Communicating Students' Understanding of Undergraduate Mathematics using Concept Maps

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The concept map data from a study of Samoan university students constructing *topic concept map*s and *vee diagrams of problems* throughout a semester is presented. Students found that, initially, concept mapping their topic was difficult. However with independent research and multiple critiques, their understanding of the conceptual structure of the topics deepened, becoming integrated and differentiated as evident from the concepts selected, valid propositions and structural complexity of the maps. Students also improved their skills in negotiating meaning, challenging and counter-challenging each others' explanations. Findings imply concept maps can facilitate the effective communication of students' understanding within a social setting.

Introduction

Working and communicating mathematically is being encouraged as part of everyday mathematical learning in schools. Research shows students' perceptions of mathematics learning reflect the way they have been taught mathematics (Thompson, 1984; Knuth & Peressini, 2001; Schell, 2001). In addition, pedagogical decisions teachers make about teaching and assessment are influenced by their mathematical beliefs (Ernest, 1999; Pfannkuch, 2001). Typically, an authoritative perspective views mathematics as a body of knowledge with classroom practices, simply a transmission of information. In contrast, cognitive and social perspectives view mathematics learning and understanding "as the result of interacting and synthesizing one's thoughts with those of others" (Schell, 2001, p. 2), suggesting mathematics knowledge is a social construction that is validated over time, by a community of mathematicians. Hence making sense is both an individual and consensual social process (Ball, 1993). Classroom practices should equip students with the appropriate language and skills to enable the construction of the mathematics that is taught, and critical analysis and justification of the constructions in terms of the structure of mathematics (Richards, 1991). Lesh (2000) argues that, "mathematics is not simply about doing what you are told" (p. 193) while Balacheff (1990, p. 2) posited that "students need to learn mathematics as social knowledge; they are not free to choose the meanings ... these meanings must be coherent with those socially recognized".

Existing problems with mathematics learning in Samoa are perceived as related to students' perceptions of mathematics, ability to communicate mathematically, and critical problem solving. Firstly, the narrow view most undergraduate students have, reflects their school mathematics experiences, found to be mostly rote learning, a problem consistently raised by national examiners. Even the top 10% of Year 13 (equivalent to Year 12 in Australia) students consistently struggle with applications of basic principles to solve inequations/equations and/or graph functions (Afamasaga-Fuata'i, 2001, 2002, 2005a,). Secondly, students justify methods in terms of sequential steps instead of the conceptual structure of mathematics. Thirdly, students may be proficient in solving familiar problems, however, the lack of critical analysis and application becomes evident when they are given novel problems. Such approaches are symptomatic of authoritative classroom practices in which students typically do not question, challenge or influence the teaching of mathematics (Knuth & Peressini, 2001). The examination-driven teaching of secondary mathematics in Samoa naturally inculcates a narrow view of mathematics (Afamasaga-Fuata'i, 2005a; 2002), As a result, problem solving skills students acquire over the many years of secondary schooling may not necessarily be situated "within a wider understanding of overall concepts" and would probably not be "long-lasting" (Barton, 2001). Against this general background, this paper reports a study, conducted over a semester, to investigate some second year university students' developing understanding of selected topics, as illustrated by individually constructed hierarchical concept maps (cmaps). Before the data are presented, the underlying theoretical framework and methodology are discussed.

