A Model of Students' Statistical Thinking and Reasoning about Graphs in an ICT Environment

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This paper, reports on the development of a theoretical framework about statistical thinking and reasoning in relation to data analysis, graphing and graph-sense making. The model developed from a review of the literature is used to construct an assessment instrument that is designed to elicit student prior learning in relation to reasoning about data in an ICT environment. The design of the assessment instrument takes into consideration the different forms of data representations afforded by the graphing software, *TinkerPlots™ Dynamic Data Exploration*.

A research project investigating ways in which technology contributes to the development of statistical thinking and reasoning for students when using a data analysis program, *TinkerPlots™ Dynamic Data Exploration* (Konold & Miller, 2005), instigated the development of an assessment instrument to evaluate student prior learning. In the process of designing an assessment instrument that was informed by previous research and based on theoretical frameworks from the literature, it was determined that the existing models of student thinking in relation to data analysis, and in particular graphing, did not take into consideration the context of technology environments. To serve the purposes of the research project it was necessary to develop a model for graphing that is influenced heavily by the learning environment afforded by *TinkerPlots™*.

A further rationale for developing the assessment instrument is to strengthen the internal validity of the research project, of which this study is a part. Watson (2002) suggests that the purpose of conducting educational research is to discover "something new" and "something important." In her paper, Watson emphasises the importance of developing theoretical frameworks that are evidence-based, exploring the work of other researchers, building on established models of cognitive development, validating theoretical frameworks and evaluation instruments, and possibly most importantly, utilising the peer review process. In developing the framework and designing the assessment instrument the ideals proposed by Watson are considered.

*ICT and statistical literacy, reasoning and thinking.*Statistical literacy is the ability to evaluate critically data in a variety of contexts and communicate this understanding in a fashion that can impact on decision-making (Gal, 2004). It can motivate citizens to be critical thinkers and can assist in the building of scientific, technological, and mathematical innovation. These skills need to be developed at school level to promote the "innovation society," which was identified by the Committee for the Review of Teaching and Teacher Education (2003) as an essential outcome for education programs. Predominantly, in today's contemporary society, ICT are seen as the "innovations." They are, however, not only innovations in and of themselves but also potential inspirations for people to be innovative.

Skills associated with data analysis as well as the construction of and reading of graphs are highly valued in the curriculum, with the inclusion of these elements throughout all the years of compulsory schooling (Australian Education Council, 1994). The modelling of data through graph creation and subsequent interpretation has been a feature of statistics education in Australia and is often performed using ICT (Ben Zvi, 2004). The use of graphing software is extremely useful for these purposes as they enable students to move efficiently from tabular and graphical representations to the visualisation of data. They also allow students to design multiple graphical representations of the same data set quickly and more accurately than with pen and paper (McGehee & Griffiths, 2004).

Sivasubramaniam (2004) suggests that the software or computing device used to display graphs takes over some of the cognitive processes employed to create graphical representations, thereby allowing students to focus on the interpretation of the graphs. He maintains the view that computing devices such as a graphing calculator distributes the cognitive process of creating graphs differently than when producing graphs with pen and paper. Hoyles, Noss, and Kent (2004) warns of the complexity that technology brings to learning experiences and notes that it places many demands on students, both mathematical and technological. They also suggest, however, together with Goos, Galbraith, Renshaw, and Geiger (2003), that the use of ICT can transform student thinking, extending beyond the immediate concerns of the learning experience to an abstraction of the mathematical concepts with student understandings being shaped by the technology used.

TinkerPlots™. The research project utilises a recently released data-analysis software tool, *TinkerPlots™ Dynamic Data Exploration. TinkerPlots™* provides educators with a graphing program that is designed specifically for students in the middle years of schooling and gives students an intuitive, informal set of operators so that they can flexibly organise data to see patterns in them — explorations that they can perform without necessarily having to first learn to make and interpret a standard set of graph types (Konold & Miller, 2005).

The interface of *TinkerPlots™* includes a stacked data card system in the plot window for the organisation of case-based data. Information about the characteristics (variables) of an individual case is presented on a single card and there is one card for each case in a data set. Data entered into the data cards are automatically entered into a spreadsheet and graphical representations can be constructed from both the data cards and the spreadsheet. The program enables students to build meaning from the data by being able to "create both simple and complex graphs by performing actions such as sorting data into categories or ordering the information according to the values of one of the variables" (Hammerman & Rubin, 2002).

