Exploring Critical Thinking in a Mathematics Problem-Based Learning Classroom

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In this study, we explored the critical thinking of 47 eleventh-grade students in a mathematics problem-based learning (PBL) classroom in November 2016. A critical thinking test was used along with classroom observations to gather the critical thinking data in five dimensions according to the Association of American Colleges & Universities (2009). The findings indicate that students' critical thinking scores in all dimensions are at an average level. The students demonstrated strength in explaining issues and analyzing influence of context and assumptions. However, students had greater difficulty in stating their positions and drawing conclusions.

Introduction

Efforts to develop thinking skills have become essential goals in mathematics classrooms (Hurst & Hurrell, 2016). Thinking in mathematics can be referred as an important "process" to foster students' mathematical problem solving (Hwa & Stephens, 2011). In particular, critical thinking is claimed to be the most important skill for problem solving, research, and discovery (Thompson, 2011) as it encourages students to think independently and solve problems in school or in the context of everyday life (Jacob, 2012). The National Council of Teachers of Mathematics (NCTM, 2000) also mentioned that the development of critical thinking generates improvement of mathematics achievement. Thus, critical thinking has become the main agenda of worldwide mathematics education.

The first question aimed toward understanding critical thinking is "What is critical thinking?" The answer is not simple since critical thinking is a complex phenomenon. Critical thinking is viewed from several distinct perspectives and thus is referred to by different definitions (Ennis, 2003; Facione, 1990; Halpern, 2006). Despite differences among these perspectives, Lai (2011) noted that definitions of critical thinking overlapped on several specific abilities, including (1) analyzing arguments, claims, or evidence, (2) making inferences using inductive or deductive reasoning, (3) making decisions or solving problems, and (4) judging or evaluating. In this study, critical thinking is defined as an ability to think objectively in order to make a decision. This definition emphasizes skills in five dimensions adapted from Association of American Colleges and Universities (AACU, 2009), consisted of (1) explanation of issues, (2) evidence, (3) conclusions and related outcomes, (4) influence of context and assumptions, and (5) student's position. The critical thinkers, therefore, should be able to comprehensively explain given issues or problems, thoughtfully select and use evidence, inquire about possible outcomes and relate them to each other in a conclusion, as well as analyze contexts, situations and others' assumptions to synthesize them to make their own positions.

Despite the importance of critical thinking, most of the teaching and learning process in school is the traditional lecture method, which is based on memorization, leading students to think less critically (Cobb, Wood, Yackel, & McNeal, 1992). The negligence of the importance of thinking skills also occurs in mathematics education in Thailand. The recent results of PISA 2015 showed that Thai students ranked 54th in mathematics from 70

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countries with a mean score of 415 points, which was significantly below the Organisation for Economic Co-operation and Development (OECD) average (2016). The PISA reports and other evidence lead to the recommendations for curriculum development in Thailand where the content, textbooks and teacher are the targets for major changes (Sunee, 2015). In these recommendations, teachers were encouraged to change their teaching behaviour from telling to questioning, to create learning activities that promotes students' participation, and to motivate students to think critically. Thus, active learning strategies such as problem-based learning were introduced.

Problem-Based Learning (PBL) is an educational approach where learning is driven by real-world problems (Othman, Salleh, & Sulaiman, 2013). Students learn by working individually and in teams to investigate, communicate, and apply essential skills to solve the problem. Therefore, the PBL environment is claimed to support students' problem-solving skills and higher-order thinking as well as their critical thinking (Roh, 2003).

In this study, students' critical thinking was explored in a mathematics PBL classroom. The learning process was adapted from Othman, Salleh, and Sulaiman's study (2013), which consists of five ladders (i.e., introduction to the problem, self-directed learning, group meeting, presentation and discussion, and exercises). The critical thinking was described in terms of the subskills adapted from AACU (2009) as mentioned before.

Methods

The participants in this study were 47 eleventh-grade students from a public high school in Chiang Mai Province, Thailand. The data were collected for four weeks during December 2016. Students' critical thinking was observed during eight PBL classes while video was recorded as supporting data. After four weeks, students were given a critical thinking test. The test consisted of two real-world open-ended problem situations and five questions to evaluate each of the critical thinking dimensions (see Table 1). The PBL lesson plans and the critical thinking test were approved by three experienced teachers for content validity.