Theoretical Framework and Relevant Studies

The difference, between an authoritative perspective of mathematics learning and Ausubel's cognitive theory of meaningful learning, socio-linguistic and social constructivistic perspectives, is the extent to which classroom discourse and social interactions are supported (Wood, 1999). That is, students learn mathematics in meaningful ways, by developing their understanding through the construction of their own patterns of meanings and through participation in social interactions and critiques (Novak & Cañas, 2006; Novak, 2002). In contrast, rote learning tends to accumulate isolated propositions rather than developing integrated, interconnected hierarchical frameworks of concepts (Novak & Cañas, 2006; Ausubel, 2000; Novak, 2002). Guiding the study were Ausubel's principles of assimilation and integration of new and old knowledge into existing knowledge structures through a degree of synthesis (i.e., *integrative reconciliation*) or reorganization of existing knowledge under more inclusive and broadly explanatory principles (i.e., *progressive differentiation*). Both the meaningful learning and social constructivist approaches support the metacognitive development of students' understanding and the active construction of mathematical thought whilst publicly presenting, for example, cmaps and vee diagrams (schematic diagrams), within a social setting. A cmap is a graph consisting of *nodes,* which correspond to important concepts in a domain and arranged hierarchically; *connecting lines* indicate a relationship between the connected concepts (nodes); and *linking word*s describe the interconnections (explanation). A *proposition* is the statement formed by reading the triad(s) "node $\xrightarrow{\text{linking words}}$ node" (Novak & Cañas, 2006). For example, the triad "Functions $\xrightarrow{\text{may be described using}}$ equations" forms the proposition, "*Functions* may be described using *equations*".

Numerous studies investigated the use of cmaps and/or vee diagrams (cmaps/vdiagrams) as assessment tools of students' conceptual understanding over time in the sciences (Novak & Canas, 2006; Brown, 2000; Mintzes, Wandersee, & Novak, 2000), and mathematics (Afamasaga-Fuata'i, 2005b; Schmittau, 2004; Swarthout, 2001); as communication tools (Freeman & Jessup, 2004); and as analytical tools to unpack teachers'/participants' perceptions (Pittman, 2002; Wilcox & Lanier, 1999). Research in secondary (Afamasaga-Fuata'i, 2002) and university mathematics (Afamsaga-Fuata'i, 2004) found students' conceptual understanding of mapped topics was further enhanced after a semester of concept mapping. Research with preservice teachers showed cmaps were useful pedagogical planning tools (Afamasaga-Fuata'i, 2006; Brahier, 2005). Workshops with science and mathematics specialists and teachers found maps/diagrams have potential as teaching, learning, and assessment tools (Afamasaga-Fuata'i, 2002;

1999). The research question for this paper is: "How can hierarchical concept maps illustrate improvements in students' understanding of mathematics topics?"

Methodology

The study required students to undertake conceptual analyses of topics (identifying relevant major concepts, principles, formal definitions, rules, theorems, and formulas) and illustrate the theoretical results on cmaps. The methodology was an exploratory teaching experiment to investigate students' developing understanding of particular topics (Steffe & D'Ambrosio, 1996), involving meeting twice a week for 50 minutes each time over 14 weeks with a cohort of students enrolled in a research mathematics course. Cmaps/vdiagrams were introduced as means of learning mathematics more meaningfully and solving problems more effectively. The content was from students' recent mathematics courses, namely, *limits and continuity*, *indeterminate forms*, *numerical methods*, *differentiation*, *integration*, *motion, multiple integrals*, *infinite series*, *normal distributions,* and *complex analysis*. The epistemological principles, namely, building upon students' prior knowledge, negotiation of meanings, consensus, and provision of time-in-class for student reflections, guided classroom practices. Hence, the study included a familiarization phase, which introduced the new socio-cultural classroom practices (socio-mathematical norms) of students presenting and justifying their work publicly, addressing critical comments, and then later on critiquing peers' presented work. Time was set aside between critiques to revise maps/diagrams. The cyclic process was: presenting (to peers or researcher) → critiquing → revising → presenting underpinned the study. Of the 13 students, 3 chose topics outside of mathematics (computer programming, cell biology, and organic chemistry). This paper reports the data from the mathematics cmaps only.

Concept Map Analysis

Although the literature documents a variety of assessment/scoring techniques (Novak & Gowin, 1984; Ruiz-Primo, 2004; Liyanage & Thomas, 2002), a modified version of the Novak scheme was adopted, which used counts of a criterion. The three criteria were the *structural* (complexity of the hierarchical structure of concepts), *contents* (nature of the contents or entries in the concept nodes), and *propositions criteria* (valid propositions).

