Video Evidence: What Gestures Tell us About Students' Understanding of Rate of Change

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This paper reports on insights into students' understanding of the concept of rate of change, provided by examining the gestures made, by 25 Year 10 students, in video-recorded interviews. Detailed analysis, of both the sound and images, illuminates the meaning of rate-related gestures. Findings indicate that students often use the symbols and metaphors of gesture to complement, supplement, or even contradict verbal descriptions. Many students demonstrated, by the combination of their words and gestures, a sound qualitative understanding of constant rate, with a few attempting to quantify rate. The interpretation of gestures may provide teachers with a better understanding of the progress in their students' thinking.

Introduction

Rate of change, with its many everyday applications, is an important concept throughout the mathematics curriculum. However it is fundamental to the understanding of early calculus: without a conceptual understanding of rate of change differentiation becomes an exercise in applying rules and executing routines. This paper reports on data, from a larger study, collected to explore the variation in pre-calculus students' understandings of rate of change. Experience (e.g., Kelly, Singer, Hicks, & Goldin-Meadow, 2002) has shown that analysing students' gestures as well as their utterances will provide greater insight into their thinking. In this paper, five gesture episodes are considered in detail. The aim of the exercise was to identify complementary, supplementary, or conflicting information conveyed by the students' gestures that was not conveyed by the oral text.

The section of the interviews that forms the focus of this paper provides data relating to students' understanding of rate of change in a non-motion context. The scenario was classified as "non-motion" because, for this example, the students were not asked to discuss change in position over time. Detailed analysis, of both the sound and images, of video-recorded interviews with individual students as they explained their reasoning about a computer-based simulation, provides insights into their thinking. Dynamic geometry (Geometers' SketchPad) was used to simulate a blind blocking sunlight coming in a window. This scenario provided a focus for each student's explanations as they grappled with the words needed to describe rate of change in the area of window exposed as the blind is raised, allowing sunlight to enter.

In the following sections, the conceptual framework is described; details of the interviews and the computer-based simulation are provided; and the manner in which the results can be analysed, by attending to gesture, is discussed.

Rate of Change

In this section we draw attention to students' likely school mathematics background related to rate of change; its importance as a pre-calculus concept; and the rationale for choosing to ask the students to discuss a "non-motion" scenario.

Proceedings of the 30th annual conference of the Mathematics Education Research Group of Australasia
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According to the curriculum advisory documents (Victorian Curriculum and Assessment Authority (VCAA), 2005) and text books (Bull, Howes, Kimber, Nolan, & Noonan, 2003) the students, in this study, would have studied rate of change in conjunction with ratio, proportion, and percentage, usually, in Year 8. The topic is included in texts for that level. Typical of these is Bull et al. (2003) who describe rate as a measure of how one quantity changes with respect to another. This relationship between two changing quantities may be described qualitatively, such as increasing quickly, or quantitatively with units, such as dollars per year.

Researchers, writing about calculus students' understanding of rate of change, commonly provide more formal or more abstract definitions. For example, Hauger (1997) stresses the importance of the unit change in the independent variable resulting in a change in dependent variable. They consider this to be a very important foundational concept for a sound understanding of derivative.

The traditional approach (Thomas, 1969; 2008) to the introduction of derivative presents students with a formal, abstract definition and rule ($\lim_{h \to 0} (f(x+h) - f(x))/h$), then requires students to manipulate the symbolic representations of functions. Some students become quite competent in this manipulation and can accurately produce the symbolic representation of the derivative, but may not appreciate its meaning and connection to other mathematical concepts studied in earlier years. Indeed, Tall (1991) asserts that although calculus is broken up into small chunks and presented in a sequential, logical (at least to the teacher who can see the whole picture) series of lessons, "students may see the pieces as they are presented in isolation, like separate pieces of a jigsaw puzzle for which no total picture is available" (p. 17). Students may not even be aware that there is a big picture.

It was evident in Pierce and Atkinson's (2003) study in which a number of students, who, when asked to prepare a worksheet for novice calculus students, based on a graphical computer simulation for the tangent to a trigonometric function, ignored rate of change and focused on a rule for differentiating polynomials! Making students aware of the big picture involves linking the new concept to their previous understandings (Hiebert & Carpenter, 1992), which, in the case of differentiation, means being aware of pre-calculus students' understandings about rate of change.

A typical abstract introduction is often then followed later by a motion (change in position over time) example where velocity is aligned with derivative. It appears to be assumed that speed is a well-understood, familiar concept on which to build an understanding of derivative. However, rate may also appear in non-motion contexts, such as the rate of change of area of a circle with change in radius.

