Mathematical Reform: What Does the Journey Entail for Teachers?

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This paper presents a case study of the journey a teacher/facilitator took to increase her mathematical content knowledge in order to implement reform-oriented teaching approaches in her mathematics classroom, and subsequently supported other teachers to do the same. In the past decade mathematic education reform has been introduced to teachers in curriculum documents and related in-service professional development programmes promoting an inquiry-based approach to the teaching and learning of mathematics to increase student achievement. Recent research findings suggest that the complex mathematical knowledge embedded in these reforms makes it difficult for many teachers to accommodate the reforms in their entirety. This was indeed the case for the teacher in this study.

Introduction

The case study reported here is part of a wider study that investigated the professional development perspectives of eight teachers and one teacher/facilitator who had participated in the long-term, school-based mathematics programme, the New Zealand Numeracy Development Projects (NDP). The teachers in this wider study found the complex nature of the reforms, for example coming to terms with understanding multiple strategies and moving away from procedural-based algorithms, led to significant shifts in their pedagogical content knowledge. Even so they struggled to accommodate the full extent of these reform teaching approaches without ongoing support (Cheeseman, 2006). Similar struggles were identified by teachers in the nation-wide government evaluations of the NDP (Young-Loveridge, 2004). In this case study the teacher/facilitator talks about her lengthy professional development journey and the types of content knowledge she gained along the way.

Background

Research over the past two decades has identified the teacher and the teaching methodology as the crucial factor for students' ability to learn mathematical concepts with understanding (Skemp, 1986; Wilson & Ball, 1996). Skemp (1986) argued that teachers as poor communicators of mathematics accounted for many students' negative attitude and anxiety towards mathematics and their resulting underachievement in the subject. Studies have confirmed that many teachers lack the content knowledge required to deliver effective teaching programmes in mathematics (e.g. Carpenter, Fennema, Fuson, Heibert, Human, Murray, Oliver, & Wearne, 1999; Hill & Ball, 2004; Shulman, 1986). The results of studies such as these led to mathematical reforms focusing on teacher knowledge and the way that knowledge is delivered so that students became fully engaged in mathematical thinking. The reform-oriented teaching approaches promoted were inquiry-based, "a process in which students reorganise their conceptual activity to resolve situations that they find

problematic" rather than procedural "a process of internalising carefully packaged knowledge" (Cobb, Wood, Yackel, Nicholls, Wheatley, Trigatti, & Perlwitz, 1991, p. 5).

These reforms required most teachers to make a major shift in pedagogy from teaching approaches that focused on a procedural approach (standard algorithms and rules) to a conceptual approach focusing on student thinking and reasoning (Stein & Strutchens, 2000; Anderson & Bobis, 2005). To make this pedagogical shift necessitated teachers extending their knowledge of mathematics to include what Shulman (1986) described as pedagogical content knowledge (PCK). Shulman (1986) sees PCK as going beyond knowledge of subject matter per se to the subject matter of teaching that includes knowledge of: how to teach mathematics, mathematics curriculum and resources, and importantly how students learn mathematics. In regard to the latter, Hill and Ball (2004) elaborated that teachers' knowledge of how students learn results from the "interplay between teachers' knowledge of students, their learning, and strategies for improving that learning". This includes the teachers' ability to understand and assess the problem solving strategies used by their students and when a new strategy is used to "determine whether such strategies would be generalizable to other problems" (p. 332). Embedded within PCK is the development of teachers' awareness of sociocultural norms whereby students feel confident to share their mathematical thinking in a non-threatening learning environment (Fraivillig, Murphy, & Fuson, 1999; Yackel & Cobb, 1996).

In New Zealand the Ministry of Education undertook a series of initiatives to provide professional development programmes to assist teachers to accommodate the reform-orientated approaches in mathematics. A small initiative began in the late 1990s with the introduction of a tertiary course (Helping Children Succeed in Maths) at the Auckland College of Education. Those teachers attending the course were introduced to the theory of relational understanding based on Skemp's (1986) research, the notion of students using their own strategies to solve number problems, and the developmental stages of children's mathematical thinking. Nation wide long-term, school-based initiatives followed, commencing with the New South Wales programme Count Me In Too [CMIT] (Wright, 1998), which was later replaced by a New Zealand numeracy project focusing on the junior school, the Early Numeracy Programme (ENP). The professional development programme was then extended to teachers of older students (8 – 10 year olds) with the introduction of the Advanced Numeracy Project (ANP). Both ENP and ANP were designed to up-skill teachers in their ability to teach numeracy by providing The Number Framework, a breakdown of the development stages of students' mathematical knowledge and thinking, and a strategy-teaching model. The Number Framework was the NDP's key tool in developing teachers' knowledge of number concepts and the processes by which these number concepts are best developed. It was intended that teachers' awareness of student mathematical mental strategies be increased and their pedagogy changed (guided by the teaching model) in order to improve student achievement (Thomas & Ward, 2002).