Table 1
Critical Thinking Test Questions

Dimension	Question
1. Explanation of issues	What is the problem in this situation?
2. Evidence	What evidence do you plan to use in solving the problem? How do you solve it?
3. Conclusions and related outcomes	What are your results and conclusions?
4. Influence of context and assumptions	Do you agree with his/her idea? Why?
5. Student's position	If you can make a decision, what will you do? Why?

The data were interpreted using both qualitative and quantitative methods. The critical thinking test was analyzed using descriptive statistics including percent, mean, and standard deviation. Furthermore, the PBL classroom observation data was described by means of descriptive analysis.

Results

Students' overall critical thinking scores were assessed by the critical thinking test via AAC&U's critical thinking VALUE rubric (2009) (see Table 2). The critical thinking score varied from 1 (low level), 2 and 3 (average level) to 4 (high level).

Table 2
Critical Thinking Test Results

Dimension	Critical Thinking Score				Mean	S.D.
	1	2	3	4		
1. Explanation of issues	4 (8%)	5 (11%)	26 (55%)	12 (26%)	2.98	0.84
2. Evidence	3 (6%)	6 (13%)	34 (72%)	4 (9%)	2.83	0.66
3. Conclusions and related outcomes	3 (6%)	31 (66%)	11 (24%)	2 (4%)	2.26	0.63
4. Influence of context and assumptions	2 (4%)	14 (30%)	24 (51%)	7 (15%)	2.77	0.75
5. Student's position	5 (11%)	23 (49%)	16 (34%)	3 (6%)	2.36	0.76

Focusing on findings from the PBL classroom observation, the five critical thinking dimensions including 1) explanation of issues, 2) evidence 3) conclusions and related outcomes, 4) influence of context and assumptions and 5) student's position are discussed respectively in the subsections below.

Additional details are provided with examples of exercise problem situations. Problem Situation 1 was an exercise problem for the first three critical thinking dimensions including explanation of issues, evidence and conclusions and related outcomes, and Problem Situation 2 was an exercise problem for another two critical thinking dimensions including influence of context and assumptions and student's position.

Problem Situation 1

You are going to participate in a water rocket competition. In the competition, each team can launch a rocket three times. The winner is the team with the highest mean of their rocket flying distances. Therefore, you invented two types of water rocket, tested them and recorded the results as follows:

Water rocket	Flying distance (meter)									
	1st try	2nd try	3rd try	4th try	5th try	6th try	7th try	8th try	9th try	10th try
Type 1	150	170	165	157	166	153	155	167	152	165
Type 2	170	151	167	155	154	171	153	161	170	148

Problem Situation 2

An educational organization surveyed learning outcomes in two neighborhood schools. Students in all classrooms of both schools were given an exam. The result showed that the schools' classroom score deviations were significantly different. Therefore, the organization conducted an interview about classroom management in both schools. The interview results are shown as follows:

First school: "Our school groups students into classes according to their entrance exam scores. Therefore, students with the same ability level are assigned to the same classroom, so they can learn in a suitable learning environment"

Second school: "Our school groups students into classrooms randomly. So, the classrooms consist of students of different ability and interest. We manage classrooms this way because we believe in diversity, and students should learn to live with others."

Question 1: What is the best classroom management policy? Why?

Question 2: What does the classroom's score deviation reflect?

Figure 1. Problem situations.

Explanation of Issues

Students were able to explain their understanding of the problem situations after the introduction to the problem in the first step of the PBL process. The students could highlight the necessary information and summarize the problem in their language. Most of the students were able to summarise the problem correctly but incompletely, as the problem's significance and supporting contexts were often ignored.

Table 3 presents examples of students' explanations of Problem Situation 1. Students with high explanatory skills described the situation including necessary information. Students with average explanatory skills usually omit details of the situation and always describe the situation's goals in an insufficient manner. Students with low explanatory skills sometimes misinterpret the problem situation including the situation's goals, leading them to misuse their evidence.