This study explores pre-calculus students' thinking about rate of change, in a non-motion context, by analysing video evidence. The next section discusses the reasons for choosing video as a data collection method.

Data Collection Methods

Data may be collected from a variety of sources such as: written tests or students' worksheets; teachers' reports; classroom observation; audio recording of interviews with teachers, individual students, pairs of students or small focus groups; and video. Each method has strengths and limitations. Even though video-recording interviews may be more time-intensive and problematic, for example, some students or their parents are less likely to give consent, the decision was made to use this method because video provides a comprehensive record for later detailed analysis.

Video enables the researcher to formulate interpretations of the gaps in the audio record. Data captured by video may provide a more comprehensive understanding of the learning demonstrated by students (Pea, 2006). Such data may include sound and images containing facial expressions, tone of voice and gestures, together giving insights into emotions and depth of understanding of concepts. Fine-grained analysis discloses insights into students' understanding not otherwise available (Alibali & Goldin-Meadow, 1993).

The next section explores how we may analyse the non-verbal data privileged by the videos.

Gesture

One of the advantages of using video for data collection is that it captures non-verbal communications. The importance of gestures, in conveying information regarding students' understanding of mathematical concepts, has become the focus of much research in recent years. Goldin-Meadow, Kim, and Singer (1999) assert the importance of teachers' gestures in the learning of mathematics in their study of eight teachers, teaching mathematical equivalence to students of age eight to ten years. Noble (2003) reports on the use of gestures in the development of the new mathematical knowledge of connecting graphs of motion with the student's own motion, for one student over three teaching episodes. Sabena (2004), who studied of secondary students understanding of the integral function, reports that gesture was instrumental to the development of this concept. Similarly, Arzarello, Robutti, and Bazzini (2005) suggest "students' cognitive activity is strongly marked by rich language and gesture production" (p. 64), as the 11- and 12-year-olds, in their study, construct "meanings related to the concept of function" (p. 55). They advocate that teachers should encourage the use of language, body-related motion, and gestures and include these in the planning of their lessons. Edwards (2005a), in her study of pre-service teachers, reports that gestures played an important role in their recall of procedures related to fractions. Williams (2005) refers to "gesture as part of an integrated communication system with language and … mathematics" (p. 146). When interactions between students are videoed and the visual images are examined, these images may record instances where one student facilitates the learning of another, and possibly their own learning, by drawing their attention to a particular aspect of a task (Rasmussen, Stephen, & Allen, 2004).

Other researchers, such as Goldin-Meadow (2004) and Arzarello and Robutti (2004), also support the claim that the use of gesture aids an individual's learning of mathematics, perhaps by replacing some of the cognitive load of problem-solving or explanation with gesture (Goldin-Meadow, Nusbaum, Kelly, & Wagner, 2001). This suggests that gesture may not necessarily be used to convey information to another people, but also performs the function of assisting gesturers to clarify their own thoughts.

Of particular interest is that gestures may convey information that differs from the information provided by speech. Gestures may provide additional, complementary information, but may also contradict speech (Alibali & Goldin-Meadow, 1993). The gesture-speech mismatch may afford teachers an opportunity to guide students towards a more correct and complete understanding of a mathematical concept (Alibali, Flevares, & Goldin-Meadow, 1997). Hence, it is important for researchers and teachers to learn more about the hidden meanings of students' gestures (Kelly et al., 2002). However, the interpretation of gesture is often difficult as the gestures may be ambiguous (Williams & Wake, 2004). Interpretation may be facilitated by the classification of gestures. McNeill (1992) defines four gesture categories: beat giving emphasis; deitic or pointing; iconic imitating physical phenomena; and metaphoric, which represent meaning of some kind, but are less easy to interpret. Edwards (2005b) refers to the need for additional categories to enable clearer interpretation of students' gestures. She suggests that the iconic classification may be divided into iconic-physical, for iconic gestures matching physical phenomena, and iconic-symbolic, for iconic gestures referring to "a remembered written inscription for an algorithm or mathematical symbol" (p. 136). Further, she proposes "the nature of mathematics as a discipline may require an even more refined categorization of gestures" (p. 138). Indeed, Arzarello and Robutti (2004) define iconic-representational gestures, as gestures that refer "to a graphical representation of a phenomenon" (p. 307).

The next section describes the methodological considerations of this study.

Method

The seven students whose data are reported in detail this paper were selected from the 25 Year 10 students from five different secondary schools interviewed for the full study. These students were selected because the videos of their interviews demonstrate clear examples of gestures that were commonly used by many of the students in the study. A Geometers' Sketchpad (GSP) file simulating two windows with blinds (Figure 1) was prepared.

