Many Dimensions: the Complex Picture of Student Encounters with a Computer Algebra System

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We studied the complex situation of first year university students using computer algebra systems (CAS) for the first time as part of their mathematics subjects. We identified four components of initial experiences with CAS. Existing questionnaires were used to identify two subgroups of students with contrasting approaches to study. When these subgroups were further split by computing experience, their scores on the four components of initial experience with CAS revealed a complex picture that was understandable from an activity theory perspective.

The significance of this issue lies with the increasing role that sophisticated software packages such as CAS are playing in the professional practice of mathematicians, statisticians, engineers and scientists. In our view, successful professional practice that makes a creative contribution requires a view of mathematics that seamlessly incorporates tool use to explore new ideas and to solve authentic problems. In universities, unfettered by state wide syllabuses, there is much interest in adapting mathematics courses to take advantage of the opportunities afforded by CAS for exploration and discovery by students, and for a more visual and experimental introduction to difficult concepts. Methods and procedures for evaluating these innovations are required.

It is our view that an activity theory approach (Vygotsky, 1986; Engeström, 1987, 1999(a) 1999(b); Engeström & Miettinen 1999) which incorporates personal histories, motivations and goals, social contexts, and tools, provides a successful framework for investigating learning as a socio-cultural activity. In the first year of university study students grapple with many changes, and in their mathematical studies they need to form new goals in the context of assessment regimes and social contexts that are quite different from school, although they will bring with them attitudes towards mathematics formed by many school years of pen and paper "drill and practice" (Crawford, Gordon, Nicholas, & Prosser, 1994). Some of their university experiences reinforce the need for learning pen and paper skills. We wanted to know how students with different approaches to study adapted to learning with, and learning about, new software tools. The research reported here is part of a larger survey and interview study carried out at a metropolitan university in Australia. (Coupland, 2004). A report of preliminary results was given in Coupland, 2000.

Background

Much of the research in this area at both upper high school and university level can be categorised as (1) descriptive reports of innovations (2) experimental and quasi-experimental studies, focussing on student achievement in constructivist terms as the main outcome, and (3) reports that classify student responses and use observations to build theory about student engagement and the roles of teachers and assessment. Selected comments about each category will be given here, and a broader review is available in Coupland (2004).

Innovations

Zand and Crowe (2001) provide an overview of university adoption of technology in mathematics teaching in Australia, USA and the UK. They point out that local innovators are usually individuals or groups who launch the use of CAS in their own teaching, and may then obtain funding for evaluation and further development. They note that there is no common agreement about the ways mathematics curriculum should be changed in the light of the new technology, and are disappointed by the lack of consensus "about the educational merits of using technology in teaching and learning mathematics". (p. 81). On the other hand, according to André Heck (2001), the pioneering work on innovations around the introduction of CAS in schools has resulted in a consensus on the most important advantage of using computer algebra in mathematics education:

> Computer algebra has the potential of making mathematics more enjoyable for both teachers and pupils because it turns mathematical entities into concrete objects, which can be directly investigated, validated, manipulated, illustrated, and otherwise explored. … abstraction, exact reasoning, and careful use of symbolism are not solely a hobby of the mathematics teacher anymore, but are immediately rewarded when using computer algebra (p. 210).

This statement signals a shift in the ownership of the process of authentication that *can* happen: not through the use of technology alone, but through the structure of learning tasks and the framing in the social context of the classroom of the activities that students undertake.

Quasi-Experimental Studies

One of the most frequently cited comparison reports is by Kathleen Heid (1988). In a college in the USA, Heid emphasised concepts and applications in her experimental classes in introductory calculus, with computers used for computing derivatives and graphing, until the final three weeks of a 15-week course. During those last three weeks the students learned the pen and paper algorithms that a comparison class, taught by a different instructor, had been learning all semester. Among the conclusions were these:

> As a group, the students in the experimental classes were better able than the students in the comparison class to answer conceptually oriented questions — an indication of a more refined ability to translate a mathematical concept from one representation to another. They performed almost as well on the final examination as the comparison class. Their performance was remarkably suggestive that compressed and minimal attention to skill development was not necessarily harmful, even on a skills test. (Heid, 1988, p. 21)