The structural criteria were in terms of integrative *cross-links* between concept hierarchies, progressive differentiation evidenced by nodes with *multiple branching* (more than one outgoing link) (which create *main branches* and *sub-branches*), and *average number of hierarchical levels per sub-branch*. The contents criteria indicate students' perceptions of mathematical *concepts* in terms of suitable labels and illustrative *examples*. *Inappropriate* entries include those describing procedural steps (more appropriate on vee diagrams), redundant entries (indicating the need for a re-organization of concepts), and linking words as concept labels (linking-word-type). The definitional-phrase invalid node, although conceptual was too lengthy, its presence signals the need for further analysis to identify "*concepts*" as distinct from "linking words". The propositions criteria define valid *propositions* as those formed by valid triads (i.e., "valid node $\xrightarrow{\text{valid linking words}}$ valid node").

Concept Map Data

The data collected consisted of students' progressive cmaps (4 versions) and progressive vee diagrams of 3 problems (at least 2 versions per problem), and final reports. Only the data from cmaps are presented here. The three criteria were used to assess students' first and final cmaps, to identify any changes. Individual results are presented first before a discussion of general themes. The cmap data for Students 1 to 5 are in Table 1 and those for Students 6 to 10 are in Table 2.

Student 1: Pene – Indeterminate Forms. Despite encountering "indeterminate forms" in first year mathematics, Pene struggled to begin a cmap. As a result of critiques, revisions and independent research, Pene's final cmap became structurally more integrated (increased cross-links from 3 to 10), more differentiated (increased multiple-branching nodes from 8 to 10 and increased average hierarchical levels per sub-branch from 6 to 8), and more compact (decreased sub-branches from 17 to 14) with main branches remaining unchanged (Table 1). However, the percentage of valid nodes (from 77% to 67%) and valid propositions (from 52% to 44%) decreased due to increased definitional-phrase invalid nodes (from 8% to 30%). An example of a definitional phrase is "$g(x)\neq 0$ *for any x in (a, b)*". Despite this, the final cmap was conceptually richer in its choice of concept labels with a structurally parsimonious, network of conceptual interconnections.

Table 1
Concept Map Data for Students 1 to 5

Student	1 Pene		2 Loke		3 Fia		4 Vae		5 Heku	
Criteria	First Cmap	Final Cmap	First Cmap	Final Cmap	First Cmap	Final Cmap	First Cmap	Final Cmap	First Cmap	Final Cmap
Contents										
Valid Nodes										
- Concepts	35 (67)	30 (65)	17 (44)	32 (56)	73 (99)	83 (83)	40 (59)	66 (99)	44 (86)	50 (74)
- Examples	5 (10)	1 (2)	19 (49)	16 (28)	0 (0)	6 (6)	10 (15)	0 (0)	0 (0)	0 (0)
Invalid Nodes										
-Definitional	4 (8)	14 (30)	1 (3)	8 (14)	1 (1)	8 (8)	12 (18)	1 (1)	1 (2)	15 (22)
-Inappropriate	8 (15)	1 (2)	2 (5)	1 (2)	0 (0)	3 (3)	6 (9)	0 (0)	6 (12)	3 (4)
Total Nodes	52	46	39	57	74	100	68	67	51	68
Propositions										
Valid Propositions	27 (52)	26 (44)	25 (69)	29 (49)	77 (96)	106 (88)	32 (51)	85 (97)	35 (66)	54 (67)
Invalid Propositions	25 (48)	33 (56)	11 (31)	30 (51)	3 (4)	14 (12)	31 (49)	3 (3)	18 (34)	27 (33)
Total Propositions	52	59	36	59	80	120	63	88	53	81
Structural										
Cross-links	3	10	0	6	9	10	4	17	6	22
Sub-branches	17	14	9	19	26	33	22	19	9	32
Average H/Levels per Sub-branch	6	8	6	8	10	9	7	8	8	7
Main Branches	6	6	5	7	5	8	4	5	6	9
M/Branching Nodes	8	10	5	8	18	19	9	18	9	19