Teacher change as a result of accommodating the reform-orientated approaches was extensive and difficult to achieve for many teachers (Cheeseman, 2006; Young-Loveridge, 2004). This parallels the reports in recent international studies (Anderson & Bobis, 2005; Cady, Meier, & Lubinski, 2006; Stigler & Heibert, 1997). Stigler and Heibert (1997) observed the challenges American teachers encountered while attempting to make changes to their deep-seated beliefs when faced with reforms and as a result only changed some practices. Anderson and Bobis (2005) investigated Australian teacher responses to the reform-oriented approaches recommended by the NSW curriculum and found that overall

teachers' agreed with the reforms but many had difficulty fully embracing them. Similarly, a longitudinal study undertaken by Cady, Meier, and Lubinski (2006) observed the development of pre-service teachers to experienced teachers and found variance in the teachers' abilities to implement reform practices in their classrooms as novice teachers. This paper examines the complexities of one teacher's journey in her attempt to incorporate and consolidate the mathematical reforms promoted by the numeracy initiatives in New Zealand into her teaching practice.

Methodology

The study used an interpretive approach to investigate the perceptions of a teacher/facilitator who participated in Ministry of Education numeracy professional development initiatives over an extended period of four years. The mode of enquiry was in the form of a 45-minute to 60-minute face-to-face interview and shorter follow-up telephone interviews. The semi-structured, open-ended interview questions were formulated as a guide for the researcher to follow. The main intention of the researcher was to listen actively so that the interview was shaped by the participant's voice (Denzin & Lincoln, 2003).

This case study was part of a wider interpretive study that explored the experiences of eight teachers who were involved in the NDP professional development programme.

Mathematical Journey

In her first year of professional development Jayne was teaching in the junior area of the school (5 – 7 year olds). She participated in the tertiary mathematics paper, "Helping Children Succeed in Maths", which introduced her to the idea of student strategies for counting, and theory about conceptual or relational thinking.

> The first year I took the paper "Helping Children Succeed in Maths" at ACE [Auckland College of Education]. I was the only teacher attending from my school which was a shame because after each session I would come to school and talk enthusiastically about all these new mathematical ideas. I suspect that most people taught like I've always done and have those same values, and the ideas I was now advocating were quite radical so my colleagues were not keen to listen. The idea that the children could think of their own strategies to solve addition problems seemed alien and their reaction was "but that means there would be more than one answer!"

Jayne took ownership of the new pedagogical ideas and practised the implementation of them with her class. The following year her junior syndicate participated in CMIT. CMIT was a long-term professional development programme (a duration of three school terms) that further consolidated the new pedagogical content knowledge she had gained from the tertiary course. An example of this knowledge consolidation was her increasing familiarity with the number framework outlining the stages of student mathematical thinking.

> Before starting the PD, I remember asking a Year three child to add 8 + 3 and she went 12345678, 123 and then counted up to 11. I thought what is she doing? Why is she doing that? Now I know that [counting all - one to one] is a developmental progression … and now you have to go from this step to teach them [children] to go further.

Her enthusiasm and success in accommodating to and implementing the PCK led to her being asked to become a part-time facilitator for ENP, which allowed her to continue to teach mathematics in her year 4 class as well as introduce other teachers of junior classes in other schools to the reform-oriented teaching practices.

> When I became a facilitator in terms of maths [knowledge] it deepened what I knew rather than changed it so much because by then I'd already changed the way I taught and the way I thought about maths. As a facilitator I became aware of the importance of the numeracy framework in focussing teachers' attention on stages of children's thinking so that you can see which stage each child is at and where to push them to next. … I used to look at my Year four children [8 yr olds] and they would all be using their fingers to count on for a problem like 8 + 5, but now we are teaching them to count smarter by rearranging the groups of numbers [making tens or using doubles strategies]. I think that knowing there is a next step was a big change in my thinking about mathematics and how children learn to add and subtract.

The following year Jayne was asked to become a full-time facilitator, this time working with teachers in both the ENP and ANP professional development courses. She talked about the increase in content knowledge in relation to strategies used by junior school students to those used by students in the senior school (8-9 year olds). She initially struggled to understand some of the more complex strategies, in particular those that involved multiplicative thinking.

