Teachers' Knowledge of their Students as Learners and How to Intervene

Jane Watson
University of Tasmania
jane.watson@utas.edu.au

Kim Beswick
University of Tasmania
kim.beswick@utas.edu.au

Natalie Brown
University of Tasmania
natalie.brown@utas.edu.au

As part of a teacher profiling instrument, 42 middle school teachers were presented with a mathematics problem dealing with fractions and wholes and asked to suggest solutions that would be given by their students. Further they were asked how they would address inappropriate responses in the classroom. The students in their classes were presented with the same question as part of a larger survey of mathematical concepts important in the middle years. This study compares the expectations of teachers and their suggested remedial actions with their years of teaching, their previous mathematics study, and the performance of students. Results suggest explicit questioning of teachers is an effective way to explore teacher knowledge for teaching mathematics.

The issue of teacher knowledge in relation to teaching mathematics and students' understanding of mathematics has long been a vexing question in mathematics education. What kinds of knowledge do teachers need in order to ensure learning for their students? Shulman (1987a, 1987b) began addressing issues of teachers' knowledge by suggesting seven types of knowledge that were required for teachers: content knowledge, general pedagogical knowledge, curriculum knowledge, pedagogical content knowledge, knowledge of learners and their characteristics, knowledge of education contexts, and knowledge of education ends, purposes, and values. These seven types of knowledge were featured and assessed in a teacher profiling instrument developed by Watson (2001) and used in relation to the chance and data part of the mathematics curriculum. She felt that the most important aspects in terms of student outcomes were associated with content knowledge, pedagogical content knowledge, and knowledge of students as learners. Among other aspects of the profiling instrument these three were addressed in questions that presented teachers with problems previously used in student surveys. Teachers were asked what appropriate and inappropriate responses students would give to these problems and how the teachers would use the student responses to devise remedial activities in the classroom. The issue of delving into these three types of teacher knowledge has often been considered delicate, in that teachers may feel threatened in particular by explicit questions about their mathematical knowledge. Often measures of teacher knowledge have been based on the number of mathematics courses completed, years of teaching, or self-report of confidence (e.g., Mewborn, 2003; Schoen, Cebulla, Finn, & Fi, 2003). These can be less reliable measures than asking teachers to produce mathematical responses for themselves in the three areas of interest.

Recent work in this area by Hill, Rowan, and Ball (2005) has focussed on "Teachers' knowledge for teaching mathematics" as an extension of the work of Shulman. By this they mean

> the mathematical knowledge used to carry out the work of teaching mathematics. Examples of this "work of teaching" include explaining terms and concepts to students, interpreting students' statements and solutions, judging and correcting textbook treatments of particular topics, using representations accurately in the classroom and providing students with examples of mathematical concepts, algorithms, or proofs. (p. 373)

The current researchers agree with this description and believe it covers the three types of Shulman's knowledge that are the objective of this study. How Hill et al. address the measurement of teachers' knowledge for teaching mathematics is discussed at the end of this paper.

In a further consideration of the above issues this study addresses the following research questions for a particular conceptually-based problem from the domain of fractions.

1. What levels of teacher knowledge are displayed with respect to content knowledge and knowledge of students as learners (Profile Question 1), and with respect to pedagogical content knowledge (Profile Question 2) for a particular fraction problem?
2. Is there an association between levels of response to Profile Question 1 and Profile Question 2, and between these two variables and years of teaching experience, and tertiary mathematics courses completed?
3. What levels of understanding do students show to the problem and do these levels reflect the expectations of teachers?

Method

Sample. The 42 teachers in this study came from 9 schools, of which 3 were primary (grades K–6), 1 was high (grades 7-10), and 5 were district (grades K-10). The 650 students in the study were all of the students in grades 5 to 8 in these schools who were present the day that surveys were administered. For the purposes of this study the number of students and outcomes for each grade level are not relevant but students were relatively evenly split among the four grades. The surveys were administered by classroom teachers, sometimes with the assistance of a researcher. Teacher profiles were completed at meetings conducted by the researchers at the individual schools. Although some teachers were traditional primary classroom teachers of a single grade or high school mathematics teachers of several grades, many had general middle school duties across classes and some also taught outside of the grades 5 to 8 because of the structure of their schools.