Key H/Levels Hierarchical Levels M/Branching Multiple Branching Count (% of total number)

Student 2: Loke – Differentiation. Loke's first cmap had relatively more illustrative examples (49%) than conceptual entries (44%). As a result of critiques, revisions and independent research, the final cmap was relatively more conceptual (increased valid concept nodes from 44% to 56% and a reduction in examples from 49% to 28%), structurally more expanded (addition of 2 more main branches), more integrated (addition of 6 new cross-links) and more differentiated (increased multiple-branching nodes from 5 to 8 and increased sub-branches from 9 to 19). However, the reduction of valid propositions (from 69% to 49%) was due mainly to increased definitional-phrase invalid nodes (from 3% to 14%). An example of an incorrect proposition is "*Differentiation* also have a *non-differentiable function*". Overall, the final cmap was more differentiated, more integrated and more conceptual than the first cmap.

Student 3: Fia – Numerical Methods. Fia's first cmap had a high percentage of valid propositions (96%) reflecting her careful organization of propositions. As a result of critiques, revisions and further research, the final cmap showed increased number of valid concept nodes (from 73 to 83) and valid propositions (from 77 to 106) but proportionally reduced (valid nodes from 99% to 89% and valid propositions from 96% to 88%) due to increased definitional-phrase and inappropriate nodes (from 1% to 11%). Structurally, the final cmap expanded (increased main branches from 5 to 8), becoming more integrated (increased cross-links from 9 to 10) and more differentiated (increased multiple-branching nodes from 18 to 19 and increased sub-branches from 26 to 33) with more compact sub-branches (reduced average hierarchical levels from 10 to 9).

Student 4: Vae – Limits and Continuity. Vae's first cmap showed inclusion of complete formal definitions as concept labels, which the first peer critique highlighted as problematic. As a result of revisions, and critiques, Vae's cmap progressively evolved into a more conceptual one (increased valid nodes from 74% to 99%) with substantially increased valid propositions (from 51% to 97%), structurally expanded (main branches increased from 4 to 5), more integrated (cross-links increased from 4 to 17), more differentiated (increased multiple branching from 9 to 18 and increased average hierarchical levels per sub-branch from 7 to 8), and more compact (reduced sub-branches from 22 to 19). Evidently, continuous revisions enhanced the hierarchical interconnections such that formal definitions were analysed substantively, with concepts appropriately linked and described to illustrate the conceptual structure of the topic.

Student 5: Heku – Motion. Heku's final cmap became more conceptual with increased number of valid concept nodes (from 44 to 50 but proportionally reduced from 86% to 74%) and increased valid propositions (from 66% to 67%). Structurally, the final cmap was more expanded (increased main branches from 6 to 9), more integrated (increased cross-links from 6 to 22), more differentiated (increased multiple branching nodes from 9 to 19 and increased sub-branches from 9 to 32), but relatively more compact within sub-branches (reduced average hierarchical levels from 8 to 7). Increased invalid nodes (from 14% to 26%) resulted mainly from increased definitional phrases (from 2% to 22%).

Student 6: Santo – Complex Analysis. With repeated cycles of presentations → critiques → revisions, Santo's final cmap (Table 2) still had the same number of main branches, average hierarchical levels per sub-branch, and cross-links, a reduction of valid nodes (from 93% to 90%) while valid propositions increased (from 74% to 79%), and becoming structurally more differentiated (increased multiple branching nodes from 24 to 34) and more compact (reduced sub-branches from 68 to 66).