> This year the development has been huge because [as a facilitator of ANP] it's been multiplication and division … it is understanding the actual strategies e.g. for a problem like 5 x 18 – can take a long time to actually understand that you are just halving that group and rearranging it [10 x 9]. Initially you just have to see it and do it [using materials]. That knowledge is then extended, to fractions and decimals, which I knew very little about and which is so abysmal in NZ anyway. Both my knowledge and strategies have increased and that would be common with most of the facilitators and a lot of the professional development had been based around that for facilitators.

Jayne's struggle to understand the more complex mathematical strategies parallels the findings of the research that reported on how teachers had coped with the increasingly sophisticated part-whole strategies introduced as part of the ANP professional development. She, as for most teachers in the research project, felt challenged but like the other teachers increased her own mathematical knowledge (Irwin, 2003; Young-Loveridge, 2004). It is crucial that teachers understand the strategies their students are using and provide guidance to extend their students' thinking. Jayne discussed the aspect of teaching strategy as another significant aspect of her increasing PCK and accommodation of the range of strategies to be taught.

> At the ANP level I had to learn firstly, what is the range of strategies children might use and secondly, how do we teach them. I can empathise with the teachers' feeling of "information overload" when learning about strategy because I often felt this too when attending the ANP facilitator professional development days. Sometimes at those PD days I would think if they say another thing I'm going to burst because I don't want to hear any more. It is a lot to take in and it's not just taking it in, it is processing it and then telling and showing that to teachers.

The nature of the NDP required a major shift in pedagogy from teaching approaches that focused on a transmission approach to a teacher facilitation approach focusing on student thinking and reasoning (Stein & Strutchens, 2001). Jayne became more conscious of the importance of listening attentively to students explain their thinking at the ANP level where the strategies were more complex. She was aware both as a teacher and a facilitator of the necessity to elicit, support, and extend students mathematical thinking (Fraivillig et al., 1999) and to model this for teachers.

> Listening to children's thinking – was a huge shift. No longer just wanting answers – asking how did you get that answer or a range of answers. Yes, I accept all the children's solutions without value judging but some ways of getting the answers are more efficient than other ways so that's the way we want to guide then depending on what the numbers are in the problem. The most efficient strategies will vary depending on the nature of the problem. For example, using an algorithm to solve a

> problem with large four to five digit numbers is fine but for a problem like 1003 – 998 you can solve it in your head. However, children do now always see that there is a quicker way to solve it and need guidance to move away from the standard algorithm to see the easier method.

Jayne found that the shift away from teaching algorithms was highly significant in her growth of PCK. This significant aspect was highlighted in the literature where it was noted that the NDP strategy-teaching model required most teachers to change their ways of thinking and learning about mathematics. This entailed a shift away from teaching rules, procedures, and algorithms to guiding students to use multiple strategies to solve a problem (Young-Loveridge, 2004). Very early on in her professional journey Jayne could see the tension caused by teaching algorithms whereby the children were just following a process and not seeing the wholeness of the numbers or looking at them contextually.

> As teachers we would get to that step where the children could count on and then we would teach the algorithm which is not so much to do with mathematical thinking but is more to do with following rules. For example, with a problem like 605 – 308, the children would not see 605 as a whole number but would concentrate on the 5 and 8 ones, each little bit of it and not have a sense of the wholeness – number sense!

Jayne had changed her practice to teach multiple strategies and delay the teaching of algorithms to students until they had a deep conceptual knowledge about operating on number. She then had to convince other teachers to move away from the standard procedures.

> As a facilitator of ANP my biggest challenge was convincing teachers to move away from teaching algorithms [standard procedures to solve all four operations]. And trying to get through that students won't be penalised because they will know so much more about number and they will have a much richer base [strategies] in their heads. And they won't understand an algorithm anyway if you teach it too early. We've been teaching procedures for years and that's fine for the basic facts, counting and number identification, and reading fractions and decimals, but operating on them involves other mathematical thinking.

The journey Jayne took enabled her to increase her mathematical content knowledge and implement reform oriented teaching approaches in her mathematics teaching practice.

Conclusion

Her dual role beginning as a self-motivated teacher and becoming a facilitator gave Jayne multiple opportunities to take advantage of the professional development associated with NDP. Her personal journey took four years and involved the challenges of working with colleagues as well as with students. It seemed for her 4 years laid a solid foundation that may be an optimum result for the numeracy teaching development programmes to be consolidated effectively. The case study outlining Jayne's accommodation of the reform-oriented teaching approaches demonstrates the difficulties faced by teachers embarking on this mathematical self-improvement journey. Recent research findings state "that teachers who are more successful than others at developing effective reform-based practices appear to be self-sustaining, generative learners (Anthony & Walshaw, 2007). This would indeed appear to be the case for Jayne.

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