Young Children's Accounts of their Mathematical Thinking

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As part of a larger study exploring teacher behaviours that challenge children to probe their mathematical understandings, children were interviewed about their mathematical thinking and asked to reflect on their learning. Fifty-three interviews were conducted in four schools with 5- to 7-year-old children. The subjects were involved in close conversation with their teachers during the mathematics lesson. Video-stimulated recall was used with a conversational interview to prompt children's recollections and reflections. Findings indicate that young children in the first years of schooling are able to recall events in their mathematics lessons to reconstruct their thinking and reflect on their mathematical learning.

Background

The theory of social constructivism underpins this research. Cobb, Wood, Yackel, and McNeal (1992) and Sfard, Nescher, Streefland, Cobb, and Mason (1998) argued that the construction of knowledge occurs within a social and cultural context where discourse is a vital component in establishing an effective learning context. The focus of this research is the meaning constructed between the teachers and children in classrooms.

There has been a long history of interviewing young children to describe their mathematical thinking (e.g., Donaldson, 1978; Gelman & Gallistel, 1978; Hughes, 1986; Irwin,1996). These interviews often involved children performing mathematical tasks to demonstrate their thinking or development. Task-based interviews have also been used to assess and plot the growth of the mathematical thinking of children over time (e.g., Clarke & Cheeseman, 2000). However there appears to be little research that reports young children's reflections on their thinking in post-lesson interviews.

Franke and Carey (1997) conducted interviews to research first-grade children's views about what it means to do mathematics in problem solving classrooms. They found that young children were in fact able to reflect on classroom events.

McDonough (2002) reported procedures that prompted 8- to 9-year-old children to articulate their beliefs about mathematics. Children found it a difficult to talk abstractly about learning, however, they "held beliefs about mathematics, learning and helping factors and could articulate beliefs when prompted" (p. 270). Although acknowledging the scarcity of research in the area, McDonough expressed little surprise that children even younger than those in her study could describe their mathematical thinking and learning after lesson of the day (McDonough, 2007, private communication).

Method

To capture some of the complexities of classrooms settings and to collect rich data, the approach termed *complementary accounts methodology* was used for this study (Clarke, 2001). Although the methodology used for the large study differed from that of Clarke, similar fundamental techniques were used. These include videotaping the whole mathematics lesson, audio taping participants' reconstructions of classroom events, and an analysis of the multiple data sets.

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In total, 53 children were interviewed on the day their mathematics lesson was conducted. The children were aged 5 to 7 years from four classes, each in a different school. The four schools were different from each other in geographic, socio-economic and cultural background and the only common characteristic was that each of the teachers was female. The selection of students was based on classroom observation notes of the researcher and where possible, the recommendation of the teacher. In some cases it was not possible to have a conversation with the teacher before children's interviews began.

The interviews were audio taped for transcription and analysis. A video of the lesson was used as a stimulus to recall sections of the lesson directly involving each child. Children were asked to recount events where they were in conversation with the teacher, to say what they were thinking at the time, and to reflect on what they had learned in the mathematics lesson. The interview was conversational in style. Although there was an interview script, it was adapted in order to elicit responses from each child. The scripted questions were:

1. I am interested in the times when teachers talk to kids in maths lessons—you know when they are really just talking to one child. I noticed that your teacher had a talk with you / stopped to work with you / asked you about your work in that maths lesson. Can you remember that? Can you tell me what happened?
2. I think that we got that on video. Would you like to see it?
3. What were you thinking about? (Maybe just watch it at first.)
4. Can you say what was happening?
5. What did you learn in maths today? Was there anything else?

These questions are modelled on those used by Clarke (2001, pp. 13-32). The original research was with secondary students, and so the language used in the questions has been simplified for young children. In fact it was not known whether children as young as 5 years old could give an account of classroom events where they were challenged to think mathematically. Hence the research question: to what extent can young children give a subsequent account of a classroom mathematical event from their perspective?

Video-Stimulus Recall

There appears to be scant literature describing the use of stimulated recall using video with young children. There are reports of Year 8 children, using video-stimulated interviews to reconstruct the learner's perspective (e.g., Williams, 2003) and reports of teachers video-stimulated recall of the events in their classrooms (e.g., Ainley & Luntley, 2005) but there seems to be no use of this methodology in mathematics education with young children.