Konold and Higgins (2003) completed a literature review on how students reason about data. They report that students should begin with graphs in which they can retrace each individual data value and should also have the opportunity to construct graphs from data using "bottom-up software" in preference to using software that produces accepted graphical representations, such as histograms, at the click of a button. The dynamic nature of *TinkerPlots™* provides the facility for students to explore graphing in these ways. Additionally, the interface of *TinkerPlots™* allows students to create multiple graphical presentations, import digital images, display tables of results, and add written commentaries into one file as they conduct statistical investigations. Students can access all the different information on one screen and can potentially build an understanding of the relationship between the data and the graphical representations.

Development of a Model of Graphing in an ICT Environment

A comprehensive consideration of the relevant literature was undertaken to identify theoretical models of statistical thinking and reasoning that were directly related to data analysis and in particular, graphing. Models developed by Friel, Curcio and Bright (2001), Mooney (2002), and Moritz (2004) were considered. Suggestions by Shaughnessy (2006) for an additional level to be added to Friel, Curcio and Bright's levels of thinking and the notion of transnumeration presented by Pfannkuch and Wild (2004) provided an extensive view of the development of graphing and graph sense-making. It was established from the literature review that although each of the models has elements that were relevant to the purposes of this study, none of the models take into consideration the way in which students learn in technological environments and how these environments may impact on the construction of mathematical knowledge. The next section will summarise the existing models and address the shortcomings in relation to this study.

Existing models. The model presented by Moritz (2004, p.523) (Figure 1) is in relation to the translation processes involved with reasoning about covariation. The arrows on the model indicate processes of translating among numerical data, graphs, and verbal statements. The translations are *graph production*, *graph interpretation* and *speculative data generation*. This model aligns closely with the learning opportunities afforded by *TinkerPlots™*. Although it was designed specifically for the statistical concept of covariation, the cyclic nature of the model whereby the learning experience can be accessed from multiple entry points is in keeping with the learning environment offered by *TinkerPlots™*. The model does not, however, incorporate elements associated with summarising data other than displaying the data in graphs.

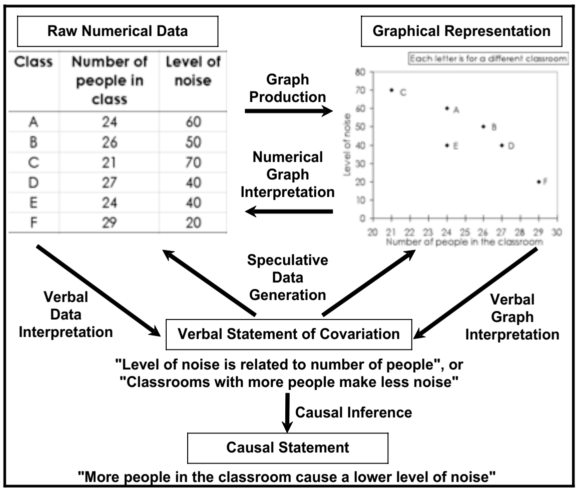

Figure 1. Translating processes involved with reasoning about covariation (Moritz, 2004, p.523).

Friel et al. (2001) identify six behaviours associated with graph reading and comprehension. Shaughnessy (2006) suggests, "each of the six behaviours seems to fit nicely with one of Curcio's three levels of graph reading (*read the data, read within the data, read beyond the data*)" (p. 84).He goes on to suggest two additional behaviours that would fall under the level of *reading behind the data*. When considered together, the additional level suggested by Shaughnessy gives the Friel et al. model greater depth in terms of the way students may develop graph sense. The levels do not, however, recognise the impact that constructing graphs may have on the development of statistical thinking and reasoning. The cognitive model provided by Mooney (2002) is similar to Friel et al.'s when considered in conjunction with the additional level suggested by Shaughnessy (2006), but it is presented as a hierarchy that does not take into account the thinking processes associated with reasoning about data or constructing graphs.