Kenneth Ruthven (2002) has challenged the contrast of concepts and techniques that Heid emphasised. He points out that the students in Heid's experimental group *were* engaged in practising techniques, but of a different kind from the conventional class. They worked on application problems and had more small group discussions. He claims that, when viewed from the French theory developed by (among others), Artigue (2001, 2002), and Lagrange (e.g., 1999a, 1999b) in which techniques have a broader scope than routine algorithms, the conceptual development of Heid's experimental group "grew out of new techniques constituted in response to this broader range of tasks, and from greater opportunities for the theoretical elaboration of these technique" (Ruthven, 2002, p. 284). This resonates with the findings of Kendal and Stacey (2001) who investigated the role of the teacher in influencing the use that students made of computer algebra systems and consequently the mix of conceptual and procedural knowledge acquired by students. They used Wertsch's notion of "privileging" to describe a teacher's individual way of teaching, including decisions about what is taught and how it is taught. Two teachers of parallel year 11 classes in introductory calculus were observed and interviewed, and their classes also took tests designed to assess competencies in numerical, graphical and symbolic aspects of differentiation. "Students of the teacher who privileged conceptual understanding and student construction of meaning were more able to interpret derivatives. Students of the teacher who privileged performance of routines made better use of the CAS for solving routine problems" (Kendal & Stacey, 2001, p. 143).

Relevant Theory Building about Student Engagement within the CAS Context

Research in French secondary schools by Guin and Trouche (e.g., Guin & Trouche, 1998) emphasises the distinction between a *tool* and an *instrument*, the latter being a psychological construct achieved when a person has obtained sufficient knowledge of the potentialities and constraints of an artefact in order to use the tool effectively in the process of an activity: in effect the person has appropriated the tool. They show how a reorganisation of classroom dynamics can help students to make the necessary integrations to achieve what they call an *instrumental genesis* with a CAS. They aim…

> … to foster experimental work (investigation and anticipation) with interactions between graphic observations and theoretical calculus, and to encourage students to compare various results of different registers… This reflection is needed in order to seek mathematical consistency in various results and will motivate students to improve the mathematical knowledge required to overcome these contradictions (such as the distinction between approximate and exact calculation, control of numerical approximations, reflection on the unavoidable discretization of the screen and the nature of representatives and calculation algorithms). (Guin & Trouche, 1998, p. 208)

In these rearranged classrooms, students worked in pairs or groups of three on problems specially chosen to provide challenges and to bring out difficulties in CAS representations. Later the teacher orchestrated a class analysis of the problem. "The teacher's role was to compare different strategies, pointing out the contribution of each group, and suggesting questions designed to make students discuss the various results found" (Guin & Trouche, 1998, p. 211).

As mentioned above in Ruthven's comments on Heid's early research, Lagrange (1999a) makes the point that conceptual reflection on techniques, not on tasks, is necessary for concept building, and that without the step of reflection, students know that their own understanding has not been enhanced: "so it appeared that many students did not consider problem solving using computer algebra as a convincing support of their understanding of mathematics, even when they liked it. They felt that their understanding developed from the techniques that they built in the ordinary context, and solving problem with CAS seemed to them very apart from these techniques." (Lagrange, 1999a, p. 6) Lagrange proposes that the use of a CAS needs to be taught with an emphasis on its own techniques, to foster student reflection.

A report by Galbraith, Pemberton, and Cretchley (2001) describes the learning experiences of students from the University of Queensland, (UQ), and the University of Southern Queensland, (USQ). The report compares results obtained from using the Galbraith and Haines scales (Galbraith & Haines, 2000), (at UQ) with results from a different set of questions designed to measure Mathematics Confidence, Computer Confidence, Math-Tech Attitudes (attitudes to technology use in the learning of mathematics), and Math-Tech Experience (views on experience with software in learning mathematics), at USQ. Observations included a very weak correlation between

Mathematics Confidence and Computer Confidence, and Math-Tech attitude and Math-Tech Experience correlating more highly with Computer Confidence than with Mathematics Confidence. The pattern of pre-post scores was quite different between the two institutions, with students at UQ reporting a decline in both Mathematics and Computer Confidence and Motivation that was not evident at USQ. In attempting to explain these results, the descriptions of what the students actually did, and how the assessment experiences were structured, became essential:

