Essential Differences between High and Low Performers' Thinking about Graphically-Oriented Numeracy Items

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This study compared the thinking of five high performing and five low performing primary students on a set of graphically-oriented numeracy items. Generally, their thinking differed in four ways. First, high performers drew on existing knowledge and skills, which low performers appeared to lack. Second, high performers used multiple cues to complete tasks, whereas low performers worked from a single cue or overlooked cues. Third, high performers used simple solution procedures correctly; in contrast, low performers used more mentally demanding procedures with limited success. Finally, high performers were more knowledgeable about everyday graphics than low performers.

Introduction

Worldwide there has been a strong and ongoing emphasis on the development of a numerate populace who can use mathematics effectively in everyday life at home, at work and in the community. Traditionally, numeracy has been characterised by arithmetical competence. However, in the digital age, numeracy also involves proficiency with the various graphics that are commonly used in mathematics (Department for Education and Employment, 1998): "numeracy also demands practical understandings of the ways in which information is gathered by counting and measuring, and is presented *in graphs, diagrams, charts and tables* (emphasis added)" (p. 110). Thus, the achievement of a numerate populace requires that all citizens use graphics effectively in mathematical situations. The students who are most at risk of being innumerate are those who struggle with mathematics. Hence, the achievement of the numeracy goal depends on our ability to educate those students who have difficulty with essential mathematics. These students are of two types. First, there are those students who have special needs due to a problem that impacts on their ability to think or to learn. These difficulties include memory problems, processing or perceptual deficits (Diezmann, Thornton, & Watters, 2003). Second, there are those students who do not have specific learning problems but nevertheless are low performers. Notwithstanding the importance of understanding how to educate students with special needs, this paper focuses on ways to support students who are low performers on numeracy items that incorporate graphics. This support will be informed by the performance of students who consistently demonstrate proficiency with these items because such students can provide an insight into the knowledge and skills that are required to be successful. Thus, this study will contribute towards addressing the paucity of literature on high and low performing Australasian students (Diezmann, Lowrie, Bicknell, Farragher, & Putt, 2004).

Background

To provide a background to the thinking and solution strategies of high and low performers on numeracy tasks, we first provide an overview on graphics in mathematics and high and low performers' use of representations in mathematics.

Graphics in Mathematics

In recent decades, there has been enormous growth in the field of information graphics for the management, communication, and analysis of information (Harris, 1996). Although there are many thousands of graphics in use, they can be categorized into six broad categories that Mackinlay (1999) refers to as "graphical languages" (Table 1). These languages are distinguished by the information that is encoded in the graphic and the relationships among the graphical elements. Knowledge of graphics is fundamental to success on many numeracy items. However, although graphics are visual-spatial rather than linguistic or symbolic representations, many primary students have difficulty interpreting graphics, such as number lines (Diezmann & Lowrie, 2006).

Table 1
An Overview of the Six Graphical Languages (adapted from Mackinlay, 1999)

Graphical Languages	Encoding Technique
Axis Languages (e.g., number line)	A single-position encodes information by the placement of a mark on an axis.
Opposed Position Languages (e.g., bar chart)	Information is encoded by a marked set that is positioned between two axes.
Retinal List Languages (e.g., saturation on population graphs)	Retinal properties are used to encode information. These marks are not dependent on position.
Map Languages (e.g., road map)	Information is encoded through the spatial location of the marks.
Connection Languages (e.g., network)	Information is encoded by a set of node objects with a set of link objects.
Miscellaneous Languages (e.g., pie chart)	Information is encoded with a variety of additional graphical techniques (e.g., angle, containment).