Pfannkuch and Wild (2004) present an extensive four-dimensional model of statistical thinking. The model is related to the way statisticians work but Dimension 2: Types of Thinking is useful when considering the way in which students use data. The section relevant to students is transnumeration.Pfannkuch and Wild describe transnumeration as changing data representations to engender understanding and communicating messages in data. *TinkerPlots*^{TM} provides students with the opportunity to construct and reconstruct their own meaningful data representations in ways that may promote transnumeration.

Whilst preferring to build on previous research without modification, none of the models presented in this section are suitable when conducting research into the ways in which students develop statistical thinking and reasoning within ICT environments. For the purposes of the research project, there is the need to develop a model of graphing in an ICT environment that takes into account the special features of *TinkerPlots*^{TM}. Relevant elements from each of the existing theoretical frameworks cited are incorporated into the new model.

Model of graphing in an ICT environment. The following theoretical framework was developed by considering the models of statistical thinking and reasoning in relation to graphing and graph sense-making presented in the previous section. The key elements extracted from each of the models are grouped into four interconnected categories: *Being creative with data*, *Understanding data*, *Thinking about data, and Generic Knowledge*. An important feature of the framework is that it recognises that there are some generic understandings inherent in all levels of data analysis, graphing and graph sense-making. This is important when considering technology environments.*TinkerPlots*^{TM}*,* in particular, provides the opportunity to engage in data analysis from different entry points, as represented in the Moritz (2004) model of reasoning about data (Figure 1). The theoretical framework developed is detailed in Table 1.

Table 1: Model of Graphing in an ICT Environment

<table>
<tr><th>Category</th><th colspan="3">Generic Knowledge</th><th>Descriptors of Category</th></tr>
<tr><td>Being creative with data</td><td rowspan="3">Speaking the language of graphs</td><td rowspan="3">Recognising the components ofdata and graphs</td><td rowspan="3">Understanding how to use the features of software and technology environments</td><td>Reducing data to graphical representations or statistical summaries.
Constructing different forms of graphs.
Translating verbal statements into graphs.</td></tr>
<tr><td>Understanding data</td><td>Making sense of data and graphs.
Understanding the relationship among tables, graphs, and data.
Identifying the messages from the data.
Answering questions about the data.
Recognising appropriate use of different forms of graphs.
Describing data from graphs.</td></tr>
<tr><td>Thinking about data</td><td>Asking questions about the data.
Recognising the limitations of the data.
Interpreting data and making causal inferences based on the data.
Looking for possible causes of variation.
Looking for relationships among variables in the data.</td></tr>
</table>

Development of the Assessment Instrument

In the case of statistics education, there is the need to conduct research that may inform pedagogical practice in relation to the ways in which students' reasoning about data analysis develops (Ben Zvi, 2004). As part of the research agenda, assessment instruments and materials, which evaluate statistical thinking and reasoning in technology environments need to be developed (Garfield & Ben-Zvi, 2004; Shaughnessy, 2006). It is, therefore, pertinent at this time to ensure the features of software available for use in statistics education and the way students access and engage in the learning experience are considered when designing these assessment instruments. In designing an assessment instrument to evaluate students' statistical thinking and reasoning it is necessary to: (a) provide opportunities for students to demonstrate an understanding of statistical concepts at varying levels, (b) consider the different data representations afforded by learning experiences in pen-and-paper and/or technology-rich environments, and (c) base the selection of items for the instrument on theoretical frameworks developed from research conducted in statistics education.

When designing teaching interventions or learning experiences utilising ICT, teachers and researchers alike are faced with the challenge of determining prior learning. Considering student learning gained from traditional pen-and-paper experiences as well as considering the possibility that students may have previously used technology in these contexts should enable both teachers and researchers to more validly determine and declare student learning outcomes.

A further challenge arises as the assessment instrument is to be administered away from the computer environment. This is necessary as the aim of using the assessment instrument is to determine students' learning in terms of their ability to reason about data. Should the assessment instrument be administered as an exercise on the computer, there is the concern that the emphasis will be on the students' ability to access the features of computer programs instead of the mathematics. Also, administering the assessment instrument on the computer may disadvantage students who are inexperienced in using graphing software. Whilst the intention is to administer the assessment instrument as a pen-and-paper activity, particular attention was given to selecting items with formats that remain authentic to the *TinkerPlots*^{TM} environment.