> For the UQ students the Maple environment was an effective gatekeeper to success in mathematics because of the central role it played in the program. Feelings about computing would likely be integrated with concern with success, even among the supremely competent, and it is most unlikely that such high stakes featured in their earlier computer experiences. … For the USQ students MATLAB was provided as a support, indeed a powerful support but not a gatekeeper to success because of the continuing priority accorded parallel approaches such as hand calculations. This meant that the computer power on offer had an element of choice, with students able to access it as they saw the opportunity and value in doing so. The students were in control. (Galbraith, Pemberton & Cretchley, 2001, p. 239)

Here again we see the importance of context, both the assessment context, and the positioning of the CAS. In the different contexts, the CAS was seen by the students as a high-stakes hurdle on the one hand (UQ), and as a support in the development of pen and paper skills, which the students still saw as "real" mathematics, on the other hand (USQ).

A summing up would emphasise the complexity of this picture of many dimensions: students' past experiences, teacher decisions, assessment requirements, classroom dynamics, hardware issues; all influence the nature of the learning environment.

The University Context for Research into Learning Mathematics

While constructivism has been a major influence in research into school mathematics education, research into student learning at university in many subjects, including mathematics, has also been strongly influenced by phenomenography (e.g., Marton, 1981; Marton & Booth, 1997). As described by John Biggs (1999, p. 12, original emphasis), a common area in these two perspectives is that "meaning is not imposed or transmitted by direct instruction, but is created by the students' *learning activities,* their 'approaches to learning'." Two kinds of approaches to learning, the deep approach and the surface approach, are not fixed characteristics of individual students, but are better regarded as an "*interaction* between the personal and the contextual". (Biggs, 1999, p. 17, original emphasis.) In activity theory, the way that a person purposefully forms and re-forms goals in response to felt needs arising from the demands of an activity, is another way to describe this interaction. Over time, some students reveal preferences for certain kinds of learning activities in certain situations, and this can be assessed by questionnaires, for example the Approaches to Study Questionnaire used in the work of Crawford and others, (Crawford, Gordon, Nicholas, & Prosser, 1998), which was in turn based on the Study Process Questionnaire of Biggs (Biggs, 1987).

A large scale study of first year mathematics students in Sydney (Crawford et al, 1998) found that students with a surface approach to study were more inclined to have a fragmented view of mathematics (agreeing with statements such as "Mathematics is a lot of rules and equations" in a Conceptions of Mathematics Questionnaire — CMQ), while students with a deep approach to study were more inclined to have a cohesive view of mathematics (agreeing with statements such as "Mathematics is a set of logical systems which have been developed to explain the world and relationships in it."). The latter group were more engaged with their studies and achieve at a higher level.

Research Design and Rationale

In the studies of CAS innovations outlined earlier in this paper, the voices of the students themselves are seldom heard. We wanted to ensure that we captured students' experiences with a CAS (*Mathematica*) by using a survey in which items were based on responses to questions asked of previous semesters' classes. A questionnaire was constructed this way, with many of the 56 items phrased just as students had written their responses to the open-ended question "Overall, how would you describe your experiences with *Mathematica*?". Two established questionnaires were also included in the survey: the Conceptions of Mathematics Questionnaire and a Study Process Questionnaire as used by Crawford et al and described above. Biographical data was also collected, along with a self-report of personal computing background. Open-ended questions were used to collect qualitative descriptions of student preferences for learning activities and opinions about the CAS experience.

The Students, Contexts for the Collection of Data, and Research Questions

The participants were first year students in three different subjects with different kinds of scheduled computer lab experiences and assessments, although the mathematical content of the subjects was similar and typical of first year calculus courses. The students were enrolled in various courses in Science, Mathematics, and Engineering. For the majority of participants, their competence with using the CAS *Mathematica* in problem solving was assessed by a demanding group assignment. Further details are given in Coupland (2000 & 2004). Participation was voluntary and the students completed the survey after one semester's experience with the CAS. Forms from 110 students were analysed.