Task. The task presented to teachers and students is presented in Figure 1. The task was the same for both students and teachers but teachers, after being presented with the question were first asked:

> "What responses would you expect from your students? Write down some appropriate and inappropriate responses (use * to show appropriate responses)" (Profile Question 1).

Approximately one third of a page was given for the response and then teachers were asked:

> "How would/could you use this item in the classroom? For example, choose one of the inappropriate responses and explain how you would intervene" (Profile Question 2).

Mary and John both receive pocket money. Mary spends 1⁄4 of hers, and John spends 1⁄2 of his. A. Is it possible for Mary to have spent more than John? B. Why do you think this? Explain.

Figure 1. Item used as a basis for the study.

Coding and analyses. Data from the teacher profiling instrument were coded in two ways. To obtain an overall picture of the range of responses, all individual responses were listed and categorised no matter how many responses an individual teacher made. These were clustered (Miles & Huberman, 1994) and frequencies recorded. As well, a rubric was devised to consider the level of response of each teacher based on the overall response presented in terms of appropriateness and structural complexity of response. These two procedures were carried out for each part of the profile item and the rubrics for the overall assessment of responses are presented in Table 1.

Table 1
Rubric for Teacher Responses to Profile Questions 1 and 2 for the Problem in Figure 1

Level	Profile Question 1	Profile Question 2
0	No response	No response
1	Response not addressing fractions or wholes	Response not addressing the mathematical content of the problem
2	Response indicating either correct fraction relation to whole *or* incorrect relationship of _ and _ to whole	A single generic idea for the problem, e.g., use money, discuss fractions
3	Response containing both appropriate and inappropriate approaches to the problem	Reference to 2 ideas without linking them
4	NA	Discussion including reference to fractions and wholes with specific examples

The teachers' number of years of teaching experience were split into three groupings: less than 5 years (14 teachers), 5 to 14 years (13 teachers), and 15 or more years (13 teachers). Two teachers did not respond to this question. The teachers' previous tertiary exposure to mathematics courses was recorded in four categories; none (9 teachers), one semester [Sem] (11 teachers), one year [Yr] (11 teachers), and more than one year [More] (10 teachers). One teacher did not respond to this question. Gender of teachers was recorded but no associations with teacher knowledge were found and gender is not considered further.

The associations between pairs of variables related to responses to Profile Questions 1 and 2, years of teaching, and mathematics background are shown in two-way tables.

The rubric for student responses to the question in Figure 1 was devised based on the researchers' previous research and was related to the appropriateness and structure of the response, in particular associated with the explanation given. The general description is presented in Table 2 and examples are presented in the Results along with the percent of middle school students in each category.

Table 2

Levels of Student Response for the Question in Figure 1

Level	Global Category	Description
3	Shows critical understanding	Critically examines information and states appropriate reasoning with concrete examples.
2	Shows understanding	Examines information; states or infers appropriate reasoning.
1	Partial understanding	Limited reasoning based on abstract relationship of _ and _.
0	Inappropriate response	Misinterpretation of question or idiosyncratic reasoning; misunderstanding of fractional values; restating of information; no response.

Results

Research Question 1: Teachers' knowledge

Of the 42 teachers, 10 (24%) did not give any possible student responses to this problem, although one teacher did make a comment about the abstract nature of the problem. Fifteen teachers (36%) did not provide a suggested way of handling the problem in the classroom. Twelve teachers (including the 9 who did not respond) gave no indication that they knew the appropriate approach to the problem. The 32 teachers who answered the first part provided 67 responses, many suggesting both the expected incorrect and correct interpretations of the problem. A summary of responses is given in Table 3.

Table 3

Suggested Student Responses to Mary and John Problem (Profile Question 1)

Responses (from 32 teachers)	Frequency
Don't know/why do? What is 1/4? How do you work out 1/2 without knowing of what?	6
John is a man & earns more/Boys get more pocket money	2
Yes, 1/4 is more than 1/2	1
Yes, Mary might have bought something more expensive	1
"No"	3
"Yes"	2
No, 1/2 is bigger than (double) 1/4	21
Yes, it depends on their starting amounts (sometimes with examples) How much money do they get?	27
Other, e.g., Some understand, others think pocket money the same; "It depends"; "I hate this sort of abstract style of thinking so wouldn't use;" "Mary is tight"	4
Total	67

The 27 teachers who addressed classroom strategies for dealing with the problem made 35 suggestions. From the response of one teacher it was not clear that the teacher understood the importance of the whole because only raw fractions (1/4 and 1/2) were discussed. Three teachers discussed the importance of critical thinking in reading the problem but did not mention the mathematics content of the problem. One teacher mentioned pocket money, again without noting the importance of the part-whole concept. The results are summarised in Table 4.

