Building Powerful Understanding by Connecting Informal and Formal Knowledge

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This paper will illustrate the importance of recognising and validating students' informal knowledge as the essential cornerstone for developing mathematical ideas. A single case story is presented to highlight a teacher's scaffolding strategies which identify and interpret a student's informal knowledge in relation to school knowledge. From here the case story continues to convey how the teacher builds student understanding as well as her own.

Informal knowledge in this paper is defined as the knowledge students bring from home and other contexts along with knowledge from school, which has not yet developed into rigorous formal knowledge in school mathematics. Students have the potential to develop powerful understandings in mathematics if teachers recognise the fundamental role that students' informal knowledge can play in the learning process. However it is often viewed as counter hegemonic to the teaching and learning process, and therefore is largely ignored because of a failure to recognise its strength and robustness.

When formal mathematics is introduced without building on or connecting to the students' informal knowledge, it may result in failure to make meaning. Depriving students of opportunities to make links between formal mathematics and the meanings they can identify, could lead to many students' learning experiences becoming one of rote, devoid of meaning and possibly leading to learning difficulties. Learning mathematics without meaningful understanding could also result in students' identity as mathematical learners being compromised. This idea resonates with research conducted by Baroody and Ginsburg (1995)

Learning with understanding does not develop in a linear progression. Students often need to return to their earlier understandings in order to make sense of the new, even when their original understandings are limited in nature. Although student's informal knowledge may vary from the ideas that they are working towards, this thinking still plays a significant role in students' understanding of complex ideas as they make meaning within these domains. (Mack 2001; Pirie & Kieren; 1994).

Supporting students to bring their informal knowledge into the classroom and connect it to formal mathematics requires the teacher to reconceptualise their own thinking of mathematics and to be inclusive of the students' different informal knowledge (Ball 2001). Therefore teachers need to be encouraged to *recognise, understand and interpret* students' informal thinking, focusing on strengths rather than limitations. This requires teachers to analyse student thinking to determine the connections already made to formal mathematics, including connections within and across the strands, strategies, concepts and skills.

This paper will illustrate teachers' work through a three stage process of *recognising, interpreting and understanding* children's informal knowledge. I will argue that when teachers adopt this approach they come to understand the richness of students' knowledge and how this is the beginning point of students' understanding of formal school mathematics. If teachers focus on the strength of students' informal knowledge and scaffold students in further thinking, it can result in students developing more positive identities as they build meaningful understandings.

Recognising Informal Knowledge

Teachers need to be encouraged to recognise students' informal knowledge, as it appears in different forms within the class setting, through students' recordings, use of language in discussions and their selection of strategies for investigating mathematical ideas. This is a very complex task because students and teachers bring differing experiences and knowledge to their perceptions of mathematics. Therefore students' interpretation of the mathematical task and ideas can be reported and recorded in a variety of ways, often very dissimilar to that of the teachers.

Interpreting Informal Knowledge

Having recognised informal knowledge, teachers need to interpret this in light of the mathematical ideas the students are exploring, to identify the connections the students have made to formal mathematics and the strength in their strategies and thinking. The teacher uses this knowledge to scaffold students to build new connections, to form new knowledge or strengthen existing connections through revisiting the ideas in a new way. The process of interpreting requires the teachers to reinvent their own thinking in order to determine the possible connections students are making or need to make. Informal knowledge provides the teacher with insight into the individual student's understanding of the mathematical ideas each is working towards. This insight will inform the teaching process because it provides the direction, possible tasks and questions required to scaffold the learning. Where students do not have an opportunity to draw on their informal knowledge in class discussions and activities, the teacher could form a false impression of students' thinking and understanding.

Building Understanding

The process of coming to understand students' informal knowledge supports teachers to deepen their own understanding of how different students construct and make meaning of mathematics. Working with students' informal knowledge not only supports teachers to understand students' thinking but can also deepen their own understanding of the interconnection and dynamic nature of mathematics itself. Often teachers have developed mathematical knowledge without fully understanding the concept that underpins the mathematical ideas. Through supporting students to move from informal to formal knowledge, the teachers deepen their own conceptual understandings. This process in itself enhances the teachers' ability to more successfully support different students' thinking (O'Toole 1997; O'Toole & Plummer 2004)

Recognising and interpreting informal knowledge and building understanding are continual and interactive processes that an effective teacher carries out throughout the learning experience. This is not linear. Often when interpreting informal knowledge, the teacher begins to recognise other informal knowledge not previously seen.