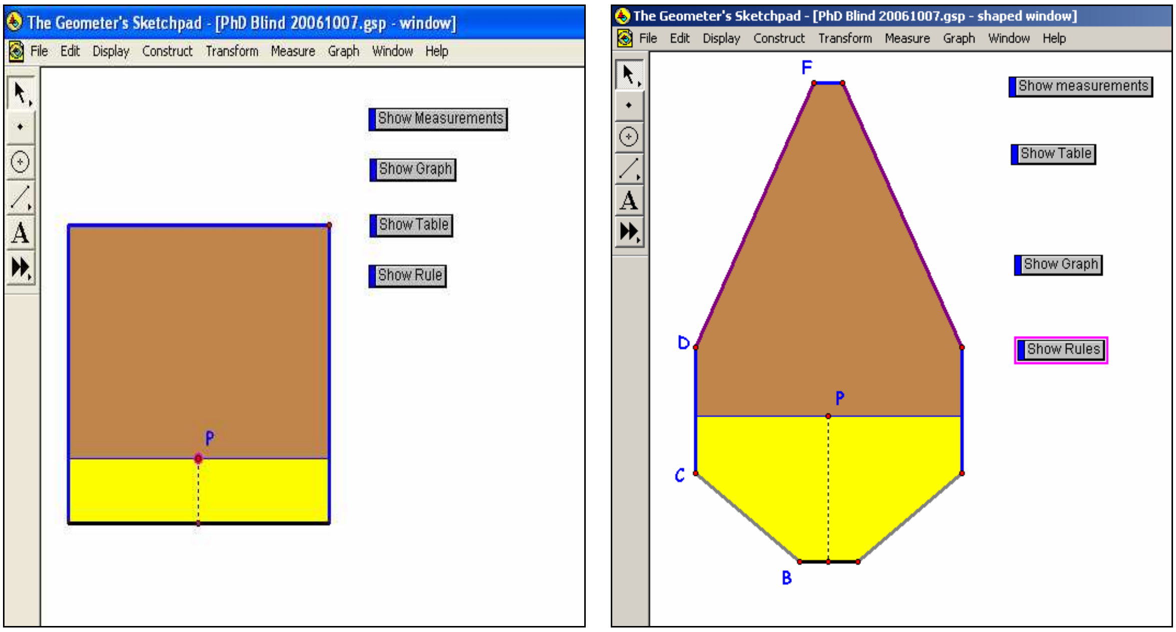


Figure 1. GSP simulation of windows.

The simulation shows two windows, one rectangular, one arched, both with blinds, which could be raised or lowered by dragging. This had the effect of changing the variables: area of sunlight and height of blind above the bottom of the window. Possible constant rate variation associated with the simulation was illustrated using multiple mathematical representations: numeric, graphic, and symbolic.

This simulation and a photograph of an arched window were used as catalysts to explore students' understanding of the constant rate of change. Similarly, the non-rectangular window was used to probe students' understanding of the differences between constant and variable rate. In this way, GSP facilitated exploration of constant and variable rate in multiple representations. These simulations, which were first trialled by a pilot group of students, provided visual material for students to point to, in order to clarify their explanations of their understanding of rate.

Students were videoed as they responded to the interviewer (first author) who prompted them to discuss the rate of change in the area of sunlight exposed as the height of the blind was changed. Students were encouraged to explain their reasoning and think aloud as they were presented with different representational forms of rate of change: the simulation, table of values, graph, and symbolic rule.

The videos were each viewed several times and the students' use of gesture was coded and checked. Coding, as is detailed below, combined McNeill's (1992) deitic and metaphoric categories with the refinements of the iconic classification of iconic-representational (Arzarello & Robutti, 2004) and iconic-physical and iconic-symbolic (Edwards, 2005a). The five episodes described below (pseudonyms used) illustrate the complex use of gesture students called upon to supplement their utterances in order to explain their thinking about rate.

Findings and Discussion

In the twenty-five video-interviews recorded, one student gestured frequently, two students only used deitic gestures to indicate locations on the screen, and the remaining twenty-three students used deitic, iconic, and metaphoric gestures especially when struggling for words to describe their understanding. The simple deitic gestures add to the audio record by clarifying exactly what the students are referring to and emphasising the feature they see as important in their explanation of rate of change.

Straight-sided arch shape for small distances	Two hands for larger distances	Deitic gestures indicating parts of the screen

Figure 2. Rate-related gestures.