High and Low Performers' Use of Representations in Mathematics

Mathematical proficiency is influenced by students' understanding of a variety of representations including graphics. According to von Glasersfeld (1987), the individual plays an important role as the interpreter or decoder of a representation: "A representation does not represent by itself – it needs interpreting and, to be interpreted, it needs an interpreter" (p. 216). Students' proficiency with representations impacts on whether they will be high or low performers. For example, students who are successful on number line items recognise that it is a measurement model and explain their solutions with reference to distance, proximity, or reference points (Diezmann & Lowrie, 2006). In contrast, some students who are unsuccessful on number line items interpret the number line as a counting model and overlook the proportional distances between marks on the line. Students' capability with linguistic representation also distinguishes high performers from low performers. For example, whereas novices (typically low performers) interpret keywords

literally and make links to a limited knowledge base, experts (typically high performers) use keywords as cues to an appropriate knowledge schema (Chi, Feltovich, & Glaser, 1981): "Experts perceive more in a problem statement than novices do. They have a great deal of tacit knowledge that can be used to make inferences and derivations from the situation to the problem statement" (p. 149). The differences between high and low performers in their interpretations of various representations extend to reasoning from the representations. An individual's reasoning must take into account the mathematical conventions that are associated with particular representations. Hence, representations are systems of organised data with inbuilt sets of rules of use. For example, reasoning about distance on a map requires attention to the scale of the map. Galotti (1989) proposes that knowledge includes an appreciation of the various rule-based systems in use in mathematics: "Experts, by virtue of their richer knowledge base and extensive experience with problems within a given domain, have a larger and more differentiated set of rules with which to reason" (p. 347). Thus, being mathematically proficient requires an extensive knowledge of various representations including graphics and the associated reasoning that is used with different types of representations.

Research Design and Methods

This study had two purposes. The educational purpose was to gain insights into the differences between high and low performers with a view to identifying specific ways to support the thinking of low performers. The methodological purpose was to establish whether a comparison between high and low performers was a fruitful avenue for gaining insights into students' thinking about graphically-oriented numeracy items, and hence, would be worthwhile implementing with a more extensive data set.

The Participants

Ten participants were identified for this study from 67 Queensland students who participated in a series of annual interviews about graphically-oriented numeracy items. These participants comprised five of the most high performing students (one boy, four girls) over two annual interviews and five of the most low performing students (two boys, three girls) for the same period. These two groups of students are henceforth referred to as "high performers" and "low performers". The students were aged between 10 and 11 years when they commenced in the study. All students attended one of two similar schools in a moderate socio-economic area of a capital city.

The Interviews

The participants were interviewed on a set of 12 items in each of two annual interviews. These tasks were drawn from the 36-item Graphical Languages in Mathematics [GLIM] test which comprises six sets of numeracy items for each of the six graphical languages (see Lowrie and Diezmann, 2005 for a discussion of the test). Examples from this test are presented in the Appendix. The two easiest items from each of the six language groups were presented to the students in the first annual interview and six pairs of items of moderate difficulty were presented in the second annual interview. (The six pairs of the most difficult items will be presented to students in a third annual interview, which has yet to be conducted.)

Data Collection and Analysis

Interview data comprised students' selections on a multiple choice task and the reasons they gave for their responses. The students attempted each pair of tasks independently, and were then asked to explain their solutions. They were probed about any difficulties that they experienced but no scaffolding was provided to avoid the possibility that support on one item might influence understanding on another item. The interviews were video-taped to facilitate analysis. These data were analysed within an inductive theory-building framework with a focus on description and explanation (Krathwohl, 1993). The tactics for generating meaning were noting patterns and themes, imputing plausibility, and building a logical chain of evidence (Miles & Huberman, 1994).

Results and Discussion

Four themes emerged from a comparison of high and low performers' responses to the 24 GLIM items.

Theme 1: The Use of Mathematical Knowledge and Skills

In interpreting items, high performers were more likely to bring existing mathematical knowledge and skills to bear on the task. Low performers were less mathematically proficient, and worked out their solutions in a more laborious fashion that typically involved counting. Though the strategies low performers selected were appropriate, their strategies were more prone to error. Differences in the use of existing knowledge and skills by high and low performers are illustrated by the following example.

On *The Pie Chart* item, students were asked to determine how many hours were spent on homework based on the information presented (see Appendix). The high performers and low performers used different strategies. The five high performers used a *fractional strategy* successfully. In contrast, four low performers used an *estimate and add strategy* with mixed success and the final low performer misunderstood the question.

The *fractional strategy* required an understanding of quarters as shown in Chloe's (a high performer) response.

> Chloe: About a quarter of it (the time) was Mathematics and that was two hours so there was four quarters … two times four is eight.

