The Teacher, The Tasks: Their Role in Students' Mathematical Literacy

Katherine Doyle
Queensland University of Technology
<km.doyle@qut.edu.au>

This paper reports on part of a larger study and examines the changing nature of mathematics teaching and tasks. Two Year 4 classes were compared after mathematical-modelling tasks were undertaken with and without top-level structuring. The results indicate that mathematical-modelling and top-level structuring tasks can advance mathematical literacy. Where students are guided through information organisation and mathematising through quality teaching, they can make sense of the mathematical world. Also evident was the vital role of the teacher in creating a positive learning environment through facilitating discourse and literacy development in mathematics students. Recommendations for teaching are given. Indications evidenced here warrant further investigation.

The nature of mathematics teaching and classroom activities is changing in an endeavour to meet the needs of today's students. The role of the teacher is changing from that of main instructor, teaching rules and correcting related exercises, to that of facilitator of mathematical activities that promote understanding of mathematics, mathematical thinking and reasoning abilities. In other words, educators today are aiming to provide students with expertise in mathematics, so that students will be equipped to use advanced thinking skills to acquire mathematical knowledge (Kulikowich & DeFranco, 2003). As a result, mathematics, as well as mathematical solutions and representations become powerful tools by which to understand the world. This paper explores two components of mathematics teaching and how they contribute to mathematical literacy and expertise: firstly, the nature of mathematical-modelling combined with top-level structuring (TLS) activities, and secondly, the role of the classroom teacher as a creator of, facilitator of, and participant in the classroom discourse community.

The Evolution of Mathematical Literacy

Mathematics has been described as an empowering tool, by which people can learn to reason and make sense of the world around them (Schoenfeld, 2002). In order to do this, Schoenfeld maintains, students must be active participants in "mathematical sense-making" activities (2002, p. 155). International educational authorities such as, The National Council of Teachers of Mathematics (2000), the United Kingdom National Curriculum (2000), and Queensland Studies Authority (2004) concur that a variety of problem-solving experiences contributes to students' empowerment and ability to function effectively in society.

The key to mathematical empowerment is mathematical literacy because it is the means by which one can actively participate in the process of problem solving, make sense of the problem, and ultimately unlock a solution. Mathematical literacy has been described by Romberg (2001) as having knowledge of the intricacies of mathematical language in order to gather and understand information on concepts and procedures. This information can then be used efficiently to mathematise various non-routine problems, for example, mathematical modelling problems.

Mathematical modelling is one problem-solving process that aims to provide conditions that facilitate growth in mathematical knowledge. Through participating in a discourse community with peers, students interpret, analyse, reason, seek relationships and patterns between elements, then explain, justify, and predict situations (Lesh & Doerr, 2003a). Through these real-world, open-ended, problem-solving experiences students' develop conceptual systems "to construct, describe or explain mathematically significant systems they encounter" (Lesh & Doerr, 2003a, p. 9). Mathematical-modelling problems involve attaining, managing, and presenting pertinent information through factual and graphic texts, as well as aurally and orally. Therefore, students require a sound level of language literacy in order to construct mathematical literacy. One means of enhancing language literacy is to employ a literacy strategy as a sense-making tool to use in conjunction with mathematical-modelling activities. TLS aids comprehension of oral, textual, or graphic information. It is described by Bartlett, Liyange, Jones, Penridge, and McKay (2001) as a procedure:

> which allows the strategic reader, listener or reviewer to form an opinion on what a writer, speaker or performer considers as essential content and if necessary, then to move on to critical or inferential analysis. Conversely, it allows a strategist as writer, speaker or performer to produce coherent text and to signal what he/she wants to be seen as essential content. (p. 67)

Harel and Sowder (2005) argue that educators must construct meaningful, rational instruction that aims to produce advanced mathematical thinking. They differentiate between mathematical thinking and mathematical understanding, but acknowledge these two as essential modes of knowledge. Meanings gained and given, justifications and assertions constitute understanding. However, generated theories and the expression of reasoning, which is specific to not just one situation, but, "a multitude of situations" (p. 31) portray thinking. These skills equate to those disseminated through mathematical modelling.