Table 2
Concept Map Data for Students 6 to 10

Student	6 Santo	6 Santo	7 Fili	7 Fili	8 Pasi	8 Pasi	9 Toa	9 Toa	10 Salo	10 Salo
Criteria	First Cmap	Final Cmap	First Cmap	Final Cmap	First Cmap	Final Cmap	First Cmap	Final Cmap	First Cmap	Final Cmap
Contents										
Valid Nodes										
- *Concepts*	165 (87)	159 (84)	32 (67)	30 (73)	41 (36)	52 (87)	34 (76)	63 (71)	100 (57)	79 (72)
- *Examples*	11 (6)	12 (6)	0 (0)	1 (2)	20 (18)	0 (0)	0 (0)	7 (8)	34 (19)	17 (16)
Invalid Nodes										
-*Definitional*	1 (1)	2 (1)	3 (6)	3 (7)	3 (3)	7 (12)	6 (13)	15 (17)	8 (5)	6 (6)
-*Inappropriate*	12 (6)	16 (8)	13 (27)	7 (17)	50 (44)	1 (2)	5 (11)	4 (4)	33 (19)	6 (7)
Total Nodes	189	189	48	41	114	60	45	82	141	92
Propositions										
Valid Propositions	148 (74)	166 (79)	15 (32)	26 (62)	48 (40)	39 (67)	28 (56)	88 (81)	110 (60)	82 (73)
Invalid Propositions	51 (26)	45 (21)	32 (68)	16 (38)	71 (60)	19 (33)	22 (44)	20 (19)	73 (40)	30 (27)
Total Propositions	183	112	47	42	119	58	50	108	183	112
Structural										
Cross-links	8	8	1	4	11	12	13	21	5	6
Sub-branches	68	66	16	13	19	16	10	24	44	35
Average H/Levels per Sub-branch	6	6	6	6	12	9	11	9	9	7
Main Branches	19	19	4	6	12	6	3	10	10	10
M/Branching Nodes	24	34	4	9	13	11	9	17	20	16

Key H/Levels Hierarchical Levels M/Branching Multiple Branching Count (% of total number)

Student 7: Fili – Multiple Integrals. Fili's first cmap illustrated sequential derivations of double and triple integrals, with critical comments targeting invalid nodes. As a result of critiques, revisions, and further independent research, Fili's final cmap became more parsimonious (reduced sub-branches from 16 to 13 and unchanged average hierarchical levels per sub-branch), more integrated (increased cross-links from 1 to 4), more differentiated (increased multiple-branching nodes from 4 to 9), more conceptual (increased valid nodes from 67% to 75%) and valid propositions almost doubled (from 32% to 62%).

Student 8: Pasi – Integration. As a consequence of the cyclic process of presenting → critiquing → revising → presenting, Pasi's final cmap evolved into a substantially more conceptual one (increased valid nodes from 54% to 87%) with increased valid propositions (from 40% to 67%). For example, a new branch illustrated the numerical limit view of integrals from successive approximations of area under a curve and linking it to the limit of the Riemann sum as a definition for the definite integral. The absence of illustrative examples was noticeable. Structurally, the cmap was more compact (reduced multiple-branching nodes (from 13 to 11), reduced sub-branches (from 19 to 16), reduced main branches (from 12 to 6), and reduced average hierarchical levels per sub-branch (from 12 to 9). Overall, the final map was predominantly more conceptual with more valid propositions and a more parsimonious, compact final structure.

