A Justification for Mathematical Modelling Experiences in the Preparatory Classroom

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Traditionally, mathematics has received little attention in prior-to-school settings, however, extensive research in the last 30 years has recognised that mathematical learning is critical to success and achievement in both school and life pursuits. Research has also documented children's capabilities for complex mathematical thinking and reasoning, and recently curriculum has been developed that addresses young children's competencies and potentials. This paper reviews current literature to provide a justification for the inclusion of mathematical modelling activities in preparatory settings. Mathematical modelling activities move beyond traditional problem solving to encourage children to develop and explore significant, real world mathematical ideas.

It is clear in this increasingly technological and global society that achievement in mathematics will have a major impact on students' career aspirations, their role in society, and even their sense of personal fulfilment (Malcom, 1999). Similarly, Hunt (1996) suggested that in the broad sectors of our society, our schools, in our research communities, and our government bodies, there is ample recognition that mathematics competence is important for entry in the work force. Many professionals in mathematics envisage new demands on their discipline in the post-industrial world (Bowman, 1999). They assume a society where children are required to have a broader range of skills and knowledge that prepares them to solve new, more complex problems. A focus on problem solving, direct experience, and understanding are the joint goals of the American Association for the Advancement of Science and the National Academy of Sciences (1999) and these associations are ambitious about young children's achievements. The period of time spanning the first 8 years of a child's life is internationally defined as early childhood (Bredekamp & Copple, 1997).

For the best part of the last century society held a bleak view on the learning abilities of young children, which reinforced the argument for limited or no mathematics instruction in early childhood. Piaget's (1952) work relating to young children's stages of development further reinforced these perceptions. Preschoolers were labelled as *preoperational thinkers,* who were not capable of thinking logically, or systematically, or of constructing abstract concepts. Progression through Piaget's stages was claimed to be a biological process and that no amount of teaching could accelerate progress through the stages. Other theories have challenged Piaget's view of learning. Bruner (1986) saw learning as a developing process that could be altered by teaching and Vygotsky (1978) emphasised the influence of learning on development.

However, in recent years much research has contributed to a drastically different view of young children. It is now recognised that young children bring to the schooling experience powerful mathematical knowledge, skills and dispositions (e.g., Baroody, 2000; Ginsburg, Balfanz, & Greenes, 2000). Clements, Sarama, & DiBiase (2003) have even gone so far as to state that "robust mathematical learning by all young children is a necessary base for later learning and is necessary to keep children from falling permanently behind in mathematics" (p.105). The importance of brain development and the opportunities for early childhood stimulation calls for well designed preschool education.As the attitudes to teaching young children mathematics have changed, so too have the research agendas of educators. An intensification of research pertaining to learning, development, and teaching has provided evidence that has contributed to the creation of policies on early childhood mathematics and the formation of appropriate mathematics curriculum for young children. These perspectives better fit the reality of young children's experience, abilities, and future.

Early Childhood Policy and Curriculum Documents

Orientations to young children and mathematics have changed considerably. For example, National Association for the Education of Young Children (NAEYC) and National Council for Teachers of Mathematics (NCTM) have affirmed that high-quality, challenging, and accessible mathematics education for 3–6 year old children is a vital foundation for future mathematics learning and produced recommendations outlining these beliefs. In recognition of good beginnings, the joint position statement created by NAEYC and NCTM (2002) lists 10 research-based, essential recommendations to guide classroom practice. For example, high quality mathematics education for young children should:

- Enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social world;
- Use curriculum and teaching practises that strengthen children's problem solving and reasoning processes as well as representing, communicating, and connecting mathematical ideas;
- Provide for children's deep and sustained interaction with key mathematical ideas;
- Actively introduce mathematical concepts, methods, and language through a range of appropriate experiences and teaching strategies (NAEYC & NCTM, 2002, p. 4)

The draft policy on Early Childhood Mathematics prepared by the Australian Association of Mathematics Teachers (AAMT) and Early Childhood Australia (ECA) (2005) advocated mathematical learning in early childhood settings. Many recommendations can be linked to those made in the NAEYC and NCTM (2002) statement. This policy stated that early childhood educators should, "encourage young children to see themselves as mathematicians by stimulating their interest and ability in problem solving and investigation through relevant, challenging, sustained and supported activities" (p. 1).