Theories of mathematics as described in Kulikowich and DeFranco (2003) provide a framework for the teaching of mathematics. Theorists, such as, Barab, Hay, and Yamagata-Lynch (2001) have argued that situated cognition, that is, "the interaction of individuals and their environments" (p. 149) shapes the setting for the attainment of knowledge, whereas others (Anderson, Reder, & Simon, 1996) have claimed that one processes, stores and organises information in one's head (the information-processing theory). Furthermore, critical theorists like Lambert and Blunk (1998) have focussed on authentic, social activity to provide valuable learning experiences. Small "classroom societies" mirror real-life social practices where professionals gather to discuss/debate and realise solutions to problems. Anderson, Greeno, Reder, and Simon (2000) have acknowledged the importance of both cognitive and social practices perspectives. They have identified the fact that there are simply different foci for learning activities. Learning can occur through both solitary and group activities. Anderson et al. (2000) distinguish two aspects of mathematics teaching: (a) the cognitive perspective where students learn structures, concepts, and procedures individually, and (b) the social, situated learning tasks whereby students can learn the intricacies of mathematical discourse, and how to participate in supportive learning practices.

Theories are many and varied. The theories cited here are only a few examples. Nevertheless, as Kulikowich and DeFranco (2003, p. 149) contend, "no one theory should dictate how to practice … educators should draw from a variety of perspectives in teaching and designing materials for the classroom".

Mathematical-modelling tasks are indicative of activities that draw on a variety of theoretical perspectives. For example, the tasks are individually and environmentally interactive: situated cognition. When coupled with TLS, mathematical modelling's alignment with the information-processing theory is enhanced. Because of their emphasis on storing and organising information, TLS provides a tool which fosters thinking skills as students organise their information in a logical and systematic manner (Bartlett, 2003). Finally, modelling tasks are social, reflecting real-world practices, whereby problem solving takes place via a process of interpreting, discussing, explaining, analysing, justifying, revising, and refining ideas (Lesh & Doerr, 2003a).

Modelling activities are a prime example of a current practice that demonstrates the changing role of the teacher. However, it needs to be emphasised that, to be worthwhile, an activity must be constructed with a true perspective on why the activity will benefit learning (Kulikowich & DeFranco, 2003), and how the activity will benefit learning. This perspective constitutes quality teaching. Furthermore, quality teachers carefully monitor students, and act on cues that indicate when and how activities can be directed to gain most benefit. Significantly, teachers need to be active facilitators of classroom discourse, supporting students' focus on meaningful content as well as their reflections on understandings about the content (Schoenfeld, 2002). In this way, teachers are leaders of functional discourse communities that promote mathematical literacy in students.

Teachers must be proactive, and at the same time modify their views to correspond with student needs (McClain, Cobb, & Bowers, 1998). A mathematics classroom should be a community "of disciplined inquiry" (Schoenfeld, 2002, p. 132). A teacher's role is to create an environment where students are supported to participate actively in "mathematical sense-making" through engaging "collaboratively in reasoned discourse" (Schoenfeld, 2002; p. 151). Therefore, students can become independent thinkers. This is what it means to become mathematically literate.

To demonstrate the issues raised in this literature review, episodes taken from a larger study on mathematical modelling and TLS are presented. These episodes provide a context in which to examine the changing face of mathematics teaching and the changing role of the mathematics teacher.

Design and Method

The section of the study reported here is part of a larger study that was designed to investigate the effects of applying TLS to mathematical-modelling tasks. The study used a design-research approach, also known as a design experiment (Bannan-Ritland, 2003). Data were sourced from video/audio taped evidence, student work samples, and teacher observations and reflections. As well, information was gathered from students' Year 3 Queensland 2004 numeracy and literacy test results. This provided an historical record, which added further credence to the final reporting. Employing multiple-method data collection validated claims and assertions from the research (Cobb, Confrey, diSessa, Lehrer, & Schauble, 2003), which used an interpretational, data analysis approach (Tobin, 2000).

The Participants

The participants consisted of 57 Year 4 students. These students were divided into two classes that formed the study's TLS group (n = 28) and the non-TLS group (n = 29). The students attended a Catholic school on the outskirts of a major Australian capital city. The school is situated in a lower to middle socio-economic area. The school principal and teachers very enthusiastically supported the study. The Year 4 teachers actively participated in monitoring the students' group work, in partnership with the researcher.

The Procedure for the Original Study

Firstly, the TLS group received instruction on and practised the application of TLS over a series of ten lessons. Both the TLS and non-TLS groups participated in a mathematical-modelling task where they had to investigate the best light conditions in which to grow beans. Using a table of data illustrating growth rates of beans growing in shade and sunlight, students analysed and compared data to lead them to a decision. Their decision had to be explained with reasons in a letter to a "farmer". Subsequently, the data collected from the students as they progressed through the modelling process was compared and contrasted.