The assessment instrument. The selection of the items for the assessment instrument was based on the theoretical framework developed in the previous section. Where possible, items that had demonstrated in previous research the ability to elicit responses from students at varying levels were selected. The formats of the items are also characteristic of data representations available in *TinkerPlots*^{TM}. The items for the assessment instrument are detailed below (Table 2) and where relevant the sources of the items are noted.

Table 2. Assessment Instrument Items

Item no.	Item description
1	What is a graph? What are graphs used for? Where have you seen graphs used?
2	Draw an example of a graph. Any type will do. Put as much detail on the graph as possible. What does the graph show?
3	Have you used the computer to draw graphs before? What programs did you use? Describe what sort of graphs you drew. What were the graphs used to show?

Table 2 cont. Assessment Instrument Items

Item no.	Item description

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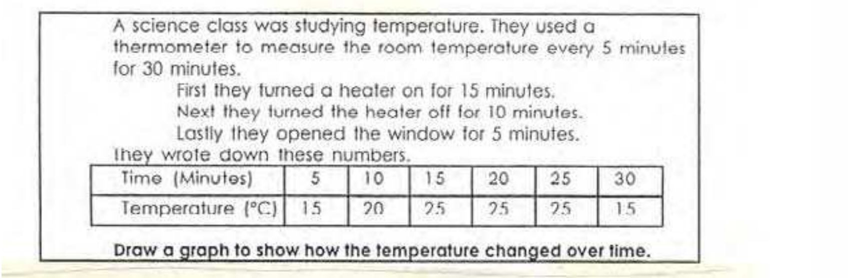

A science class was studying temperature. They used a thermometer to measure the room temperature every 5 minutes for 30 minutes.
First they turned a heater on for 15 minutes.
Next they turned the heater off for 10 minutes.
Lastly they opened the window for 5 minutes.
They wrote down these numbers.

Time (Minutes)	5	10	15	20	25	30
Temperature (°C)	15	20	25	25	25	15

Draw a graph to show how the temperature changed over time.

Moritz (2003b, p.234).

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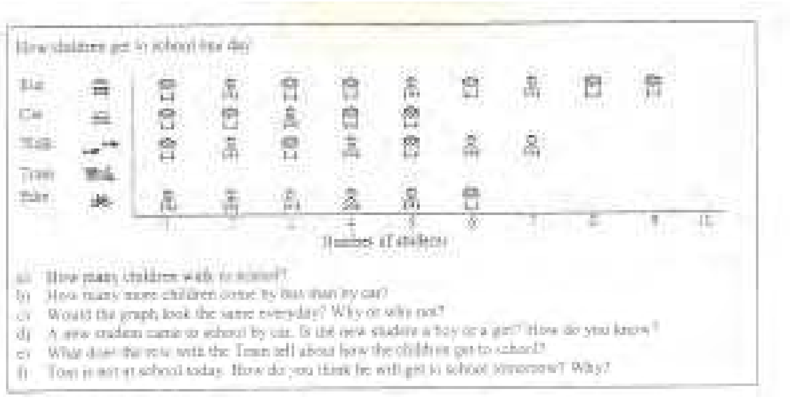


Watson & Kelly (2003, p.722).

6 The information about individual students is on separate cards, like the ones below. What questions about the group of students could be answered by using the information on the cards?

Student 1	Student 2	Student 3
GenderMale	**Gender**Male	**Gender**Female
Height 1.3m	**Height** 1.3m	**Height** 1.2m
Eye Colour Blue	**Eye Colour** Brown	**Eye Colour** Brown
Hair Colour Blonde	**Hair Colour**Brown	**Hair Colour**Brown
Right/Left	**Right/Left**	**Right/Left**
Handed Left	**Handed** Right	**Handed** Right
Shoe Size6	**Shoe Size**7	**Shoe Size**5
Hobby YuGiOh	**Hobby** Cricket	**Hobby** Soccer
Pet Budgie	**Pet** Dog	**Pet** Dog
Brothers0	**Brothers**0	**Brothers**2
Sisters 1	**Sisters** 0	**Sisters** 1

Adapted from Chick & Watson (2001).

