaching "would interest them". For example Lorry reported:

They just do not want to do it, or they can't, or they are just too lazy or they have just given up hope.

The students were asked "If the average student who 'mucks up' received the teaching that they had experienced in the algebra class, could they understand the algebra work?" Most students responded "Probably".

It was clear that student responses to questions about the learning environment could be grouped into two themes: those related to teacher scaffolding and student collaborative learning, and those related to the use of the concrete materials and games. The following responses made during the final interview were typical of student comments on teacher and peer support in the algebra class.

This teacher helps me understand a lot more.

The teacher explains it more and my friend can help me as well.

We help each other a lot more in this class.

These student comments above are substantiated by data collected through prolonged video observation of the class during the intervention. An increased level of cooperative learning increased over time was observed as the students became more familiar with other and the teacher's ways of providing appropriate assistance.

When asked about the role of the materials in facilitating their understanding of algebra all but two students (Tammy and Simon) responded with comments similar to those shown below (Interview data).

It shows you what is actually happening and what is going on, it is not all in your head. The cups and counters helped us to make sense when we had to write down the x and y's.

The activities in the algebra class are a lot harder than the ones in the textbook and it actually makes you have to think. And it is fun.

With the text book I had to work harder to find the information. With the cups and counters it makes it easier to understand.

The cups and counters are very important. Without them it is just "What the!" but with the cups and counters and envelopes you can see what you are doing and so you can learn heaps more.

You have to think about what does that equal before you can do it.

I feel like a maths nerd, which is good (Girl).

Tammy reported that she did not really use the materials. When asked to explain this she responded,

I used them at the start, but I did not really understand them. But then the teacher drew them on the board, and I did not need them (physical materials). The diagram was enough.

Simon who was asked to explain why he had responded on his survey that he worked less in the algebra class explained,

Well, with the cups and counters and games, it is easier to understand, that's why I voted I worked less in this class.

The student comments with respect to the algebra games were similar to those above. The following comment by Kingsley summed up the class evaluation of the algebra games.

I understood it more with the games. It is actually showing you how to put it into action. It is showing you things. You have to try harder to find it rather than just finding an answer by adding or multiplying. You have to find the equation, and you have to do it with cups and counters and a diagram.

Classroom observations supported student comments in that they demonstrated a high proportion of time on task while working with the games and much of the discussion between students centred around the underpinning mathematics.

With respect to student drawings of their activities and feelings in the different classes some common themes emerged and were subsequently expanded upon by the students. Figure 1 represents one student's (Harry) report of his perceptions about learning in the two classrooms. Harry explained that in his usual mathematics class he started with enthusiasm. This enthusiasm waned over time.

This is like the path of a ball, it bounces around. It loses momentum and eventually just sits and does nothing. I was trying to do my work that was just confusing, so eventually you just lose motivation. You just do nothing.

Harry explained his feelings about the algebra class as follows, "It is like the opposite of the box. These (lines) are clear straight and easy to understand. It is like the algebra class." The underlying themes evident in Harry's drawings were common to all student sketches.

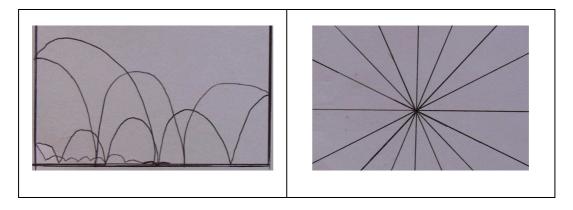


Figure 1. One Student's (Harry) Comparison of his usual Mathematics Lessons (LHS) with the Algebra Lessons (RHS).