Here we report on these questions that were part of the larger study (Coupland, 2004):

- How do students respond to their experience with a CAS as part of their mathematics subjects?
- What relationships exist between students' personal histories, their levels of engagement in mathematical learning, and the range of experiences they report concerning using a CAS for the first time?

Analysis and Results

From the *Mathematica Experience Questionnaire*, 29 items were judged appropriate for a principal component analysis that yielded four components, with further details given in Coupland (2004). The components, with their leading items, were

1. I gained a useful tool (I have used *Mathematica* to investigate mathematical questions of my own that arose in other subjects.)
2. It was worth the time spent on it (*Mathematica* takes so long to learn that it is not really helpful. REVERSED)
3. I found it easy and fun (I found that *Mathematica* was easy to use.)
4. I enjoyed learning with others (because of *Mathematica*, I had interesting conversations with others about mathematics.)

The scores on the subscales of the Conceptions of Mathematics Questionnaire were similar to those in the literature. Cohesive conceptions of mathematics were positively correlated with deep approaches to study and negatively correlated with surface approaches to study; while fragmented conceptions of mathematics were positively correlated with surface approaches to study and negatively correlated with deep approaches to study.

Our analysis now turns to the way that these questionnaire results help to illuminate the interactions between personal backgrounds of the students, their levels of engagement in mathematical learning, and aspects of their experience with the CAS.

From the scores on the surface approach and deep approach scales, we located those students below the mean on one of these aspects and above the mean on the other. These two groups of students we called “High Surface Low Deep” and “High Deep Low Surface”. Using the students’ self-report of their computing experience we then divided these groups again into those with High computing experience and those with Low computing experience, and tracked the scores of these four subgroups on each of the four components of CAS experience described above. The results are shown in Figure 1.