Secondly, the non-TLS group received instruction on and practised the application of TLS over a series of ten lessons. The two groups then participated in a further modelling task. In this experience, students viewed data on distance, time, and number of attempts made by paper planes in a contest. The students were asked to write a letter to judges to explain the best way to decide on a winner. Data collected from this episode was compared and contrasted, any changes in the non-TLS group's capacity to engage fully in the modelling task was noted, taking into account that this was the second modelling experience for the students.

Mathematical Modelling: Teaching and the Role of the Teacher

When the data from this research were analysed, some unexpected findings became evident. These were particularly of interest because of their potential impact on mathematics teaching and outcomes. Of specific interest, were the effects of the interactions of individual teachers with students as the students participated in their group tasks. As a result, this paper reports on these findings in the light of mathematical modelling and TLS as interactive, sense-making, social components of a discourse community that should positively contribute to mathematical literacy for students. Equally, the findings are discussed with special attention to the teacher role and the effects teacher interaction had in enhancing or diminishing students' mathematical literacy.

Results and Discussion

There are results on the impact of TLS on mathematical modelling reported elsewhere, such as Doyle (2006). Two major assertions can be drawn from the data analysis here. Firstly, with reference to mathematical-modelling tasks coupled with TLS, the analysis indicated that students participated in an active discourse community. Students were able to mathematise as they investigated and analysed data, made connections, explained, and justified their ideas, an indication of students' acquisition of mathematical literacy. Secondly, emerging from these data, was the fact that the teacher plays a vital role as creator, facilitator, and participant in the discourse community. Of major significance were (a) the role of the teacher as listener, and (b) the role of the teacher as questioner. Listening to student discourse proved to be crucial in the teachers' ability to impact positively or negatively on the classroom discourse community. As well, the way in which questions were directed to students influenced the discourse community. When focused, sharp questioning occurred, the result was positive in that the students remained on task and their mathematising was enhanced. When questioning were unclear and prolonged, the students became confused, which detracted from their mathematising.

In the following text example, the students demonstrated that they were comparing and measuring as they participated in the modelling task. They were using the organised language of TLS to explain that they were "comparing" the weights of the beans. The teacher's role here was to focus the students on their mathematising. The students explained their perceptions. The teacher's questioning had a positive impact supporting the students' sense-making of the situation.

Megan: We need to write down Weeks 6, 8 and 10 and rows 1, 2, 3, and, 4 for sunlight and then we'll move on to shade.

Teacher: What is happening here?

Jeff: We're *comparing the weights*.

Teacher: So what is happening when you compare the weights?

Jeff: We're mainly measuring the weights of butter beans after they're in sun and shade.

Teacher: What are you comparing – weeks or rows??

Jeff: *We're comparing like* in row 1, week 1 they have 9 kilos in the sun. They're not growing too well *but in the shade* they are growing heavier.

Teacher: That's row 1 but is that the same for everywhere else?

Jeff: No, not really.

(Students continue to work out and write their results under two headings: The results of the weights of the butter beans in the shade/and in the sunlight.)

Jeff: *We are comparing the results* of the butter bean plants. After 10 weeks we have some results.

Other examples of the teachers' positive role occurred throughout the modelling investigations, such as, the teacher in the following text encouraged mathematical thinking and the need for justification.

Ben: So, sunlight has more kilos *compared to* the shade.

Teacher: And is that true for all of it? You have to make a decision and you have to check that information really carefully.

The teacher's intervention in the next excerpt was necessary to counteract the students' over reliance on their prior knowledge. Where prior knowledge often plays an important part in the construction of new knowledge, in this instance the student was relying on it whilst ignoring the mathematical data and evidence. The teacher had listened to the students as they discussed and was correctly cued to this situation. As a result, the teacher was able to redirect the students' attention to focus on the data to provide mathematical evidence for their explanations.

Kiesha: Well, a plant needs sunlight and shade. If a plant gets too much shade it will die or if a plant gets too much sunlight, it will die.

Teacher: What is this (the table) telling you? This is going to prove your discussion. Why is it saying sunlight is best?

Matt: Its got more kilos than in the shade.

Teacher: Oh! OK, so it has more kilos compared to?

Matt. The shade. So, sunlight has more kilos compared to the shade.

On the other hand, there were examples of instances where teacher intervention detracted from the students'mathematising during the investigation. This was most likely due to the teacher not having listened into the students' discussions accurately. In this next excerpt, the students were reasoning with time, distance, and also realising from the table that "scratching" needed to be taken into account when deciding on a winner.

Isobella: I chose E because it goes for a long distance. It goes for longer seconds and it has no scratches.

Eden: I chose Team E because its scores were higher than the others and it didn't get scratched and it hoes for the longest time." Both girls reasoned with time, distance and took scratches into account.