Table 4

Suggested Classroom Strategies for use of Mary and John Problem (Profile Question 2)

Responses (from 27 teachers)	Frequency
"Discuss fractions"	3
"Explain" how much money/give examples	3
Relate to pocket money/budgeting	3
Use different amounts to see which is larger; emphasise starting points	16
Use different concrete materials (pie charts, number line, paper, cakes)	4
Importance of critical reading of problem	3
Get students to explain answers/brainstorm	2
Prepare lesson on equal opportunity	1
Total	35

For the rubric assessing teachers' overall responses to the two questions, Table 5 summarises the outcomes for the 42 teachers in terms of level of response. Whereas 71% of teachers could suggest at least one appropriate or inappropriate solution that would be given by students, only 43% could suggest both appropriate and inappropriate strategies. For classroom use of the problem only 24% suggested a mix of strategies that would indicate pedagogical content knowledge of the type recommended by Shulman (1987b) or Hill et al. (2005). The differences in these percents reflect to some degree the differences in frequencies of response in Tables 3 and 4.

Table 5

Levels of Response for Overall Response to Profile Questions 1 and 2 (n = 42)

Level	0	1	2	3	4
Profile Question 1	10	2	12	18	NA
Profile Question 2	15	3	8	6	10

Research Question 2: Association of variables

The association of teacher responses to the two Profile Questions is shown in Table 6 in relation to the rubrics described in Table 1. As can be seen, providing student responses was apparently easier for teachers than providing ways that the problem could be used in the classroom, as only one teacher was able to provide two generic ideas for using the task in the classroom without being able to suggest either an appropriate or inappropriate response involving fractions. This teacher, however, was one of the 12 who did not provide explicit evidence of understanding the appropriate solution, responding, "I would relate the question to their pocket money and ask them how much they receive a week. Then I would ask what _ is, what a quarter is and discuss which is greater." Fifteen teachers (36%) were able to provide at last one of an appropriate and inappropriate response as well as two or more ideas for using the task in the classroom.

Table 6

Association of Levels of Response by Teachers to Profile Questions 1 and 2

Profile	Profile Question 1			
Question 2	Level 0	Level 1	Level 2	Level 3
Level 0	10	0	4	1
Level 1	0	0	2	1
Level 2	0	1	2	5
Level 3	0	1	1	5
Level 4	0	0	3	7

The lack of association of the number of years of teaching experience and the levels of response to the two teacher questions is shown in Table 7. The distribution of experience is relatively even across the levels of response, with a tendency for the most experienced teachers (≥ 15 years) to be split into the extremes, either declining to answer the question or providing high level responses.

Table 7

Association of Years of Teaching Experience and Levels of Response to the Two Profile Questions (n=40)

	Profile Question 1			Profile Question 2		
Years of teaching	<5	5-14	≥15	<5	5-14	≥15
Level 0	2	2	4	5	4	4
Level 1	1	1	0	0	2	1
Level 2	6	5	1	3	5	0
Level 3	5	5	8	2	2	2
Level 4	NA	NA	NA	4	0	6

The further lack of association of the teachers' reported tertiary mathematics background and the levels of response to the two teacher profile questions is shown in Table 8. It would be very difficult to make a claim from the data presented that more exposure to mathematics in pervious tertiary study leads to higher level responses to the questions related to the problem in Figure 1.

Table 8

Association of Teacher Tertiary Mathematics Background and Levels of Response to the Two Profile Questions (n=41)

	Profile Question 1				Profile Question 2			
Tertiary study	None	Sem	Yr	More	None	Sem	Yr	More
Level 0	2	1	3	3	2	3	5	4
Level 1	2	0	0	0	2	1	0	0
Level 2	2	4	4	2	1	2	3	2
Level 3	3	5	4	5	2	1	2	1
Level 4	NA	NA	NA	NA	2	4	1	3

Research Question 3: Student performance

The responses of students to the question in Figure 1 are summarised in Table 9. The responses at Level 2 and Level 3 combined represent appropriate responses to the task (31.4%) but the percent at Level 3 represents the students who went on to provide a concrete example. As this was not specifically asked for in the task statement, it is not known if this is a reasonable estimate of the percent of students who could provide examples, but probably not. Of significance is the percentage of students (29.9%) who did not have an appropriate understanding that a fraction must be associated with its whole in order to work out comparisons. Another 38.8% of students either did not respond or gave an answer not related to the relationship of "$\frac{1}{4} < \frac{1}{2}$" or to the relationship of fractions to wholes.