The NCTM (2000) has stipulated that, "the foundation for children's mathematical development is established in the earliest years" (p. 73) and has created standards and principles to promote this theory. The Queensland Studies Authority (QSA) developed similiar curricula to support a new phase of early childhood education in Queensland State schools. The Early Years Curriculum Guidelines [EYCG] (QSA, 2005) has been produced to support teachers and children participating in preparatory year programs. Children participating in the state preparatory program must have turned 5 by June 30 in the year they enrol in a preparatory program. Curricula developed for Queensland preparatory students highlight early mathematical understanding, social learning, and learning contexts. Mathematical ideas are to be developed via a range of learning contexts, such as, investigations and real-life situations. Learning is viewed as a social experience and children are encouraged to communicate, investigate, focus on thinking, apply and begin to transfer understandings, and make connections. The preparatory year curriculum acknowledges that "children live in a changing world which is increasingly more complex" (EYCG, 2005, p. 1). These curriculum materials developed for the new millennium present opportunities for children to initiate, engage in, negotiate, and investigate mathematical concepts, skills, and processes. Activities presented to children in the early years settings need to reflect the changing views of children's interests and powers.

Problem Solving in Early Childhood

Problem solving has been defined as the "hallmark of mathematical activity and a major means of developing mathematical knowledge" (NCTM, 2000, p. 116). It has received particular attention in recent curricula documents. Young children are natural problem solvers who question, investigate, and explore their surrounding as they make sense of their world. In the early years children should have frequent opportunities to "formulate, grapple with, and solve complex problems that require a significant amount of effort" (NCTM, 2000, p. 52). The NCTM recommended that young children be encouraged to develop a wide range of "problem-solving strategies, to pose (formulate) challenging problems, and to learn to monitor and reflect on their own ideas" (2000, p. 116). The draft policy on Early Childhood Mathematics prepared by the AAMT and ECA (2005) similarly stated that early childhood educators should, "recognise, celebrate, and build upon mathematical learning that young children have developed and use the children's methods for solving mathematical problems … as the basis for future development" (p. 1). Likewise NCTM's, *Principles and Standards for School Mathematics* agree that "problem solving is an integral part of mathematics learning" (2000, p. 52).

A primary goal of mathematics teaching and learning is to develop the ability to solve a wide variety of complex mathematics problems. Traditionally, problem solving in mathematics education has been defined as getting from givens to goals when the path is not clear or obvious. For most problems that occur in the mathematics curriculum the problem solvers goal is merely to produce a brief answer to a question that was formulated by others. Subsequently, problem solving has been treated as an isolated topic completely separated from the learning of substantive mathematical concepts (Lesh & Zawojewski, in press). In our increasingly technology and information-based society, students need to develop abilities to function in a world that demands more flexible, creative, and future-orientated mathematical thinkers and problem solvers. Mathematical modelling is a powerful form of problem solving which incorporates activities that move beyond traditional views of problem solving.

A variety of mathematical models and modelling perspectives have been formulated and described in the literature (e.g., Greer, 1977). The definition adopted here is that models are "systems of elements, operations, relationships, and rules that can be used to describe, explain, or predict the behaviour of some other familiar system" (Doerr & English, 2003, p. 112). Mathematical modelling activities are thought-revealing activities that require the children themselves to develop mathematical interpretations of situations. That is, students "must mathematise situations" (Lesh, Hoover, Hole, Kelly, & Post, 2000, p. 594). Whilst working the modelling tasks children draw the mathematical ideas out of the problem situation using skills such as thinking, communicating, justifying, revising, refining, and predicting (English, 2003). A mathematical modelling perspective emphasises "students learning mathematics *through* problem solving and learning problem solving *through* creating mathematical models" (Lesh & Zawojewski, in press, p. 28). English and Lesh (2003) have emphasised that it is not just reaching the goal that is critical, but also the interpretation of the goal, the nature of the goal and the steps taken towards the solution.The modelling process is often seen as "building a link between (a) mathematics as a way of making sense of our physical and social world, and (b) mathematics as a set of abstract, formal structure"(Greer, 1997).