Table 3

Examples of Students' Explanations of Problem Situation 1

Level	Students' Explanation
High	Find rocket with the greatest flying distance and the highest reliability for the competition by using flying distance data to make a decision.
Average	Find rocket with the highest mean of flying distance.
Low	Find rocket that flies the greatest distance using first three of the greatest distance records.

Evidence

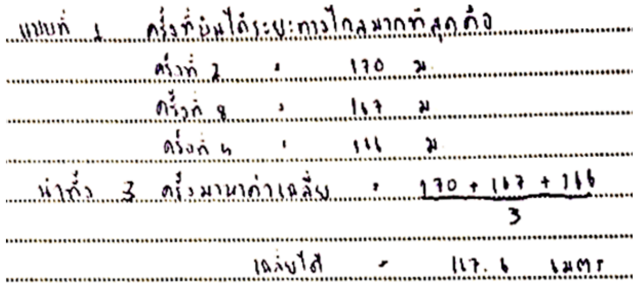
Students examined the situation, interpreted information and made their plan individually in the second step of the PBL process, self-directed learning. The students listed relevant information including given data, existing mathematics knowledge, and sometimes listed alternative hypothesis or approaches. Additionally, in the third step of

PBL, group meeting, the students shared individual work with their groups. The students discussed the problem goal and presented evidence to derive a group strategy and draw up a solution.

Table 4 presents students' selections and their use of evidence in Problem Situation 1. Students with high skill in selecting and using evidence always found multiple approaches and synthesized them to solve the problem. Students with average skill in selecting and using evidence were entirely focused on one approach, and they would give up when the approach failed. Students with low skill in selecting and using evidence could not interpret evidence in the situation, and thus they always came up with inappropriate approaches.

Table 4
Examples of Students' Selection and use of Evidence in Problem Situation 1

Level	Students' selection and use of evidence
High	Students used experimental records to calculate measures of central tendency including mean, median and mode.
Average	Students used experimental records to calculate one of the measures of central tendency and used it to make their decision.
Low	Students used experimental records in inappropriate approaches, and used them to make their decision as shown in Figure 1.



Type 1: The greatest distances are
2^{nd} try: 170 meters
8^{th} try: 167 meters
10^{th} try: 165 meters
Use the distances to calculate mean

$$= \frac{170 + 167 + 165}{3}$$

Mean is $= 167.6$ meters

Figure 2. Example of inappropriate approach: Three of the greatest records were used to compute the mean of expected distance. The left image shows a student's work, and the right shows its English translation.

Conclusions and Related Outcomes

Students concluded and presented their findings to the class in the fourth step of the PBL process, group meeting. The groups' discovered evidence, working processes, outcomes and conclusion were demonstrated in the presentation and discussion while other students in the audience wrote summaries of the presented idea. After the presentation and discussion, students evaluated and concluded the ideas presented by the different groups to identify the best practices for the problem situation.

Table 5 presents students' outcomes and conclusions in Problem Situation 1. Students with high skill in drawing conclusions found possible outcomes and evaluated them to reach a conclusion. Students with average skill summarized their findings in a conclusion. They also created an alternative plan if the existing outcome could not reach the desired conclusion. Students with low skill formulated a conclusion from a single finding. Most of the conclusions were oversimplified and therefore were incomprehensible.

Table 5
Examples of Students' Outcomes and Conclusions of Problem Situation 1

Level	Students' Outcome and Conclusions
High	Means of rockets' flying distances are 160 meters for both types. Medians of rockets' flying distances are 161 and 158 meters. Modes of rockets' flying distances are 170 and 165 meters. We choose the first rocket type according to the mode because the difference between the modes (5) is greater than that between the medians (3).
Average	Because both rockets' flying distances have the same mean, we therefore need to find other method which is median. The calculated median of the first rocket is higher than that of the second type. Thus, we choose the first rocket type.
Low	We can choose either rocket because mean of 10 recorded flying distances of both rocket types are equal.

Influence of Context and Assumptions

Students examined their colleagues' thoughts and hypotheses in group meeting. The students analyzed and evaluated individual works to synthesize a group solution. Once a student proposed his/her idea, the others criticized the idea based on the problem's context, and their ideas and perspectives.

