Authentic Learning in a Year 8 Classroom

Kathy Blum
Queensland University of Technology
k.blum@qut.edu.au

Even though school mathematics has been subject to many reforms, the delivery of the high school mathematics curriculum in most schools has changed little since public schooling began. Mathematics is presented as a collection of abstract procedures and, consequently, both student understanding and affect are poor. Year 8 mathematics was traversed through authentic learning experiences, during which students established socio-mathematical norms. The implications from this study are that student-directed learning fosters deeper understanding, and improved affect, compared to traditional methods.

In many high school mathematics classrooms, information is transmitted by the teacher and memorised and regurgitated by the students (Uhl & Davis, 1999). Because the learning of mathematics has traditionally been interpreted as a linear activity, mathematics is decontextualised and deconstructed so that the building blocks can be strengthened, by a great deal of repetition of standard exercises, before they are assembled into the edifice of mathematics (Boaler, 2000; Wiggins, 1993). Mathematical meaning is thus jettisoned, causing the teacher to transmit mathematics as a collection of operations and algorithms that are applied mechanically to standard problems to calculate the correct answers. Understanding and creating mathematics are seen as the prerogative of experts only (Mukhopadhyay & Greer, 2001). Hence many students believe that memorisation is the only way to learn mathematics (Boaler, 2000). Because high school mathematics assessments usually test only the lower order cognitive processes, memorisation of standard algorithms is often the fastest way to achieve a high regurgitation score (Skemp, 1976; Pesek & Kirshner, 2000). Many educators denounce this practice and instead suggest that more interest and learning can be evinced by longer-term, theme-based projects (Greeno, 1997).

Another major source of misunderstanding in mathematics classrooms derives from the misfit between the student's background or habitus and the school mathematics curriculum or classroom field (Zevenbergen, 2001). For example, Zevenbergen described how the triadic dialogue, consisting of teacher's question, student's response, and teacher's evaluation of the student's response, did not work as well with the students from working class backgrounds as with those from higher social strata. Other educators have identified different interpretations of mathematics problems and test questions by students from different class backgrounds (e.g., Lerman & Tsatsaroni, 1999; Lubienski, 2000). More students from the higher rather than the lower socio-economic classes, who have the requisite cultural capital, can postpone gratification for a few years, and accept the high school mathematics deal: Memorize the fruitless, abstract algorithms transmitted by the teacher and regurgitate them during assessments in order to gain entry to the high status tertiary courses (Dowling, 1998).

Understandably, high school mathematics classrooms are experienced as uncomfortable, alien places by many students, a dysfunctional background for learning (Boaler, 2002). Educators such as Goldin (1998) regard "the affective system of representation as the most fundamental to understanding the structure of mathematical ability in students and adults" (p. 155). In other words, if the experiences in mathematics classrooms evoke positive emotions, then the learning will be stronger and the stored positive memories will increase the accessibility of particular problem solving strategies in the future. The opposite is true if only negative feelings are aroused in mathematics classes.

My research follows in the tradition of research reported by educators such as Boaler (2000) and Greeno (1997), but also differs in that it is an effort by a sole teacher to engender student-directedness, through authentic learning experiences, in one mathematics classroom. As such, it offers valuable insight for other sole practitioners wishing to do the same.

Authentic Learning Experiences

The authentic learning experiences are well-structured tasks, of interest to most students, and, perhaps most importantly, they enable student-direction. They encompass observations, investigations, problem solving, experiments, modelling, performances, and artistic interpretations. As did the Productive Pedagogies of the Queensland School Reform Longitudinal Study (Ladwig, Lindgard, Mills, & Land, 2001), they offer intellectual quality, connectedness to the students' world, a supportive environment and recognition of individual difference.

Authentic learning experiences, as the word authentic denotes, comprise processes and strategies that are involved in normal life activities. Thus students should have information and help from experts, access to tools, and discussions with peers, as do problem solvers or performers in the real world (Wiggins, 1993).

In many authentic learning experiences manipulatives are used to build models and aid understanding. Some educators believe that actions on mathematical objects are crucial, whether these objects be manipulatives such as blocks, or computer images such as patterns or straight lines, because, as students use the tools, they naturally begin to hypothesise, to make predictions, and to test them, and deep understanding flows from this creative process (Driver & Scott, 1996; Greeno, 1997). These mathematical tools "allow the establishment of practices and the taken-as-shared basis for mathematical communication in explanations and justifications" (Hershkowitz & Schwarz, 1999, p.151). Some educators believe that mathematical tools may assume a focus of control in authentic learning: that engaged participation seems to depend on the availability of supportive tools, from blocks to computer programs (Greeno (1997; Hershkowitz and Schwartz, 1999).

Uhl and Davis (1999) commend such learning experiences because the understanding that results "permeates the brain, the heart and the soul" (p. 71). Goldin (1998) concurs that positive affect is strengthened, and also commends the opportunity to practise reflection and decision making, which are vital for the successful completion of tasks that require higher order cognitive processes.

Therefore, in an effort to enhance mathematical understanding and attitude for students from all backgrounds, I organised a Year 8 mathematics program in which almost all learning derived from authentic learning experiences.