Modelling problems are complex situations where the problem solver engages in mathematical thinking beyond traditional school experiences where the products generated are needed for some purpose or to accomplish some goal (English, Lesh & Zawojewski, in press). Modelling activities are specifically designed for small group interactions, where students are required to develop shareable and reuseable products that involve descriptions, explanations, justifications, and mathematical representations. A typical modelling scenario would ask children to construct a model that will enable a client to make a decision about a real world situation. Modelling activities encourage children to develop and explore significant mathematical ideas, together with powerful communication and representational processes and skills.

Traditionally, mathematical modelling has been a topic designated for secondary classrooms. However, research conducted by English and Watters (2004) demonstrated that younger children can and should deal with mathematical modelling tasks. The mathematical modelling activities conducted in the primary school years by English (e.g., 2005) follow a standard format. Children are firstly presented with a scenario which describes a context, and poses readiness questions to ensure comprehension has occurred. During this session the children are presented with a problem relating to the story. Secondly, the children spend time in small groups working the problem and creating their mathematical model. Thirdly, the children present the models they have created to an audience, namely, their class peers. The presentation phase of the modelling activities provides opportunities for children to articulate and clarify their ideas, justify and argue their thinking and develop communication skills. The audience has the chance to critically question their peers developing further valuable life skills. Mathematical modelling activities are in-depth investigations that provide opportunities for children's deep and sustained interaction with key mathematical idea.

Research has concluded that young children are mathematically capable and early childhood programs and curricula now reflect these beliefs. An analysis of the features of mathematical modelling tasks; recommendations and policies of early childhood mathematics; and early childhood curriculum suggest that mathematical modelling could potentially provide powerful learning opportunities for young children. Links can be established between mathematical modelling and high quality early childhood mathematics. These include;

- Mathematics as problem solving
- The use of real world context
- Ascertaining children's knowledge base
- Social collaboration
- Development of communication skills

Mathematics as Problem Solving

The draft policy on early childhood mathematics produced by AAMT and ECA

(2005) suggested that young children's mathematical development be assessed "through means that are sensitive to the general development of the children, their mathematical development… and the nature of mathematics as an investigative, problem solving and sustained endeavour" (p. 2). Mathematical modelling tasks can help teachers recognise and reward students with a broad range of mathematical ideas, skills, and abilities. As stated in Principles and Standards for School Mathematics, "Learning with understanding is essential to enable students to solve the new kinds of problems they will inevitably face in the future" (NCTM, 2000, p. 21). In the early childhood environment, modelling tasks provide teachers with the opportunity to map their children's mathematical knowledge and plan future experiences.

Modelling tasks are meaningful problem-solving situations that allow for multiple solution approaches. The activities can be solved at different levels of sophistication which enables all children irrespective of their knowledge base to participate.

Real-World Context

In early childhood contexts, children's ability to 'mathematise' situations is highlighted as a desirable development. Mathematisation is the process "of generating mathematical problems, concepts, and ideas from real-world situations using mathematics to attempt a solution to the problem derived" (Perry & Dockett, 2002, p. 89). It has been recommended by NAEYC and NCTM (2000, p. 4) that mathematical activities should "enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds." Mathematical modelling activities are couched in authentic contexts that provide real situations in which children can develop their mathematical thinking. The modelling problems need to be interpreted by the children and described in mathematical ways. When tasks are based on children's own personal knowledge and experience, children have an opportunity to make sense of a situation and mathematise the activity. The use of real world contexts also assists the development of another recommendation made by NAEYC and NCTM, namely that is "enhance children's natural interest in mathematics and their disposition to use it to make sense of their physical and social worlds" (2002, p. 4).

Ascertaining Children's Knowledge Base

Young children are capable of dealing with great complexity in their mathematics learning and teachers need to ascertain what children know. Once teachers recognise what children know, and how they know this, they can better plan challenging experiences for the child (Perry & Dockett, 2002). The NAEYC and NCTM (2000) joint position statement recommends that mathematics experiences should "build on children's experience and knowledge, including family, linguistic, cultural, and community backgrounds; their individual approaches to learning; and their informal knowledge" (p. 4). Mathematical modelling activities provide teachers with the opportunity to gather useful information about their students' conceptual strengths and weaknesses. In early childhood settings, children have had no formal schooling but many have extensive and varied informal mathematics experiences. When teachers observe children working the mathematical modelling tasks opportunities arise to establish the nature and extent of young children's experience and mathematical knowledge base. This knowledge can help teacher's better plan effective and worthwhile learning experiences.