## **Discussion and Conclusions**

Survey responses, student diagrams, interview data, and sustained classroom observations in the specialist algebra class indicated high student engagement on mathematical tasks, with productive discourse between students and teacher, and high quality collaborative learning behaviours. Students helped each other and discussed activities and games. Sfard, Nesher, Streefland, Cobb, and Mason (1998) have reported that getting students to talk about mathematics in a meaningful way is challenging. The results of this study indicate that these students placed high value upon the instructional discourse. This discourse was based on the use of materials to build algebraic concepts, as recommended by a variety of researchers (e.g., Becker & Rivera, 2005; Booker et al., 2004; Quinlan et al., 1987; Lowe et al., 1993), and emphasised the links between concrete and visual representations and explicit algebraic language. No students reported that mathematics was inconsistent with their identity formation, an issue reported by other researchers (e.g., Khoon & Ainley, 2005; Watt, 2005). Rather, students reported an increased confidence in their capacity to understand algebra. This is an educationally significant finding because expectation models (e.g., Ethington, 1992; Wigfield & Eccles, 2000) indicate that success and positive perceptions about mathematical study are likely to encourage students to undertake studies in advanced mathematics.

The results from the interview data where students described and commented upon their learning in the non-streamed classes confirmed what many authors have reported, that the school mathematics tradition of talk and chalk from the front of the room and reliance on worksheets and textbooks with a focus upon repetitious symbolic manipulation played a significant role in their perception that mathematics is dull, boring, and hard and was a collection of rules that frequently made little sense (e.g., Barrington, 2006; Boaler, 2000; Kaput, 1996; Thompson & Fleming, 2003; Watt, 2005). In addition, the students reported that this pedagogy did little to foster their deep thinking about the mathematical ideas. Such findings support those of other authors with respect to standard algebra activities (e.g., Kaput, 1995; Stacey & Chick, 2004). Some students also reported that their disenchantment with the learning activities in normal classes was linked to issues of behaviour management and consequently limited help from the teacher for their learning. Although these results raise the issues of classroom management strategies and streaming of mathematics classes according to ability, such issues are largely beyond the scope of this

paper. The authors are aware that the intervention class was smaller than normal classes that have about 25 students and that this would have impacted upon teaching dynamics. The students commented that the absence of students who were overt in disrupting the normal classes made the class "much better". Without exception they did not want to go back to their mixed ability classes. However, it is noteworthy that most students acknowledged that the choice of activities in this class would "probably" have helped most students in unstreamed classes to learn algebra. This finding encourages the authors to extend the teaching models to the mixed ability classes. The students and Jane reported that the pedagogy enacted in unstreamed classes appeared to condemn students to failure in mathematics.

The students in this study were highly articulate in explaining why they valued the instructional discourse. First, it was apparent that the learning activities helped create a classroom environment in which the teacher was able to provide learning support to individuals and small groups of students. This was the case because the students found the activities valuable and engaging and being on task, did not disrupt the class or their peers. Second, student comments emphasised the importance of physically manipulating the materials and linking the material representations, pictorial displays and symbolic representations. Some students reported the use of materials to be useful in early phases of learning, but once the procedural rules were understood, no longer needed to manipulate the materials physically. These results are consistent with those of researchers who recommend this approach to teaching (as above) and in contrast to the descriptions of teaching of algebra in most classrooms (e.g., Kirshner & Awtry, 2004; MacGregor, 2004; Stacey & MacGregor, 1999) and the teaching of mathematics in general (e.g., Barrington, 2006; Gregg, 1995; Hollingsworth et al., 2003; Perry et al., 1999; Thompson & Fleming, 2003).

Teachers are central to effective reform (e.g., Doerr, 2004) and this study indicated that there are good instructional discourse models upon which to build engaging conversations that would help students to develop perceptions that they can learn algebra, that it is not mostly a collection of unrelated rules and symbolic manipulations but rather an inquiry based upon ideas. It also provided data that students can improve in their perceptions that mathematics learning can be fun. This conclusion highlights the potential importance of the nature of the instructional pedagogical discourse used by Jane in her intervention for the professional development of pre-service and in-service teachers. In particular, the use of concrete materials, games and explicit language should underpin middle school mathematics teaching and learning in order to foster students' positive perceptions of algebra.

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