Because little was known about how young children would respond to video-stimulated interviews, some piloting occurred. In the pilot stage, young children responded to the video of the mathematics lesson in a very different way from that of their teachers. When teachers were shown excerpts of the lesson they were able to jump into the moment and to talk about what was going on and even reconstruct their thinking at the time. Young children though, would watch the video as a passive observer and if asked at the end of the event to talk about what was happening there, they would give a look as if to say "What do you mean? You just watched what was happening!" They seemed to feel that the video required no explanation or interpretation. After a while it became clear that the best way to prompt recall was to play a little of the beginning of an incident of interest to set the scene for the child then to pause the video and to ask, "Do you remember that bit, what was happening there?"

If a child had no recollection of the event, the entire video episode involving them in conversation with the teacher was played and used as a stimulus to help them describe their thinking or reflect on their learning. In general, the video was used as a starting point and it was paused as soon as the child had remembered the event.

Children of 5 to 7 years old are often asked to talk about a piece of work in class, especially when reporting back to the class at the end of the lesson. So, during piloting each child was interviewed with their work sample as well as the video. However having the work in their hands tended to focus their reflection on the output of the lesson and the details of what was on the paper rather than what they were thinking so the technique of having work samples available to the child was discontinued. If a child asked for the work sample to help them to explain it was provided to them.

Data Coding and Analysis

Interviews were digitally recorded. Seventeen interviews were transcribed in full. An analysis of the transcripts resulted in the data being considered in terms of the children's recall of an incident or task, description of events, explanation of their thinking, and description of their learning. Categories of response emerged as nodes in the data (see Table 1). Descriptors of response were listed in increasing levels of sophistication, with 0 being the least and 3, 4, or 5 as the most sophisticated responses. The category "missing" was used where the question was not asked. This happened because a feature of semi-structured interviews is that the interviewer tries to follow the child's previous response.

The remaining 36 interviews were coded directly from the audio files. In general, the highest level of the particular category was coded when evidenced anywhere in the interview. Codes were then entered into a statistical analysis program (SPSS) to produce descriptive statistics.

Reliability of Coding

To improve internal reliability, interviews were re-coded. This was done to examine whether there was consistency between researchers and whether similar conclusions could be reached about children's behaviour (Goldin, 2000, p. 531). An independent person coded a 20% sample of the audio data. This person was skilled at listening to young children describe their ideas as she came from a primary teaching background and mathematics education research. All points of difference were discussed and an agreed understanding of the data was reached. The following matters were raised:

- transcripts would have helped the coder;
- the broad categories that emerged from the data seemed appropriate;
- some descriptors required clarification to better define distinctions in levels of response;
- examples would help the coder/listener/reader;
- the distinction between evidence of description of thinking and correct thinking was reiterated; and
- evidence of a higher level of code was taken as the default.

Based on the combined critical analysis, further interviews were transcribed in full (17 in total) and category descriptions were refined. The entire data set was coded again applying the new protocols without any reference to the previous coding. The results of this second coding form the data reported here.

Table 1
Categories of Response to Aspects of the Interview

Aspects of interview	Categories of response
Recall of the incident/task	no recall
	could talk about the event only after of the entire video excerpt was replayed
	recall with the video paused just before the event of interest or with the video playing in the background with no audible sound
	recall spontaneously with little or no assistance of the video extract
Description of events	no description of interaction with teacher
	describe actions
	describe outcomes only, e.g., a work sample, "I stuck the cats onto the paper."
	describe the event from their perspective
	describe their reasoning and/or justify their thinking
Explanation of their thinking	no explanation
	"account for" the videotape e.g., make up a "story" of the event
	explicit description of thinking
	explain/reconstruct thinking, reasoning, justifying, evaluating thinking
Description of their learning	unable to specify learning
	learned nothing
	learned a behaviour not mathematics e.g., "to share"
	remembered factual information e.g., number facts
	learned how to do something e.g., "to count by 6s"
	described learning at a conceptual level, expressed as a mathematical principle or an insight, e.g., "I can count by 1s, 2s, 3s, 4s, 5s, 6s, 7s, 10s, and 100s and 1000s …once I can count by ten I can count by all the rest. Like 10, 20, 30, 40, 50, and it always has a zero on the end."