Social Collaboration

Mathematical modelling for children is inherently a social experience (English, 2004; Zawojewski, Lesh, & English, 2003). Vygotsky's theory recognises that learning is a process that occurs within social interactions emphasised by social collaboration and negotiated meanings (Klein, 2000). A social constructivism perspective which underpins many early childhood curricula documents (e.g., EYCG, 2005) is informed by Vygotsky's learning theories. Modelling activities are designed for small-group work in which children develop effective communication and teamwork skills whilst the teacher adopts a facilitator role. Interactions occur between the child and other children, child and teacher, and child and problem. Zawojewski, Lesh, & English (2003) also state that "peer interaction has the potential to amplify the interest and motivation of the children involved, increasing the potential mathematical power" (p. 343).

Development of Communication

The recommendation to "Encourage young children to justify their mathematical ideas through the communication of these ideas in ways devised by the children that display appropriate levels of mathematical rigour" was made by AAMT and ECA (2005, p.2). Features of mathematical modelling include the group development of the model and the group presentation of the mathematical models created by the children. As Zawojewski, Lesh, and English (2003) suggested the activities have embedded in them social dimensions which reflect teamwork and communication. Within the group children discuss, debate, refine ideas, listen and collaborate with their peers. When the group presents their final model to fellow classmates they communicate their mathematical ideas and understandings. Opportunities exist for critical questioning, justification, and debate to occur between the group and their audience. As children work interactively in solving the modelling problems they communicate their thinking and generate meaningful mathematical ideas and processes. Participating in challenging tasks contributes to the development of mathematical competence and confidence (Stipek, 1997).

Further Links Between Mathematical Modelling and Early Childhood

Mathematical modelling activities have other qualities that lend themselves to early childhood contexts. For example:

- Modelling tasks are developed around themes that interest children and are of importance to children. Activities stimulate children to investigate and explore the problem posed. Children will bring their own personal meaning to the problem and test and revise their interpretations.
- Modelling tasks are open-ended (Lesh, Hoover, Hole, Kelly, & Post. 2000). There is no predetermined right answer therefore nearly all students are successful at some level. Activities are structured to encourage children to build mathematical models based on what their group deems important which may be different to that of another group. All children have the potential which can (and should be) identified and promoted (Ball, 1994).
- Children express their models using external notation systems. The systems of representation can vary, for example, written symbols, verbal accounts, paper-based diagrams, or pictures. Children can choose the medium that they are most

comfortable with and can best describe their ideas. Children in early childhood settings may not have developed writing skills however they can still participate in the development of models.

- The teachers' role in modelling activities is to facilitate and support children's mathematical development — not directly teach it. Yackel (1995) suggested that teacher's roles include facilitating discourse among children while they engage in collaborative problem solving and to support children's developing understandings.
- Bredekamp and Copple (1997) suggested that the domains of children's development — physical, social, emotional, and cognitive — are closely connected. Development in one area influences development in other areas. Engagement in modelling activities can potentially assist the development of the whole child and contribute to the integrated nature of early childhood curricula.

For mathematical modelling activities to be successful learning experiences for young children in early childhood settings careful consideration needs to be given to their formation. The problem context will need to be meaningful and challenging to the children. Key mathematical ideas presented in the task must be relevant to the children's developmental levels. A variety or representation modes need to be available for the children to choose from. The medium that children use to present their models will depend on their abilities. For example, some groups may choose to draw a picture, make a 3D model, or provide a verbal account. Collaborating with class teachers in the development of modelling tasks will ensure that tasks best meet the needs, interests, and developmental levels of the children participating.

Research has shown that young children are capable of and interested in a variety of mathematical experiences that are complex and challenging. Recommendations and policies detailing high quality early childhood mathematics and curricula developed for the new millennium advocate children's engagement in meaningful experiences that develop powerful mathematical ideas. The ability to problem solve, reason and converse mathematically are critical in a future-orientated society. The impact that mathematical modelling can have on early mathematical development is significant. We need to provide young children with greater access to rich, challenging, mathematical experiences that will help them flourish in a global future-orientated world. Mathematical modelling experiences fit current early childhood learning perspectives and provide unbounded opportunities for children's mathematical growth. Future research studies investigating the development of a modelling program for young children are recommended.

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