Table 9

Student Responses to the Mary and John Question (n=650)

Level	Examples	%
3	Mary could have got more money than John e.g., John got $20 and spent $10 of it and Mary got $60 and spent $15 of it.	3.5
2	Mary may have got more money than John.	27.9
	$\frac{1}{2}$ could be more than $\frac{1}{4}$ if Mary gets more than John.	
	Mary gets more than John.	
	Mary could be older than John and get more money.	
1	If John get more pocket money she would still have spent less because spend means spend.	29.9
	$\frac{1}{2}$ means more than $\frac{1}{4}$.	
	It's lots more to have $\frac{1}{2}$ than $\frac{1}{4}$.	
0	John spends half, $\frac{1}{2}$ means $\frac{1}{2}$ of John's money so Mary can spend over half.	25.9
	The thing she wanted cost more. John spends $\frac{1}{2}$.	
	$\frac{1}{4}$ is bigger than $\frac{1}{2}$. Mary spends $\frac{1}{4}$.	
	4 is bigger than 2.	
	No Response	12.9

In comparing the responses summarised in Table 9 with those expected by teachers in Table 3, of the teachers who responded (n=32), 66% noted the inappropriate response involving $\frac{1}{4}$ and $\frac{1}{2}$ given by 29.9% of students, whereas 84% noted the appropriate response given by 31.4% of students (Levels 2 and 3). Of the total of 67 responses presented, 19 (28%) were of the type that would have been coded at Level 0 for the students (38.8% including non responses). Most of these responses were provided by teachers who also provided appropriate and/or inappropriate responses; as can be been in Table 2 only two teachers gave only this type of response.

Discussion

Limitations. The reasons why 9 teachers did not respond to this question are unknown. It may have been a lack of understanding of the task or fatigue in answering the profile. Considering the stronger background of some of the teachers who did not respond, it may have been that these teachers felt the task was beneath them or perhaps they even felt intimidated if they had not ever thought about such questions. The task, however, is fundamental to the understanding of fractions and wholes and the researchers believe it is a legitimate task to ask of all middle school numeracy or mathematics teachers.

Task choice. The task in Figure 1 was chosen for the student survey and teacher profile because of its requirement to consider both fractions and wholes, without an explicit presentation of the wholes. That one teacher reported he/she would never use a problem like this with students was disturbing to the researchers. The plan of the researchers is to use the information from this study as a basis for professional learning in the larger project of which it is a part.

Findings. Of interest for this report are several initial findings. For this group of teachers, there is no association of years of teaching or tertiary background in mathematics with the levels of response to suggestions of potential student answers (Profile Question 1) or to levels of response to suggestions of classroom use for the problem (Profile Question 2). This suggests that a direct method of enquiry, such as used here, is going to be more effective in gauging teachers' content, pedagogical, and student knowledge with respect to specific mathematics topics. The fact that most of the responding teachers were aware of appropriate and inappropriate student responses is encouraging. Less encouraging is the percent of teachers who did not or could not respond to the profile questions and the minority of those who did who could provide multiple responses both for student responses and for classroom use (36%).

Further research. Encouraging support for further research in this area of teacher knowledge is found in the research of Hill et al. (2005), whose measurement of teachers' knowledge for teaching mathematics showed it was positively associated with their own students' mathematics outcomes. The examples of items used to measure teacher knowledge, however, were multiple choice and although related to content and diagnosis of student errors, did not appear to delve into pedagogical content knowledge. The items also appeared, from the limited examples, to be related to factual and procedural knowledge, rather than to conceptual knowledge. The current researchers hope that those interested in teachers' knowledge for teaching mathematics will also consider tasks like the one presented here that allow teachers freedom to compose responses and show their understanding of three aspects of teacher knowledge. It is difficult to conceive of a multiple choice question that could do this. The rubric employed also permits the description of teacher's progress, not just a right/wrong response.

Acknowledgement. This research was funded by ARC Linkage Grant No. LP0560543.

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