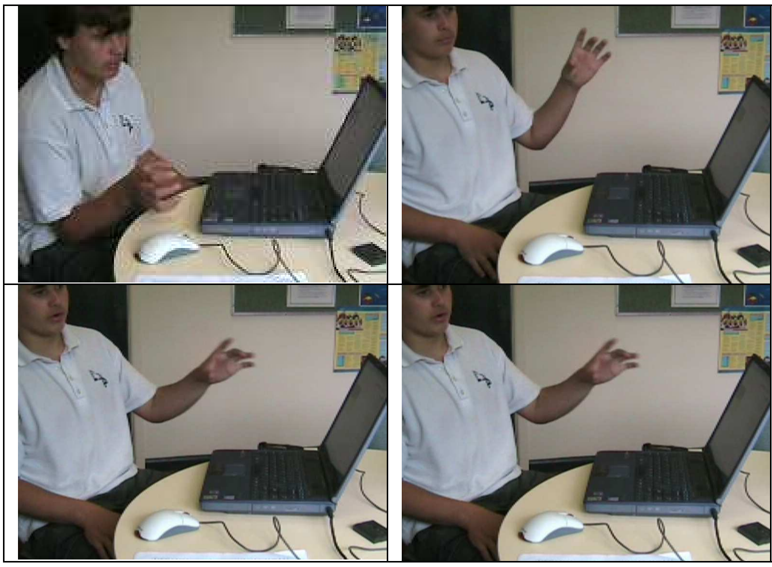
Many students used one hand to form a straight sided arch to represent small distances and two hands held apart for larger distances (Figure 2). These are examples of iconic-physical gestures (Edwards, 2005a). Many students employed what Rasmussen et al. (2004) chose to call a "slope hand gesture" (Figure 6) representing the shape of the linear graph. Rasmussen et al. (2004) found that this was commonly used by students to infer constant rate.

In addition to noting specific static forms of rate-related gestures seen in Figure 2, five gesture episodes were examined in greater detail. "Moving slope gesture" (Figure 3), where a hand was held straight and rigid with the arm pivoted at the elbow, when Annie was describing what the graph would look like if the window were narrower, indicated a change in constant rate in the same manner as Rasmussen et al. (2004) describe.

Figure 3. Moving slope gesture.

The next example demonstrates this student's thinking about the variables involved in this constant rate context.

> Researcher: what does the table tell you about the rate that the height is changing?
> Jason: it goes up three point two meters [pause] every half a meter



It seems that Jason is using the same straight-sided arch shape others used to indicate a small distance, but in a different way.
He seems to demonstrate a unit measurement by making a straight-sided arch shape with his right hand and matching that with three movements of the same-sized, straight-sided arch shape with his left hand.

Figure 4. Speech-gesture mismatch.

In this episode (Figure 4), Jason's gestures and words do not match. He indicates the 0.5 m, from the table, with his right hand as he says "it goes up three point two meters" and makes just three movements with his right hand whilst saying the words "every half a meter". This may suggest that Jason is uncertain about which variable, area or height, is involved in the unit change. These arch gestures could be classified as iconic-symbolic, in this case, as they appear to represent a unit of measurement. However, this episode also suggests that he has some notion that rate involves a change in one variable related to the unit change of another variable. Such a gesture-speech mismatch may provide an opportunity for guidance by a teacher (Alibali & Goldin-Meadow, 1993) to clarify the variables.

Interestingly John, in Figure 5, when he was looking at the simulation of the non-rectangular window and describing the rate in the rectangular section of the window, also uses the same movement of the straight-sided arch gesture as Jason, as he talks about constant rate when referring to the rectangular section of the window.

Changing at a constant pace	…the rates will be going up by the same amount

Figure 5. Constant rate gesture.

The next example demonstrates the manner in which gesture can be used to supplement words. There were many instances, in the data, of gesture being used in this way, as shown in Figure 6.

Researcher: what do you think the graph [of the arched window in this photo] might look like? Sue: I don't think it would be a straight line because [pause]	Researcher: What makes you think that? Sue: because this window's not square hmm I dunno because its up the top umm curved there	Sue: [pause] umm a straight line means its always the same

Figure 6. Gesture supplements words.