Teachers need to develop a framework for ongoing analysis in order to recognise, interpret and understand students' informal knowledge to more effectively facilitate student learning. This framework needs to include possible learning connections that teachers may observe as they facilitate different students' learning. The descriptions would include possible connections students may be making among informal knowledge and formal school mathematics, real life situations and mathematical ideas within and across strands.

In this paper I begin to explore the notion of *possible learning connections* through presenting a single case story that is part of a much larger project. This story centres on Ruby, a year 5 student as she works from her informal knowledge to build meaningful understanding of the relationship that lies behind the formula ($b \times h$) for measuring the area of a rectangle.

Methodology

The data for this study was collected using a collaborative action research model that was undertaken by a practitioner researcher (teacher) and researcher. The model involved fortnightly visits to the school, approximately two hours in duration, nine in total for this study. Each visit entailed one hour classroom observation followed by about 45-50 mins collaborative discussion between the teacher and researcher, debriefing and sharing notes from observation sessions. The nature of the data included field notes taken by the researcher from classroom observation and incidental interviews, work samples of students' recordings, teacher case notes, audio and video tapes. To ensure rigour many strategies were used, including recording verbatim dialogues from classroom observations.

The data from the research was systematically analysed to explore the connections Ruby made, as she built meaningful understandings of area measurement from her informal knowledge. The data was coded according to informal knowledge, connections to measurement concepts, connections to other mathematical areas and linking between informal and formal knowledge.

Ruby's Story[1]

Ruby experienced difficulties in most subject areas, but especially in mathematics. At the beginning of the year, Ruby was a learner who experienced difficulty in completing tasks without considerable support from her teacher. Initial observations of Ruby's understanding conveyed that she had difficulty with parts of units in both area and linear measures; confusion between area and perimeter attributes and difficulty with number knowledge linking with measurement.

At the beginning of the unit, Ruby was tracing around shapes using grid paper and then counting units individually to measure and compare areas of different shapes. As she continued to collect her data, the teacher, Sylvia observed Ruby covering a rectangle with transparent centimetre square grid and beginning to count the rows within the rectangle in groups of 10. Counting aloud and to herself, that is, 10, 20, 30, … 180. Ruby then recorded her results and described her strategy of why she counted in groups to determine her area measure (see Figure 1).

[1] Ruby's story is an extract from O'Toole, & Plummer 2004

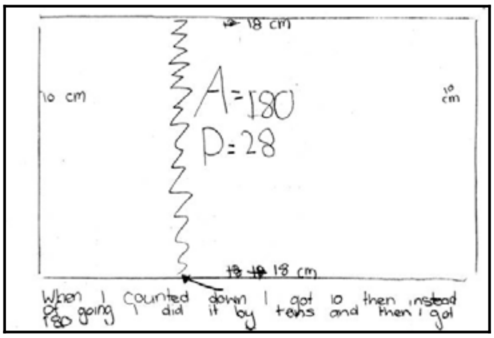

Figure 3. Ruby: Counting by tens to find the area

Based on her observation of Ruby's thinking Sylvia decided to set a new challenge. The intention was to encourage Ruby to refine and build onto the use of group counting in an attempt to find the rows and columns, as a move towards connecting her thinking to the area formula. She asked Ruby to construct 20 different rectangles and to collect data on the linear dimensions and the area. Sylvia's question was: Can you find a pattern in your data that will support you to find the area more efficiently? The work sample in Figure 2 illustrates some of the data Ruby collected.

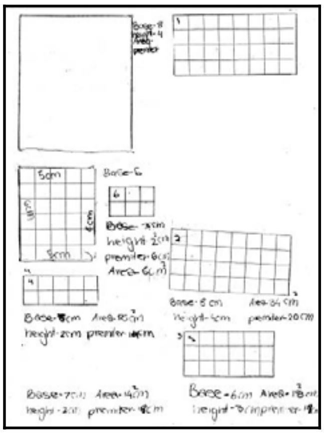

Figure 4. Part of Ruby's data for the area of rectangles

Ruby's data extended over several pages as she found it difficult to see a pattern. Sylvia encouraged her to put her data in a table (see Figure 3), but she still failed to see the pattern.