Student 9: Toa – Normal Distributions (ND). Toa felt challenged to construct a cmap that included ND, Poisson distributions (PD) and binomial distributions (BD). He wrote: "(it was) hard to think of a concept to start the cmap and then link the others right down to the end when it introduces (BD, PD and ND)." The first peer critique commented the cmap had "too many useful concepts … missing", and the "concepts used were paragraphs". In subsequent revisions, he "tried to break down those paragraphs into one or two concept names" and "re-organized concept hierarchies", eventually resulting in a final cmap that was more conceptual (increased valid nodes from 76% to 79%) with increased valid propositions (from 56% to 81%). Structurally, the final cmap became more expanded (increased main branches from 3 to 10), more integrated (increased cross-links from 13 to 21), more differentiated (substantial increases with multiple branching nodes from 9 to 17 and sub-branches from 10 to 24) and more compact within sub-branches (reduced average hierarchical levels from 11 to 9). Shown in Figure 1 is part of Toa's final map (example of a good cmap) showing examples of integrative crosslinks between two branches (proposition "*Normal Distribution* can be approximately used for *Binomial Distribution* → *Normal Distribution*"), multiple branching nodes (*bell-shaped curve* and *parameters*) and integrative reconciliation of a number of nodes merging into a single node (nodes x, $n - x$, p, n, $q = 1 - p$, with merging links to *Probability Function*).

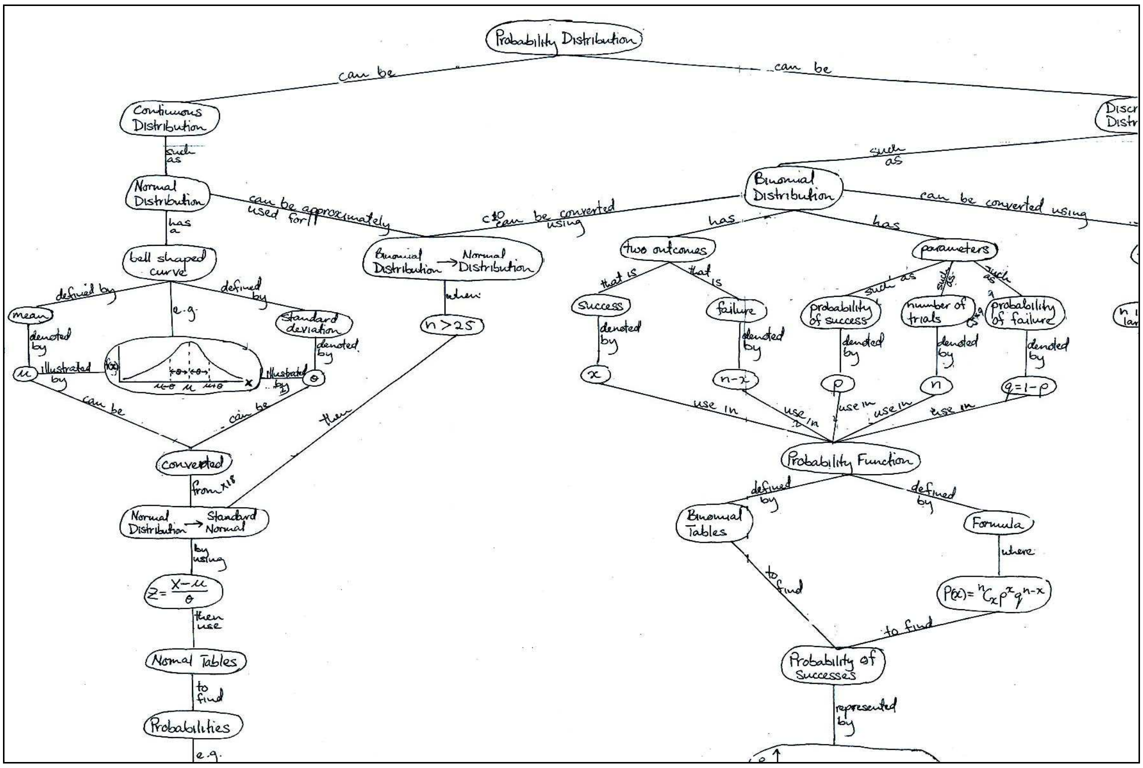

Figure 1. Partial final concept map – Toa.