It appears Sue is grappling with the difference between graphs for constant and variable rate but does not have the words to express her understanding. She is using her hands to indicate what she is thinking. The iconic-representational gesture in the first frame (Figure 6) appears to represent the shape of the graph of constant rate. In the middle of Figure 6, her gesture is iconic-physical as she is showing the physical shape of the top of the window. Finally, in the right frame of Figure 6, the "slope hand gesture" is repeated, indicating her understanding that constant rate will result in a linear graph. Sue has identified the key difference in the two scenarios presented by a rectangular window and an arch window. Her gestures communicate her understanding, demonstrating her awareness that the graph for the curved section of the window would not be the same as the rectangular section. The distinction between the iconic-representational gesture and the iconic-physical gesture indicates that her thinking had not yet progressed to transferring her understanding of the physical situation into a graphic, mathematical representation. Her gestures provided additional information not available in her words. This presents an ideal opportunity for a teacher (Alibali & Goldin-Meadow, 1993) to assist by supplying suitable words to describe her correct thinking and extend her understanding of variable rate to include the graphical representation.

The final example demonstrates a student's thinking about the shape of the graph for variable rate, as she is considering the graph for the rectangular window. The deitic gestures have been used to supplement words rather than just indicating an aspect of interest on the screen.

Claire: It's going at a average pace. It's equal um even pace equals yeah its even. Researcher: The rate's even?	Claire: yeah like if it was like uneven like it would be like say like pull up this much of it this much light showing	Claire: like it would probably go like that

Figure 7. Gesture for variable rate.

In this episode (Figure 7), Claire also uses the pointed fingers of both hands to indicate a larger distance, similar in meaning to the manner in which other students have used two straight hands. In Figure 7, the diagram illustrates the up and down motion of the index finger of her right hand. It appears that Claire is contrasting "even" rate by trying to demonstrate "uneven" rate with these deitic gestures. She seems to be associating variable rate with this collection of linear segments, varying the slope from positive to negative in an upward trend.

The gesture episodes observed, offer insights into students' understanding of rate of change, which were absent from their words alone. The images in Figure 2 show some commonly used rate-related gestures. In Figure 3, Annie used the "moving slope gesture" to indicate change in constant rate. Jason's gestures, in Figure 4, suggested confusion between the variables, involved in the rate, which was not evident in the written transcript. John (Figure 5) uses the same shaped gesture as Jason, but matches his words to the gesture. In Figure 6, Sue was unable to verbalise why the graph would not be straight for variable rate, but demonstrated by the repeated use of the "slope gesture" that constant rate would result in a linear graph. Claire's statement, in Figure 7, "like it would probably go like that" could not have been interpreted, whereas her deitic gestures suggest she does not fully understand either constant or variable rate. Gestures augmented the verbal descriptions to give greater depth to the researcher's understanding of the meaning of the students' utterances.

Implications

The concept of rate involves an understanding of quantities and their measurement. The episodes, described in this paper, demonstrate examples of gesture related to constant rate; gesture related to variable rate; gesture supplementing utterances; gesture contradicting utterances; and gesture consistent with the classification of other researchers. For the students in this study, gestures provided an intermediary stage. They were able to articulate qualitative, but not completely correct quantitative, descriptions of rate; gesture enabled them to communicate their understanding by using non-standard units (e.g., Figure 4).

Analysis of the video evidence showed that these students had a sound conceptual understanding of constant rate of change but some students had difficulty in verbalising this. The use of gesture enabled many students to communicate ideas related to the less abstract graphic and numeric representations but most students, although able to describe operations with the symbolic representation, could not link this to rate of change. Some students were able to use gesture to supplement their utterances relating to variable rate, but none could describe their thinking with words alone. Indeed, many students demonstrated little conceptual understanding of variable rate in this non-motion context.

Students' gestures provided a rich source of evidence from which to evaluate their understanding of rate of change. Such evidence is not always available in written tests where only the words are valued. Attention to gestures may enable teachers to comprehend better the depth and accuracy of students' understanding of mathematical concepts and allow teachers to target interventions appropriate for individual students. For example, when students' words and gestures match it is likely that they have a clear understanding of the concept. Such students are ready to explore more advanced concepts. When students cannot find words to express themselves, but can demonstrate concepts through gesture, there is an opportunity for the teacher to build on their understanding by targeting vocabulary and symbolic representations. In the case where students use gestures that contradict their utterances, there is an indication that the students do not, as yet, fully understand the concept. Such a mismatch may alert the teacher to the need, both, to further probe the students' understandings, and also to provide suitable tasks to help the students clarify their understandings. Attending to gesture as well as words helps the teacher more accurately chart their students' growth in understanding of rate of change. The examples included in this paper highlight the advantages of including analysis of gesture in the repertoire of both teachers and educational researchers.

Acknowledgements. This study is part of the RITEMATHS project, led by Kaye Stacey, Gloria Stillman, and Robyn Pierce. The researchers thank the Australian Research Council, our six partner schools and Texas Instruments for their financial support of this project, and especially the teachers and students who took part in this study. Special thanks to Gary Asp for sharing his GSP skills.

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