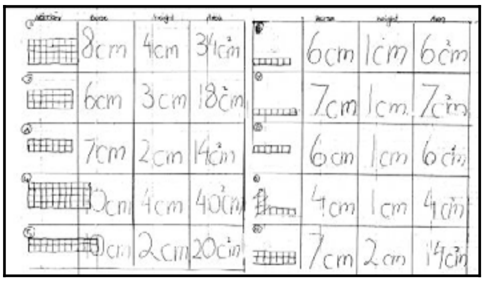

Figure 5. Ruby's recording of her area data

Excerpt 1 is the dialogue between Ruby and Sylvia after Ruby had reorganised her data into the table and was trying to search for a pattern.

Ruby	I think I have found a pattern but not in all of them. I found it in 1 cm by 6 and 2 cm by 7. You see 1 times 6 is 6 and seven 2s are 14.
Sylvia	What does the times mean?
Ruby	Groups of? 7 groups of 2 is like plussing 7 + 7 … it's 14.
Sylvia	Can you see any groups in your diagram?
Ruby	Yes, 2 groups of 7 [Ruby points to the base line and moves her fingers up and down the groups.]
Sylvia	Now look back at your table data. Can you see any other examples where this might be happening?
Ruby	No.
Sylvia	What about this one? [Pointing to the data for a rectangle that has height 4 base 1 and area 4.]
Ruby	No.
Sylvia	How are 4 and 1 different to the 1 and 6, 6 you mentioned before?
Ruby	It's less than 6 and 1.
Sylvia	Yes, but what was the pattern you saw with the 1 and 6 and 6?
Ruby	Oh, 6 groups of 1 is 6.
Sylvia	So what would it be for this one? Does that same pattern happen here?
Ruby	Oh, yes, it's 4 times 1.

Realising that Ruby's number skills were not supporting her to see the pattern, Sylvia encouraged Ruby to reflect on her knowledge of 'groups of' and 'times'. Then she refocused Ruby on the visual patterns in the diagrams in her table of data, attempting to support Ruby to link her knowledge of 'groups of' to the pattern in her table.

Sylvia:	Can you look back through your data that are not by 1 and see if your pattern still holds?
Ruby	[pointing to one of her diagrams] It's 10 and 4 … 40.
Sylvia	Does your pattern work here?
Ruby	I think so. I think there might be 10 groups of 4.
Sylvia	Can you cut out this rectangle out of grid paper and then cut out the groups.

Sylvia left Ruby to investigate further, returning to her later when Ruby had cut a 10 by 4 rectangle into 10 columns of 4.

Sylvia	What have you done now?
Ruby	Cut out the 10 groups of 4.
Sylvia	Can you see any other groups?
Ruby	Yes, 4 groups of 10.
Sylvia	Where are they?
Ruby	Here [running her finger along the rows that she had already cut out].
Sylvia	What measure of your rectangles tells you about the groups? Look back through your data.
Ruby	The height and the base tell you the groups, that' s why you can times them.
Sylvia	Is that so for all your data?
Ruby	Yes.

Ruby started to make the connections, but to ensure that she understood the pattern and the relationship between the linear measurement of a rectangle and its area, Sylvia gave her another task.

Sylvia	Can you draw up some rectangles on blank paper and work out the area? Before you do, could you think carefully through what you have just discovered and record all that you just found out so you can reflect on this information later?

Here Ruby has described the pattern she could now see occurring in the different rectangles. She used her descriptions for one rectangle to make a generalisation for other rectangles. At the end of the session, Ruby recorded her reflections and new understandings (see Figure 4).

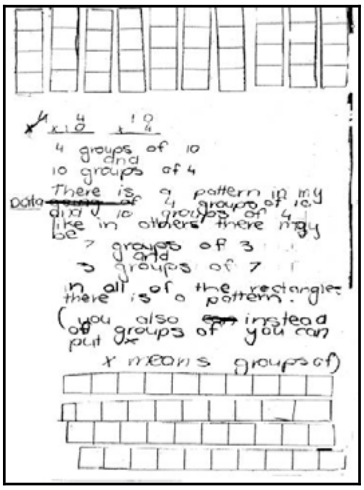

Figure 6. Ruby's recording of her understanding of how to calculate the area of a rectangle

Once Sylvia saw that Ruby recognised the connection, she focused Ruby's attention on the relationship between the linear measures and the grid of columns and rows that formed the groups. Figure 5 shows Ruby's work samples from the sessions indicating how she used her own informal ideas and language to build the understanding of the conceptual ideas behind the formula $b \times h$.