Results and Discussion

Recall of Events

Using videotape of events involving each child in the mathematics lesson of the day to stimulate the recall and an account of the episode from the view of the child was largely successful. This is evident from Table 2, which summarises the categories of responses of children's recall of events, where only 2% of children were unable to recall the events of the lesson. Some children needed to watch the entire replay of the videotape where they were in conversation with the teacher in order to talk about it (23%). Many children, having watched the video of the lesson leading up to the event, could recount their version of what had unfolded after the videotape was paused (30%). In addition almost half of those interviewed could recall a conversation with the teacher before the video was replayed.

Table 2
Categories of Response of Children's Recall of an Event

Category of response	Frequency as a percent (*n* = 53)
No recall	2 (1)
Recall with video replay of the event	23 (12)
Recall with video paused or with no audible sound	30 (16)
Recall spontaneously	45 (24)

Description of Event

An analysis of the children's descriptions of events revealed an interesting three-way split of responses (see Table 3). Some children described only what they did (23%). The following example illustrates this category of response. James could be seen on the video interlocking blocks but saying nothing:

Interviewer: So what was happening here?

James: My brain was counting and I wasn't. [James, J2.3:25]

Other children offered a description from their point of view (36%). For example, Ali explained his counting of five groups of five teddies saying, "It goes 10, 20, 30, 40, 50. You have to count the ears" [Ali, G1, 7:30]. It is hardly surprising that 36% of children who could remember the event described it from their point of view. In fact what was interesting was that such a large proportion described the event with some reconstruction of their reasoning at the time (28%). This was perhaps the most interesting group of responses. For example, Jessica was explaining how to weigh a dog, Joey, who would not stand on bathroom scales:

Interviewer: Can you tell me about your good idea for maths today please?

Jessica: I thought of holding Joey on the scales. I would know how much Joey weighed. So I hopped on the scales with him and I holded him. And then we took away 19 [from 28] because I was 19 and he was 9 and so that was 9 kilograms and that's what he weighed [Jessica, J3, 0:35].

Table 3
Children's Descriptions of Events

Category of response	Frequency as a percent (n = 53)
No description of interaction with teacher	4 (2)
Describe actions	23 (12)
Describe outcomes only, e.g., a work sample	8 (4)
Describe the event from their perspective	36 (19)
Describe their reasoning and/or justify their thinking	28 (15)
Missing	2 (1)

Explaining Thinking

Table 4 shows the number of children who could explicitly describe their mathematical thinking was high (85%).

Expecting children to be able to communicate their thinking has been an element of mathematics curriculum definition for years (Australian Education Council, 1991; Board of Studies, 2000). Certainly based on classroom observational data from the classrooms of the children interviewed here it is a clear expectation of their teachers that they explain their reasoning. The teachers frequently ask; "How did you work that out?", "What do you think?", "Why are you doing that?", and "How do you know?"

It should be said that these children had been learning mathematics in the classrooms of "highly effective" teachers of mathematics (McDonough & Clarke, 2003) for 8 months. Perhaps this would account for their readiness to describe their mathematical thinking. Whether children in other classrooms can explain their thinking with this frequency is a question that might be explored by further research.

An example of the type of response that shows a child reconstructing and evaluating his thinking is when Tom offered a thinking strategy for his classmates who could not count by four. His idea was to use a count by two.

> Interviewer: Now Mrs A says that's a really complicated way to work it out I can't really hear what you were saying. She was looking at a page that had 8 legs and 4 things on each leg. How were you trying to work that one out?
>
> Tom: Oh a different way. You know, when there's 8 legs and I was thinking if people didn't know how to count by 4, I was splitting 4 in half to make two on each side. Then I did 2 X 8 equals 16 then I have to count by 2s up to 32 what it equals. I have to count by 2s 16 times [Tom, G1, 1:00].