However, the teacher interrupted and asked to be shown the proof for their claims. Also, a number of questions were asked in a row.

Teacher: How do you know? Where did you work it out? Show me how you worked that out. Show me the numbers.

Students: The numbers?

Teacher: Show me the best number from the others. Make sure you can prove it.

Although the intention to ensure the students had evidence for their claims was good, the timing was unfortunate and only succeeded in misdirecting and confusing the students. It took them away from their sense-making, mathematising, and empowerment. Isobella's final comment ratifies this. She moved from a confident participant to believing that they were all "wrong in their answer". As well as being interrupted, the students were also faced with four directions in a row. This appeared to baffle the students even more. This is demonstrated in the subsequent conversation.

Kristy: I don't get it though.

Eden; What are we meant to do?

Kristy: I don't get what he said.

Isobella: Well, we're not doing it right because he told me to pick the number of each that would be the best one so I circled 13 because it was the biggest out of all of them so that's why I chose E.

Following is an illustration of what teachers can unwittingly cause in the classroom. The students were confidently examining and interpreting the data, analysing, and justifying their ideas. Firstly, the students appeared confident and empowered. This conversation was indicative of the type of discussion that the researcher had witnessed occurring in the classroom.

Eryn: What about Team D?

Kristy: Yeah, *but it has scratches*.

Eden: E doesn't have any scratches.

Kristy: Neither did C and neither did B.

Isobella; I chose E *because* it has 13, the highest number out of ll of them and that's why I chose E.

Kristy: What about C? It goes 9, 11, 11

Isobella: Eden, why did you choose E?

Eden: *Because there were no scratches*. It had the highest number in metres *and because its seconds* were more and so…

Students were examining the data, accounting for variables, looking for patterns, considering length and time, and generally finding mathematical justifications for their explanations. However, despite an urging from the researcher not to interrupt the students as they were actively participating, and were on task, the teacher stopped all groups. This teacher thought it would be better for the students to think individually about their decisions and then share with the rest of their group. The justification for this was that there would be better outcomes for the research because after personal reflection, the students would have more ideas to discuss. As a result, the students' behaviour diminished. When they returned to discussion, they were not on task. They read out their written ideas. It appeared that fellow group members did not listen to these readings.

Conclusion and Implications

Mathematical-modelling tasks coupled with TLS demonstrate how activities can be successful in promoting mathematical literacy. These tasks go beyond traditional views to provide students with opportunities to acquire advanced-thinking skills that are interpretive, organisational, and communicative as students encounter a variety of narrative, graphic, and factual texts (English, 2004; Lesh & Doerr, 2003b; Lesh & Yoon, 2004). The overall study not only clarified this claim, but also opened other windows of opportunity to investigate such issues as the role teachers have in impacting positively or negatively on students' acquisition of mathematical literacy.

The research reported here informs mathematics educators in two specific ways. The first way is to reiterate the view of Harel and Sowder (2005), that mathematical thinking will be best produced if meaningful and rational tasks are constructed. Students must be guided to think mathematically through the activities provided for them, and by the expertise of the teacher. These vital roles of the classroom teacher were demonstrated in the results reported here. Students were given tasks that encouraged mathematical thinking, but the teacher, in certain instances needed to guide the students to mathematical understanding (Harel & Sowder, 2005).

The second way is to impress upon educators the essential role of the teacher to (a) construct quality activities that benefit learning, and (b) act appropriately on indicative cues to benefit learning (Kulikowich & DeFranco, 2003). The episode where the teacher interrupted the whole class was an example of where this teacher could have modified personal views (McClain et al., 1998) to benefit the learning community. This, as well as the other example cited, demonstrates that perhaps we, as educators, all have lessons to learn on how our decisions impact upon our students. Further research investigating teacher impact on students learning could benefit mathematics teaching.

Mathematical modelling with TLS has given a prime example of how tasks can be constructed to reflect a diverse theoretical basis. These interactive tasks are established in situated cognition (Barab et al., 2001). As students are required to store and organise information, the tasks build upon information-processing theory (Anderson et al., 1996). They are social activities (Lambert & Blunk, 1998). They reflect both cognitive and social practices (Anderson et al., 2000).

An environment for mathematical sense-making (Schoenfeld, 2002) must be created. A quality teacher provides the means by which to do so. Mathematical modelling with TLS provides a task by which to do so. Such an environment encourages students to make sense of situations as they participate in a supportive discourse community to advance their mathematical literacy.

References

Anderson, J., Greeno, J. G., Reder, L. M., & Simon, H. A. (2000). Perspectives on learning, thinking, and activity. *Educational Researcher, 29*(4), 11-13.