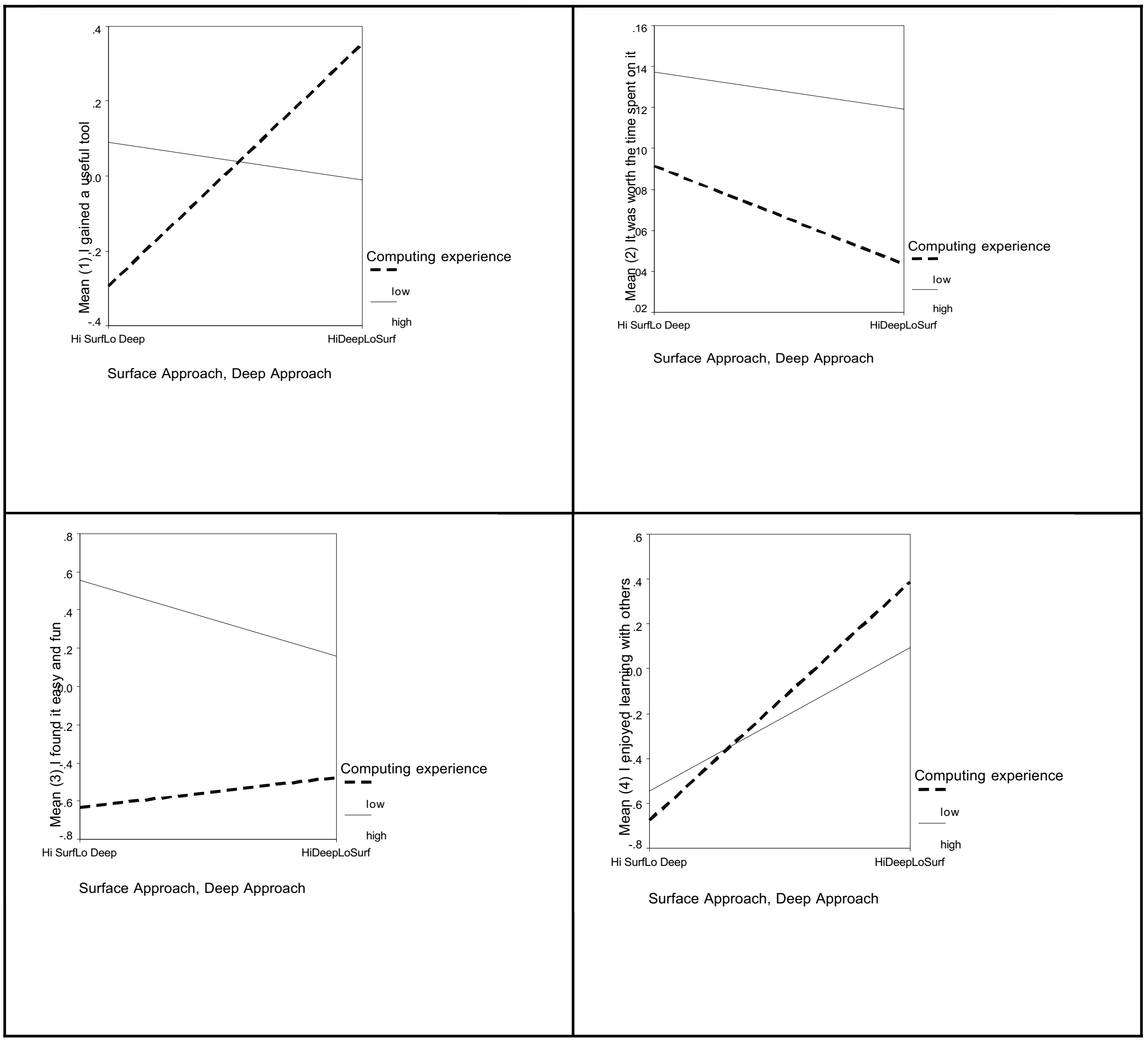

Figure 1. Standardised mean scores on four components of initial CAS experience, for groups with different levels of engagement and different computing backgrounds.

On the first component, (I gained a useful tool), "High Deep Low Surface" students, that is highly engaged students, were more likely to overcome any disadvantage due to low computing backgrounds than "High Surface Low Deep" students. Many of these students were motivated by the assessment demands of a difficult group assignment, as judged by their comments on open ended questions about their CAS experiences. This trend went the opposite way, and was not as pronounced, for students with high computing backgrounds.

Students with high computing backgrounds were more likely to agree that it was worth spending time on CAS activities than were students with low computing backgrounds.

On the third component, (I found it easy and fun), students with high computing background scored more highly than students with low computing background, for both "High Deep Low Surface" students and for "High Surface Low Deep" students.

On the final component, (I enjoyed learning with others), the more engaged students scored more highly regardless of computing background.

It would appear that our second and third components reflect the Galbraith et al findings reported above that "… attitudes to mathematics and to computers occupy different dimensions" (Galbraith et al, 2001, p. 239). Our first component is closer to revealing the extent to which instrumental genesis has been achieved, and we note the importance here of student engagement and their opinions of assessment demands.

Interpretation and Conclusions

We interpret these results as indications of the importance of fostering engagement in learning, which activity theory predicts will only occur when students have a purpose for that engagement. Appropriating the tool for one's own use is not automatically done just because one finds the tool easy to use. Appropriation occurs for a purpose. That purpose might be to use the CAS to avoid tedious calculations, or to learn to use the tool if it is judged to be useful to one's future career goals, or if it is essential for assessment.

In activity theory the construction of the internal plane *through* external activity embodies the idea that our conceptual knowledge inescapably carries with it the flavour of the activities that were its source. Vygotsky's sentence "the central moment in concept formation, and its generative cause, is a specific use of words as functional tools" (Vygotsky, 1986, p.107) could, we believe, be usefully adapted to include the specific use of symbols, diagrams, algorithms, and software, as functional tools.

Personal socio-cultural histories *and* motivations *and* social contexts influence students as they form and reform their goals for engaging in learning activities. The educational significance of the study is that it highlights the importance of context when students form their goals. A major part of the context of university study is assessment. When the solitary performance of pen and paper algorithms dominates assessment, students will have little time and purpose for learning to use a complex new computer tool for a demanding and creative group assignment. We pose these questions: first, which of these kinds of activity is more valuable for the mathematical futures of our students? Second: if both are valuable, how do we achieve the most fruitful balance?

References

Artigue, M. (2001). The teaching and learning of mathematics a the university level: Critical questions for contemporary research in education. Notices of the American Mathematical Society, 46(11), 1377-1385.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. International Journal of Computers for Mathematical Learning, 7, 245-274.