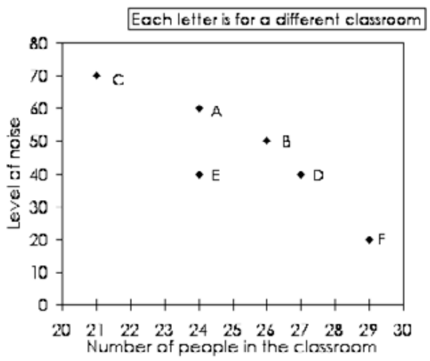
7 Some students were doing a project on noise.
They visited 6 different classrooms. They measured the level of noise in the class with a sound meter.
They counted the number of people in the class.
They used the numbers to draw this graph.

Q1. **Pretend you are talking to someone who cannot see the graph. Write a sentence to tell them what the graph shows. "The graph shows…**

Q2. **How many people are in Class D?**

Q3. **If the students went to another class with 23 people, how much noise do you think they would measure? (Even if you are not sure, please estimate or guess.)**
Please explain your answer.

Q4. **Jill said, "The graph shows that classrooms with more people make less noise".**
Do you think the graph is a good reason to say this?θ YES or θ NO
Please explain your answer.



Moritz (2003, p.525)

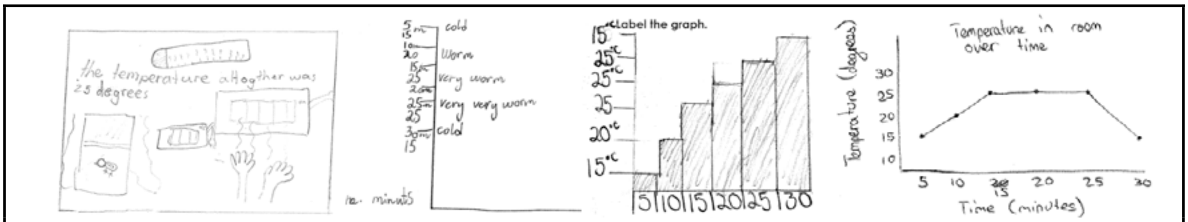
Discussion and Conclusion

An objective of developing a theoretical framework to guide the construction of an assessment instrument is to increase the internal validity of the research for which the assessment instrument was designed. This was achieved by addressing the shortcomings of existing models of statistical thinking and reasoning in relation to data analysis, graphing, and graph sense-making. Additionally, it was important to ensure that each of the key elements in the theoretical framework organisers (Table 1) can be addressed in multiple items of the assessment instrument (Table 2). Table 3 demonstrates the clear connections between the theoretical framework and the assessment instrument items.

Table 3. Summary of Model of Graphing in an ICT environment and Assessment Instrument Items

Category	Item no.						
	1	2	3	4	5	6	7
Being creative with data		✓		✓			
Understanding data	✓	✓	✓	✓	✓	✓	✓
Thinking about data				✓	✓		✓
Generic knowledge	✓	✓	✓	✓	✓	✓	✓

Many of the items in the assessment instrument were drawn from previous research in statistics education. In order to provide students with the opportunity to demonstrate an understanding of statistical concepts at varying levels, only items from the literature that reported specific examples of student learning were considered. The research reports detailing Items 4, 5, & 7 provide examples of student responses to the items. As an example, Figure 2 shows some student responses to Item 4. Item 6 in the assessment instrument provides information in data cards. This data representation along with data in spreadsheets (tabular form) and graphical representations are available in *TinkerPlots*^{TM}. Facets of all these representations are included in the assessment instrument.



Non-statistical　Single aspect　Inadequate coordinate　Appropriate coordinate

Figure 2. Examples of Student Responses to Item 4. (Moritz, 2003b, pp. 240-244).

Validating the instrument. The next step in the research process is to validate the assessment instrument and make modifications if necessary. The assessment instrument is designed with the specific features of *TinkerPlots*^{TM} in mind, however, it is hoped that the instrument will be considered for use outside the confines of the research project for which it is developed.

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