Anderson, J., Reder, L. M., & Simon, H. A. (1996). Coming to terms: How researchers in learning and literacy talk about knowledge. *Educational Researcher, 24*(4), 5-11.

Bannan-Ritland, B. (2003). The role of design in research: The integrative learning design framework. *Educational Researcher, 32*(1), 21-24.

Barab, S. A., Hay, K. E., & Yamagata-Lynch, L. C. (2001). Constructing networks of action-relevant episodes: An in situ research methodology. *The Journal of the Learning Sciences, 10*, 63-112.

Bartlett, B. J. (2003). Valuing the situation: a referential outcome for top-level structurers. In B.J. Bartlett, F. Bryer & D. Roebuck. (Eds.), *Reimaging Practice: Researching Change* (pp. 16-37). Griffith University, Brisbane: Proceedings of the 1st International Conference on Cognition, Language and Special Education. School of Cognition, Language and Special Education.

Bartlett, B. J., Liyange, I., Jones, S., Penridge, J., and McKay, K. (2001). Science Education through Literacy and Language. In F. Bevan, C. Kanes & D. Roebuck (Eds.) *Knowledge Demands for the New Economies Volume 1*. Brisbane: Academic Australian Press, pp. 64-70

Cobb, P., Confrey, J., diSessa, A., Lehrer, R., & Schauble, L. (2003). Design experiments in educational research. *Educational Researcher, 32*(1), 9-13.

Doyle, K. (2006). Organisational structure for mathematical modelling. In P. Grootenboer, R. Zevenbergen & M. Chinnappan (Eds.), *Identities, Cultures and Learning Spaces:* (Proceedings of the 29th annual conference of the Mathematics Education Research group of Australasia, Vol. 1, pp. 187-195). Sydney: MERGA.

English, L. D. (2004). *Mathematical modelling in the primary school.* (Proceedings of the 27th annual conference of the Mathematics Education Research Group of Australasia, Vol 1, pp. 207-214). Sydney: MERGA.

Harel, G., & Sowder, L. (2005). Advanced mathematical-thinking at any age: Its nature and its development. *Mathematical Thinking and Learning, 7*(1), 27-50.

Kulikowich, J. M., & DeFranco, T. C. (2003). Philosophy's role in characterizing the nature of educational psychology and mathematics. *Educational Psychologist, 38*(3), 147-156.

Lambert, M., & Blunk, M. L. (Eds.). (1998). *Talking mathematics in school: Studies of teaching and learning.* Cambridge: Cambridge University Press.

Lesh, R., & Doerr, H. M. (2003a). Foundations of a models and modelling perspective on mathematics teaching, learning and problem solving. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modeling perspectives on mathematics problem solving, learning, and teaching* (pp. 3-33). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Doerr, H. M. (2003b). In what ways does a models and modelling perspective move beyond constructivism. In R. Lesh & H. M. Doerr (Eds.), *Beyond constructivism: Models and modelling perspectives on mathematics problem solving, learning, and teaching* (pp. 519-556). Mahwah, NJ: Lawrence Erlbaum Associates.

Lesh, R., & Yoon, C. (2004). What is distinctive in (our view about) models and modeling perspectives on mathematics problem solving, learning and teaching? In H. W. Henn & W. Blum (Eds.). *ICMI Study 14: Applications and Modeling in Mathematics Education* (pp. 151-159): Dortmund, Germany: University of Dortmund.

McClain, K., Cobb, P., & Bowers, J. (1998). A contextual investigation of three-digit addition and subtraction. In L. J. Morrow & M. J. Kenney (Eds.), *The teaching and learning of algorithms in school mathematic., 1998 yearbook*. Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston, VA: National Council of Teachers of Mathematics.

Queensland Studies Authority. (2004). *Mathematics: Years 1 to 10 Syllabus*. Retrieved 24/05/05, from http://www.qsa.qld.edu.aqu/yrsato10/kla/mathematics/index.html

Romberg, T. A. (2001). *Mathematical literacy: What does it mean for school mathematics?* Retrieved 15/03/07, from http://www.wcer.wisc.edu/ncisla/publications/articles/OctMathWASB.pdf

Schoenfeld, A. H. (2002). A highly interactive discourse structure. *Social Constructivist Teaching, 9*, 131-169.

Tobin, K. (2000). Interpretive research in science education. In A. E. Kelly & R. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 487-512). Mahwah, NJ: Lawrence Erlbaum Associates.

U. K. National Curriculum. (2000) *Mathematics: The National Curriculum for England*. Retrieved 18/03/07, from www.nc.uk.net