A few children could not explain their thinking and another few gave an explanation of their thinking as if telling a story. In examining the knowledge that experienced mathematics teachers access to operate effectively, Ainley and Luntley (2005, p. 78) made a distinction that may be pertinent here. Teachers were shown episodes of videotapes of their classrooms and in these interviews some teachers gave an "account for" rather than an "account of" their actions. The children who made up a story to suit the occasion may be doing the same thing or perhaps there is a different mechanism at work. No definitive statements could be made based on the evidence collected here. All that can be said is that 3 (6%) children made up a fiction to match the video.

Table 4
Children's Explanation of Their Thinking

Category of response	Frequency as a percent (*n* = 53)
None	6 (3)
"Account for" or gave an invented story	6 (3)
Explicit description of thinking	43 (23)
Reconstructs thinking, justifies, reasons, evaluates	42 (22)
Missing	4 (2)

Specify Learning

Only 15% of children did not know what they learned in the mathematics lesson (see Table 5). The category of "nothing" proved unreliable because it became clear that young children translated "What did you learn today?" into "What new things did you learn today?" and these two questions are quite different. Therefore this category is not discussed. Some children talked about behavioural learning, for example, "to share." Or they referred to non-mathematical things, for example the learning context, "talking about tools and building" [Michael, Jk2]. Totalling the first 3 categories of Table 5 shows that 30% of the children did not specify mathematical learning.

The three categories of most interest were those that made distinctions between learning factual information (15%), learning how to do something (23%), and learning at a conceptual level (21%).

About one third of the children who remembered facts talked in terms of numbers. For example, Annie who had been talking about measuring with a piece of string when asked what she learned said, "I learned that 9 + 11= 20." Although it is not possible to be certain from these data, it raises a question as to what these young children think constitutes mathematics learning. Is learning mathematics equated to remembering numbers? Lindenskov (1993) found that students' learning can be influenced by their everyday knowledge of what mathematics is. She was also struck by "the students" perceptions of details, even small ones, both in the teaching and in her/his own learning" (1993, p. 153). Certainly the children interviewed for this research described their learning in detail. For example, Tom talked about his learning in the following exchange.

Tom: I think I might have leant some new times tables.

Interviewer: Oh so you sort of had to figure some out?

Tom: Yes.

Interviewer: In which times table?

Tom: I think some were in the, I think some were like 9 X 6. I didn't know that but then I knew it because I just counted by 6 nine times [G1: 6:36].

Some children learned how to do something, for example Jordan, who "learned how to count by nines." Another substantial proportion of the children (21%) reflected on their learning at a conceptual level. For example, Tahani reflected on a lesson where the teacher intended to introduce multiplicative thinking, saying she learned "about groups, to make groups and to count them altogether and I learned to count by 6s." Another example was

Lucas who said he learned "how long things were and how short they were … by counting the blocks."

Table 5
Children's Learning

Category of response	Frequency as a percent (*n* = 53)
Unable to specify learning	15 (8)
Nothing "new"	9 (5)
Learned behaviour/ not mathematics	6 (3)
Remembered factual information	15 (8)
Learned how to do something	23 (12)
Specified a conceptual level of understanding	21 (11)
Missing	11 (6)

Conclusion and Implications

It can be concluded that young children could give an account of mathematical events from their perspective. Children could recall at least part of their conversations with the teacher during the day's mathematics lesson. These interactions appear to have some lasting effects. If, as we assert, interactions that challenge children to think about their mathematical understandings are a critical factor in their learning, then knowing that many young children spontaneously remember these conversations and can reconstruct their thinking is an important finding.

The sophistication of their descriptions of events in the classroom was fairly evenly split between recounts of actions, descriptions of the event from the child's perspective, and a description that involved some recount of their reasoning. It was impressive to find that such a large proportion of five- to seven-year-old children (42%) could reconstruct their thinking and justify it.

It is assumed that the experiences offered to children in mathematics classrooms contribute to their learning. These data indicate that 59% of children could talk about their learning as a result of the lesson – some at a factual level, some at a procedural level, and some at a conceptual level. Further research might investigate factors that influence different levels of understanding reported by young children.

It is also important for researchers to know that video-stimulated recall can be successfully used with 5- to 7-year-old children.

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