Table 6
Examples of Students' Analysis on Influence of Context and Assumptions in Problem Situation 2 on the Question "What does Classroom's Score Deviation Reflect?"

Level	Students' analysis on influence of context and assumptions
High	It reflects knowledge diversity in classrooms. From the interview, the first school manages classrooms by students' entrance scores which causes student at the same level to be together, and thus classrooms in the first school have low score variation. On the other hand, the second school mixes students in each classroom which causes diversity, and thus classrooms in the second school have high score variation.
Average	It reflects how many scores each classroom earns. The classroom which has low score variation is where the student scores are in the same interval and the classroom which has high score variation is where the student scores are greatly different.
Low	It reflects the teachers' principles and teaching methods because the teaching method of the first school is specified in each classroom, but the teaching method of all classrooms in the second school is the same.

Table 6 presents students' analysis on influence of context and assumptions in Problem Situation 2 on the question "What does the classroom's score deviation reflect?" Students with high context analytical skill considered given information and found a relation to their perspective. Students with average skill could use sufficient information to reach their answers. The answers, however, ignored some important information. Students with low skill were aware of existing information, but could not analyze it to reach an answer.

Students' Positions

Students expressed their thoughts through the group meeting. The students brainstormed their ideas to analyze problem objectives, evidence and hypotheses. The students' ideas and perspectives were usually unmatched and therefore were discussed to reach a group decision. The students also demonstrated their positions in the presentation and discussion step. The presenting group illustrated their problem solution to the class, and sometimes the others criticized and suggested alternative opinions or refinements.

Table 7
Examples of Students' Positions in Problem Situation 2 Toward the Question "What is the Best Classroom Management Policy? Why?"

Level	Students' Positions
High	I agree with the first school's principle because we can study effectively with classmates at the same level. Moreover, in my experience, putting diverse students together make the students unmotivated because the low achievers stick together, and there is no challenge for the high achievers. However, I disagree with the use of entrance score because it cannot always define students' abilities. I prefer using students' grades to arrange classrooms in every year. This method will motivate students to study with their friends.
Average	I agree with the first school's principle because it helps teachers to teach systematically and helps students with similar ability level to learn together. The second school's principle, on the other hand, makes high achievers learn less effectively while waiting for low achievers who also get depressed.
Low	I agree with the second school's principle because bringing diverse students together creates opportunities for high achievers to help the lower ones. Low achievers can develop themselves while high achievers can practice their skills and knowledge.

Table 7 presents students' positions in Problem Situation 2 toward the question "What is the best classroom management policy? Why?" Students with high skill in stating a position specify their position considering multiple viewpoints. The chosen path and unchosen paths were discussed in order to reach their decision. They mentioned the limitation of their decision and sometimes came up with alternative ideas. Students with average skill in stating a position considered both chosen and unchosen viewpoints and made their decision. Students with low skill in stating a position considered only a single viewpoint and made their decision, which thus was too simplistic and biased.

Discussion and Conclusion

This study explored students' critical thinking in five steps of a mathematics PBL classroom. The findings revealed that PBL learning processes allowed students to express their critical thinking in all of the dimensions as "students interpret the problem, gather needed information, identify possible solutions, evaluate options, and present conclusions" (Roh, 2003, p. 1), and the PLB tasks also gave opportunities for the students to share and evaluate their thoughts and opinions in a group as "these tasks require an open exchange of ideas and engagement by all members of the group" (Hmelo-Silver, 2004, p. 241).

Additionally, the critical thinking test results indicated that the students showed the highest score in the explanation of issues dimension and the lowest score in the conclusions and related outcomes dimension. These results are compatible with AACU's current report (2017) that demonstrated explaining of issues as the strength and drawing conclusion as the weakness of students. Furthermore, the results showed that the students also had difficulty in the student's position dimension as they omitted available viewpoints and specified their position inadequately. This finding was not specifically mentioned in the AACU report. Therefore, future work should focus on confirming students' weakness and strength, and supporting students' critical thinking in the specific dimensions.

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