By identifying the Mathematics portion of the pie chart as a quarter, Chloe reduced the question to a simple multiplication calculation, which she easily accomplished mentally. That is, two hours of Mathematics multiplied by four (for a quarter of the pie chart) is eight hours of homework in total. Thus, as typical of the other high performers, Chloe's success was due to her ability to use existing knowledge and skills to achieve the correct answer. None of the low performers recognised the opportunity to use a simple fractional strategy or mentioned that Mathematics was a represented by a quarter of the pie chart.

The *estimate and add strategy* was used by four of the five low performers. Two were successful and two were unsuccessful. Although this strategy had the potential to be successful, it required students to estimate the number of hours in each segment of the pie chart accurately and to sum these values to determine the total hours shown on the chart.

An inherent pitfall in applying this strategy was to accurately estimate the value of each portion of the chart, as shown in Bree's (a low performer) explanation.

> Bree: Whenever I count, I get to nine … Mathematics is two hours ... each half of Science (is) two, Reading and History … an hour each, and I counted that (Art) as an hour. That's why I (got nine).

Bree used only whole number values when estimating sections of the pie chart. She incorrectly identified Science as 4 hours (actually 3½ hours) and Art as 1 hour (actually ½ an hour). Bree added these incorrect estimates for Science and Art to her correct estimates of two hours for Mathematics and one hour each for Reading and History to reach a total of nine hours instead of eight hours. Similarly, Mike (a low performer) also overestimated the value of some sections of the pie chart. However, two other low performers, Nellie and Helen, correctly estimated values and were successful in their use of the estimate and counting strategy.

Thus, a key difference between these high performers and low performers on *The Pie Chart* was the high performers' selection of an effective but simple strategy incorporating their existing mathematical knowledge of fractions and their multiplication skills. Pie charts are Miscellaneous graphics that encode information through the use of angles (Mackinlay, 1999). In the *fractional strategy,* high performers showed their ability to recognise the value of a key portion of the chart as a quarter of the total time and to use this knowledge efficiently in solution. In contrast, in the *estimate and add strategy,* low performers typically estimated the values of all of the portions, sometimes erroneously, and added these times. This approach was more mentally demanding because half hours needed to be recognised and the addition involved multiple addends including fractions.

Theme 2: The Use of Cues

A further difference between high performers and low performers was their use of cues within the task. High performers were aware of and used multiple cues to solve problems, whereas most low performers were not. The importance of using more than one cue is illustrated by students' responses on the following item.

The Scale item required students to find the mass of an apple by referring to a graphic depicting a traditional set of kitchen scales (see Appendix). On the face of the scales there were three cues in the form of values marked in grams: zero at the top, 100 in the middle, and 200 at the bottom. Between the labelled numbers were unlabelled marks that each represented 10 grams. Use of at least two of the number values was needed to appreciate that the vertical scale was arranged in ascending order.

The five high performers and one of the low performers successfully identified that the scale indicator was at the 170 gram mark. Four of the five high performers noted that the unlabelled mark halfway between 100 and 200 was 150, and proceeded to count in tens to 170. Cody was one of these high performers who used this *midpoint strategy* to successfully find the mass of the apple.

> Cody: What I did then is like, do 150, and then went 160, 170.

One low performer, Mike, used exactly the same process as four high performers and found the halfway mark and counted on. Recall that low performers were selected as students who were consistently low performers over 24 interview items. As in Mike's case, this did not preclude them from being successful on a few items. Elise, the fifth high performer, was also successful but her *count all tens strategy* was less efficient. She counted on in tens from 100 grams to 170 grams making no reference to the halfway point between 100 and 200 grams.

In contrast to the successful students (five high performers, one low performer), the unsuccessful students (four low performers) did not detect the ascending order of the scale. These unsuccessful students used a single number value as a cue and then attempted to identify the mass of the apple. Nellie's response was typical of other unsuccessful students in that she focused on the "200" value, which was close to the mass indicator, and incorrectly assumed that the scale was in descending order.

> Nellie: I put 230 grams because the arrow was near 200 and then I just counted steps up.

Nellie was efficient in counting by tens from the 200 mark to reach 230 grams, but because she did not account for the directionality of the scale, she counted forwards rather than backwards. Thus, the key difference between all high performers and most low performers was the ability to identify the directionality of the scale. Detecting that the scale was ascending required attention to at least two number values, which acted as cues for directionality.