Student 10: Salo – Infinite Series. The first peer critique targeted the high number of inappropriate nodes (33) with subsequent critiques focussing on the need to improve linking words and appropriate placement of progressively-differentiated concepts. Salo's final cmap became more conceptual (increased valid nodes from 76% to 88%) with increased valid propositions (from 60% to 73%). Structurally, the final cmap was more integrated (increased crosslinks from 5 to 6) and more compact with less differentiation (decreased multiple branching nodes from 20 to 16 and decreased sub-branches from 44 to 35, with a lower average hierarchical levels per sub-branch from 9 to 7) whilst main branches remain unchanged. Overall, the final map was more conceptual with a more enriched network of interconnections and structurally more integrated and more compact than the first cmap.

Discussion

Findings suggested that students' progressive cmaps became integrated and differentiated as students continually strove to illustrate valid nodes and meaningful propositions, in response to concerns raised in social critiques and in anticipation of future critiques. Hence the re-definition of socio-mathematical norms appeared to affect the nature of students' cmaps substantively, particularly as students had to justify their displayed connections, negotiate meanings with their peers, and reach a consensus to revise or not. For example, half the students showed increases in valid nodes, propositions, and structural complexity by the final cmap. There was a marked shift from simply providing formulas, procedural steps, excessive illustrative examples, and entire paragraphs, to seeking out more integrated and differentiated conceptual interconnections, which reflected the impact of the social interactions on an individual's evolving understanding. Also, students necessarily had to reflect more deeply, as individuals, about the conceptual structure of topics than they normally did. Because of the need to communicate their understanding competently in a social setting, over time and with increased mapping proficiency, students became more parsimonious in their selection of concepts and more astute in describing the nature of the relationships between connecting concepts more correctly to minimize critical comments. From students' perspectives, they realized that mathematics has a conceptual structure, the socially validated body of knowledge, which underpins its formal definitions and formulas. By searching for missing relevant concepts to make the cmaps more robust and comprehensive, students eventually realized that an in-depth understanding of topics required much more than re-stating a definition or formula. Concept mapping required the identification of main concepts, an integrated understanding of connections between relevant concepts, visually organising this understanding as a meaningful hierarchy of interconnecting nodes with valid linking words that form valid propositions as socially warranted by a community of mathematicians.

Over the semester, students eventually appreciated the utility of cmaps as a means of depicting networks of conceptual interconnections within topics and of highlighting connections between concepts, definitions and formulas. However, attaining this greater conceptual understanding of mathematics was hard work and required much reflection, social negotiation, and individual research on their part. The findings suggested that with more time and practice students can become proficient and adept at constructing cmaps whilst simultaneously deepening and expanding their theoretical knowledge of the structure of mathematics. Challenges faced by the students included the importance of getting quality feedback from their peers, sustaining students' motivation to seek more meaningful connections by revising inappropriate nodes and incorrect linking words and reorganising concept hierarchies, and developing their self-confidence in presenting mathematical justifications and counter-arguments during social critiques. The progressive quality of students' cmaps over the semester confirmed that students' ways of learning mathematics are very much influenced by the expectations and beliefs of the teacher, the prevailing socio-mathematical norms of the classroom setting, and the socially-validated structure of mathematics. Findings also extended the literature on the impact of social negotiations of meanings, interactions and critiques on the development of students' conceptual understanding of topics, which in this study, was greatly facilitated with the visual mapping of students' progressive conceptions on hierarchical cmaps over time. Finally, using the metacognitive tools promoted a higher level of self-reflection and lateral thinking that generally motivated students to analyse their perceptions of mathematics knowledge critically and specifically encourage deeper, conceptual understandings of topics.

Implications

Findings from the study imply that the concurrent use of concept mapping and social critiques as part of the culture and practices within mathematics classrooms has the potential to promote the development of mathematical thinking, reasoning, and effective communication, which are most desirable skills to succeed in mathematics learning. Doing so as early as the primary level would be an area worthy of further investigation.

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