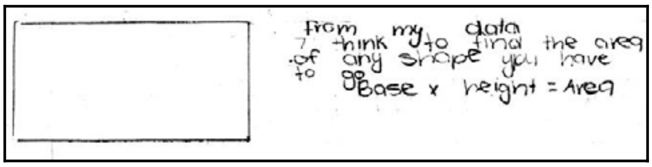

Figure 7. Ruby's recording of her conceptualisation of the rectangle area formula

Here Ruby has linked her idea of columns and rows to the linear measures of the rectangles. She has worked out the area of rectangles using blank paper and her ruler. In this sample Ruby is now using appropriate units for both linear measurement and area. She has generalised her new insight of base times height as applying to any shape. However, after further investigations, she came to understand that $b \times h$ would only apply to other rectangles.

There was a further practical investigation towards the end of the unit. Students were required to consider 4 differently shaped birthday cards and determine which card would need the greatest amount of cardboard to produce.

Ruby selected the formula to calculate the rectangular shaped cards. For the other cards, including the triangular shaped card, she used the centimetre-squared grid and counted the squares. Ruby was observed counting the whole units, and then working out the parts of units by piecing them together to make whole units. She was able to use standard measures for both linear measurement and area. Ruby demonstrated an understanding that when measuring base and height of a rectangle in centimetres, she would record her area measure in centimetres squared.

Analysis and Discussion

Ruby seemed to be strengthening and building new connections as she attempted to make meaningful understanding of the formula for a rectangle. Ruby was constantly moving back and forth between different interrelated conceptual ideas of mathematics, within and across different strands. These included measurement, number, space, data and pattern. In order to make sense of her thinking, Ruby was also making connections between her informal language and the formal language of mathematics.

Ruby could see equal groups of unit squares across the rectangle surface and used the idea to more efficiently count the units in groups of 10. Yet she did not draw on this idea when pattern searching her data, because it seems she had not yet linked group counting to the concept of times or multiplication. Ruby made the connection through the use of her diagrams that she had included in her table. This appears to have supported Ruby to make the connection between groups of and times and connect the visual pattern of the groups to the pattern in her recorded measures in the table.

It is not that Ruby was forming one connection and then another. Some of these connections seemed to be occurring simultaneously, or it was difficult to see which connection was coming before the other. For example, Ruby appeared to be making simultaneous connections between group counting and multiplication, as well as the connection between the linear measures of the rectangle and the equal groups she recognised and used to group count. Ruby also indicates in her recording that she is making connection to the principle of commutitivity.

Ruby has begun to make the connection between the linear measures, describing the number and the size of groups she saw in the centimetres squares covering the area of the rectangle. At this point, her connection is still forming and she believes that this may be so for all shapes. Ruby now needs to connect her measurement ideas to spatial concepts, to think about the properties of the rectangle and to understand why this relationship works. She could then explore the properties of other shapes in relation to area.

For Ruby, future connections will require other mathematical ideas from within measurement as well as across strands like algebra. Central to Ruby making meaningful connections was the opportunity for her to record her understandings using her own informal language and strategies. Ruby also needs to continue to connect this to conventional language used in formal mathematics.

Ruby's story illustrates the complex web of possible connections students may take as they build on to their existing informal knowledge, constructing and making meaning of formal mathematics. It demonstrates that the connections need to be made within and across the different areas of mathematics. These networks are complex with multiple connections that vary in direction and could possibly be described as almost limitless in variety. Students working from their informal knowledge will build different connections in order to construct new knowledge, although there would be similarity between some students.

As further research continues, it is hoped that an emerging framework of *possible learning connections* will assist teachers in ongoing assessment of their students throughout the learning process. It will enable the teacher to analyse the sorts of connections different students may be making, to determine which connections are strong and which may need strengthening and what new connections could be formed by building onto existing connections. In doing so, the framework of *possible learning connections* will support the teacher in task design and thinking through possible questions to further challenge the student's thinking. In supporting teachers to reconceptualise their thinking so that they can facilitate, assess and support a range of different students' learning, the framework needs to be viewed as dynamic and evolving therefore encouraging teachers to take ownership as they work with student learning to see and document new connections.

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