The Scale item used an Axis graphic to encode information by the placement of a mark on some form of number line (Mackinlay, 1999). Although number lines are commonly used in primary texts and tests, they are difficult for some students. On the (US) National Assessment of Educational Progress, many fourth graders' success using a scale was no better than chance accuracy on a multiple choice item (1 out of 4, 25%) (National Center for Education Statistics, 2003). Here, we have identified directionality as problematic but students also have difficulty with Axis graphics because they interpret the number line as a counting model rather than a measurement model (Diezmann & Lowrie, 2006).

Theme 3: The Solution Approach

A further difference between high performers and low performers was their solution approach. When approaching a task, more high performers than low performers were methodical. They typically broke tasks into components and dealt with these components systematically. In contrast, low performers tended to attempt items more holistically. These differing approaches are illustrated in the following example.

In *The Puzzle* item, students were asked to select which of four puzzle pieces would complete the picture of three triangles (see Appendix). The solution piece needed a portion of each triangle to match the partly shown triangles in the picture. Every high performer was successful on this item whereas only two of the five low performers were successful.

Four out of five high performers selected the correct response by using a *component strategy* involving pieces of the puzzle. Rita's response was typical.

> Rita: That bit there can fit into this one, that bit can fit into this one, and that can fit into there.

Rita's response suggests that she examined the sections of the triangles and decided which piece would fit into the larger puzzle. All high performers who chose this strategy were successful but only one of two low performers using the same strategy was successful.

The other approach used by students was a *perceptual strategy*. This strategy was used successfully by one high performer and one of three low performers. Jacob (low performer) used this strategy successfully and like his high performing counterpart made his choice based on what "looked" right.

> Jacob: They all looked in place.

On this item, there was overlap in strategy use by high performers and low performers. Students' success using these strategies revealed two points of interest. First, some

strategies are more likely to lead to success than others. Overall, the success rates were 83.3% for the *component strategy* (5 out of 6 students) and 50% for the *perceptual strategy* (2 out of 4 students). The *component strategy* was selected by 60% of students (40% high performers; 20% low performers) and the *perceptual strategy* by the remaining 40% of students (10% high performers; 30% low performers). Thus, high performers more than low performers selected strategies that were more likely to lead to success. Second, irrespective of which strategy the high performers selected they were more successful than low performers. All high performers who employed the *component strategy* were successful compared with 50% of low performers. Additionally, the one high performer who used the *perceptual strategy* was successful compared to only 33% of low performers. Thus, high and low performers differed in both their selection of a strategy and in its execution.

The Puzzle item used a Retinal list graphic, which encodes information in various ways including shape, size, and orientation (Mackinlay, 1999). The *component strategy* accommodates each of these visual-spatial characteristics when puzzle pieces are tested systematically to check their fit in the large puzzle. In contrast, the *perceptual strategy* relies more on an overall impression of the goodness of fit of a particular piece rather than whether the shape, size, and orientation of the piece is correct for the puzzle.

Theme 4: Knowledge of Everyday Graphics

Everyday graphics add authenticity to numeracy tasks. However, it cannot be assumed that students are familiar with these graphics or can use them effectively as shown in the following example.

In *The Calendar* item, students were asked to find a certain date on the supplied calendar (see Appendix). Unlike the other items discussed in this paper, there was limited difference in the success rates for high (100%) and low performers (80%). However, high and low performers differed in two ways in their use of the calendar.

First, more high performers (80%) than low performers (40%) used an efficient graphically-oriented strategy. Four high performers and two low performers successfully used a *count back by weeks strategy* in which they read off the numbers in the Thursday column, thereby capitalising on the spatial organisation of the calendar. Anna's (high performer) response is typical of these students.

> Anna: One week was 22, and two weeks would have been 15, and three weeks would have been the eighth.

A less efficient strategy – *count back by days strategy* – was used by two low performers. Although this strategy was used successfully, it was inefficient because the students failed to capitalise on the spatial organisation of the calendar when they counted by days instead of by weeks. The final high performer successfully used a *subtraction strategy* to calculate 21 days earlier. No low performers attempted this strategy.