Biggs, J. (1987). *Student approaches to learning and studying*. Melbourne: ACER.

Biggs, J. (1999). *Teaching for quality learning at university.* Buckingham, U.K.: Society for Research into Higher Education & Open University Press.

Coupland, M. (2000). First experiences with a computer algebra system. In J. Bana (Ed.), *Mathematics education beyond 2000* (Preceedings of the 23rd annual conference of the Mathematics Education Research Group of Australasia, Fremantle, pp. 204 – 211). Sydney: MERGA.

Coupland, M. (2004). Learning with new tools. Unpublished PhD thesis, University of Wollongong, Wollongong, Australia. Available at http://www-library.uow.edu.au/theses/

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1994). Conceptions of mathematics and how it is learned: The perspectives of students entering university. *Learning and Instruction, 4*, 331-345.

Crawford, K., Gordon, S., Nicholas, J., & Prosser, M. (1998). Qualitatively different experiences of learning mathematics at university. *Learning and Instruction, 8*(5), 455-468.

Engeström, Y. (1987). *Learning by expanding: An activity-theoretical approach to developmental research*. Helsinki, Finland: Orienta-Konsultit Oy.

Engeström, Y. (1999a). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen, & R. Punamaki (Eds.), *Perspectives on activity theory* (pp. 19-38). Cambridge, U.K.: Cambridge University Press.

Engeström, Y. (1999b). Expansive visibilization of work: An activity-theoretical perspective. *Computer Supported Cooperative Work, 8*, 63-93.

Engeström, Y., & Miettinen, R. (1999). Introduction. In Y. Engeström, R. Miettinen, & R. Punamaki (Eds.), *Perspectives on activity theory* (pp. 1-16). Cambridge, U.K.: Cambridge University Press.

Galbraith, P., & Haines, C. (2000). *Mathematics - computing attitude scales* (Monographs in Continuing Education). London: City University.

Galbraith, P., Pemberton M., & Cretchley, P. (2001). Computers, mathematics, and undergraduates: What is going on? In J. Bobis, B. Perry, & M. Mitchelmore (Eds.), *Numeracy and beyond* (Proceedings of the 24th annual conference of the Mathematics Education Research Group of Australasia, Sydney, pp. 233-240). Sydney: MERGA.

Guin, D., & Trouche, L. (1998). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning*, *3*(3), 195-227.

Heck, A. (2001). Variables in computer algebra, mathematics and science. *The International Journal of Computer Algebra in Mathematics Education, 8*(3), 195-221.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education*, *19* (3), 3-25.

Kendal, M., & Stacey, K. (2001). The impact of teacher privileging on learning differentiation with technology. *International Journal of Computers for Mathematical Learning, 6* (2), 143-165.

Lagrange, J. (1999a). *A didactic approach of the use of Computer Algebra Systems to learn mathematics*. In P. Kent, J. Monaghan, & N. Zehavi (Eds.), Papers of the CAME (Computer Algebra in Mathematics Education) meeting at the Weizmann Institute of Science, Israel, (pp. 4-15). Retrieved May 19, 2001, from http://metric.ma.ic.ac.uk/came/events/weizmann

Lagrange, J. (1999b). Complex calculators in the classroom: Theoretical and practical reflections on teaching pre-calculus. *International Journal of Computers for Mathematical Learning, 4* (1), 51-81.

Marton, F. (1981). Phenomenography: Describing conceptions of the world around us. *Instructional Science*, *10*, 177-200.

Marton, F., & Booth, S. (1997). *Learning and awareness*. Mahwah, NJ: Erlbaum.

Ruthven, K. (2002). Instrumenting mathematical activity: reflections on key studies of the educational use of computer algebra systems. *International Journal of Computers for Mathematical Learning, 7*, 275-291.

Vygotsky, L. S. (1986). *Thought and language.* Cambridge, MA: M. I. T. Press.

Zand, H., & Crowe, W. (2001). *Aspects of the influence of technology on teaching mathematics in the last decades.* In W. Yang, S. Chu, Z. Karian, & G. Fitz-Gerald, (Eds.), Proceedings of the 6th Asian Technology Conference in mathematics, Melbourne (pp. 77-86). Blacksburg, VA: ATCM.