Second, one low performer demonstrated a lack of understanding of the basic structure of a calendar. Helen appropriately chose the *count back by weeks strategy*. She started counting at 29 but the three "weeks" she counted were the Thursday, Friday, and Saturday columns. Helen selected her answer, the third of May, from the top of the Saturday column.

Helen: I worked it out because… it's one week (indicating the Thursday column), I counted the weeks until the 29th May…

Interviewer: So tell me why you think it's the third (of May)?

Helen: I went back from the 29th and I counted three weeks and it ended up there (3 May).

During the solution process, Helen made four errors in calendar use. Her first error was to treat the columns incorrectly as weeks rather than the rows. Her second error was to count forwards rather than backwards starting at the Thursday column and finish at the Saturday column. Her third error was to count the commencement column as the first week before the 29th May. This meant that she only counted two "weeks" before the initial date instead of three "weeks". Recall her concept of the representation of a "week" as a column on the calendar was incorrect. Helen's identification of the commencing location as one week is another example of primary students' lack of understanding of how to interpret the measures on a graphic. Diezmann (2000) reported that many similarly-aged students incorrectly identified the ground height on a diagram of a tree as one metre. Helen's final error was to select the answer from the top of the Saturday column rather than its base. This step violated her own reasoning that the columns were weeks when she moved up the rows. However, this anomaly might have occurred because the only multiple choice answer option in the Saturday column was "3 May", which was at the top of the Saturday column.

The Calendar is a Miscellaneous graphic that uses a variety of graphical techniques to communicate information. The conventions for using a calendar typically include representing the weeks of a month in seven labelled columns – one for each day of the week – and showing blank cells in the first and last weeks of the month before and after the first and last days of the month if necessary. The four high performers and two low performers who used the *count back by weeks strategy* capitalised on the spatial organisation of the calendar in their solution. In contrast, the spatial structure of the calendar was not recognised by the two low performing students who used the *count back by days strategy.* Though they were successful, these students' strategy is inappropriate because it fails to take into account the structure of a calendar. Similar to using the columns on a hundred board to count forward and backward in tens, students should use the columns on a typical calendar to count forward and backward in weeks. Because a calendar is an everyday graphic, both the low performers who used the *count back by days strategy* and Helen, who made multiple errors in calendar use, need to learn how to use a calendar efficiently.

Conclusion and Implications

Educationally, the comparison of these high and low performers' thinking about the use of graphics in mathematics was instructive in three ways. First, low performers need to develop adequate mathematical and graphical knowledge to be successful on numeracy tasks. Hence, teachers should support low performers to identify any related mathematics that could be used in the solution and to check on their interpretation of the graphics. Second, low performers should be encouraged to draw on implicit information embedded in the graphic to generate further information – which high performers seem to do intuitively. Thus, low performers need to capitalise on the multiple cues within a graphic and reason from this visual-spatial information. Visual reasoning differs substantively from sequential reasoning (Barwise & Etchmendy, 1991). Hence, explicit instruction may be

required, such as teaching students how to interpret and reason from a family tree. Third, because some strategies are more likely to lead to success than other strategies, it would be helpful in discussions with students to compare the range of strategies used in terms of the efficiency of strategies and the likely errors using particular strategies. Overall, the comparison of these high and low performers indicated that to become more successful on graphically-oriented numeracy tasks, it is essential that low performers develop and use mathematical and graphical knowledge, generate information from graphics, build repertoires of strategies, and select and use these strategies judiciously.

Methodologically, the comparison of high and low performers' thinking has been fruitful because it provides a means to explore how different approaches to thinking contribute to success. Thus, conceptually high performer-low performer comparison acts as a thought-revealing tool for researchers in a similar way to model-eliciting tasks acting as a thought-revealing tool for teachers and students (see Lesh, Hoover, Hole, Kelly, & Post, 2000 for a discussion of thought-revealing activities).

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Appendix

The Pie Chart (National Centre for Educational Statistics, 2003, Year 4, Q. 3)

The Scale (Queensland School Curriculum Council, 2001, p. 31).

The Puzzle (Educational Testing Centre, 2002, p. 8).

The Calendar (Queensland School Curriculum Council, 2002, p. 9)