Methodology

My study could be described as action research, one cycle comprising: the planning of authentic learning experiences; the observation of the Year 8 students (NG = 13, NB = 14) as they negotiated the tasks; and the reflection on and analysis of the data from several sources before another series of authentic learning experiences were created (McKernan, 1991). Over 30 hours of video recordings and the activity booklets, in which students recorded their progress, methods, and evaluations of the tasks, provided copious, rich sources of data for analysis. Other valuable insights came from student journals, student interviews, and the teacher's diary. The activity described in this paper was a part of this overall corpus of data.

Composition of the student groups (about 3-4 per group) and the choice of tasks available to the groups were associated factors which I varied during the research cycles. Initially a few students with serious behaviour and/or learning problems formed dysfunctional groups which tended to disrupt other groups and deny them my services as facilitator. The latter problems were ameliorated by adding more structure to the learning experiences to facilitate student self-direction, and also by using more tasks for which practising with manipulatives was integrally related to progress. The significance of Greeno's (1997) belief that mathematical tools are crucial in assuming the focus of control, necessarily vacated by the teacher during authentic learning, became clearer to me as the research proceeded.

The most important research tool for gaining insight into how students establish sociomathematical norms and so construct meaningful concepts was the video recorder. The efficacy of this tool also increased as the action research cycles progressed. I gradually realised that understanding cannot be hurried, nor can it appear by demand at the commencement of video recording. The best mathematising was captured on video recordings when the camera was focussed on the same group for a relatively long time, even over a few days.

In order better to convey the learning that transpired in the yearlong authentic journey by my Year 8 students and me, I offer an intimate, description of a representative snapshot of student learning. I have been influenced by educators who employ an arts-based approach to educational research (e.g., Barone & Eisner, 1997; Wolcott, 1997).

My perception of my students' learning and attitudes underpins my research data analysis, and while it will differ from anyone else's perception of the same phenomena, it is in this difference, the rich details of different perceptions, that more understanding of students' learning may come. Barone and Eisner (1997) wrote: "The very categories and procedures that we believe to be legitimate in science may themselves create a profound bias because of what such procedures neglect" (p. 90). The main aim of educational research is to further understanding so that educational practice can be improved. I believe that the style in which my research is reported will make it more accessible for classroom teachers who can identify the similarities, the shared reality in my narrative. From the particular, the teacher can draw parallels and applications to her/his own classroom. In this way generalization does not have to be in the narrow meaning of the positivist, statistically-significant study: It could be that "a singular story, as every true story is singular, will in the magic way of some things apply, connect, resonate, touch a magic chord" (Wolfe, cited in Peshkin, 1997, p. 25).

Results

One particular authentic learning experience (How Many Cubes?), appearing after many action research cycles, illustrates well the sort of mathematising in which the students engaged. It qualifies on many counts as authentic:

- the task being intelligible and interesting, the students directed their own learning;

- the mathematical tools, the blocks, provided a strong focus for the task, and a natural starting point in building bigger cuboids from the blocks;
- the constraints were authentic: students were free to work at their own pace; could discuss the problem with friends; and seek confirmation or help from the teacher;
- progress could not be made simply by applying learned rules: students were required to act on tools and collect empirical data; and choices had to be made.

How Many Cubes?

The transcript of a video recording of Kate and Nicole[1] engaged on "How Many Cubes?" is an example of how students collaborating together can gradually construct shared meaning; it is also an enjoyable experience, with moments of excitement.

This task first asks how many different cuboids can be made with 24 cubes. The students practice the actions on the mathematical objects: building cuboids with multicubes. Nicole and Kate explain and justify in terms of their actions on mathematical objects. Among the socio-mathematical norms that are established are the following:

- The naming of a cuboid in the manner: length x width x height.
- Constitution of different cuboids with the same volume.
- Constitution of the family of cuboids, A x B x C, where at least one of the dimensions A, B, or C is 1.

Figure 1 shows a sequence in which the students are practising the construction of the cuboids according to their length x width x height notation.

Kate: The length is 2, width is 2, the height is 4. This is the height. That's the height, … and so the length is 2.
Nicole: Is that what you mean … 2…*(indicating the lengths and sides on Kate's block construction*) and that 3?
Kate: But the width has to be 3. So then you'd have 3 coming out here.
Nicole: Like that. Yeah that's right.
Kate: Like this. (*holding her block prism, and adding blocks*)
Nicole: 3 by 2 by 4 (*Nicole deliberately put her prism on the ground, seemingly satisfied that she has successfully completed part of the activity*.)

Figure 1. Construction of a 3 x 2 x 4 cuboid.

The girls afforded a good example of an ambiguity in one system of representation, the abstract, symbolic factors, requiring a quick referral to the multicube construction to provide clarification. The dialogue in Figure 2 records a classic moment when they discover that a 2 x 3 x 4 cuboid, the shape they have just made, is the same as a 2 x 4 x 3 cuboid, the shape they made previously. The humour associated with this discovery is also good for their affective representation (Goldin, 1998).

[1] All names are pseudonyms.

Kate: Now, that looks like the shape we've just made.
Nicole: It is the shape we just made cause the height is 3 actually … because it is the same one.
Kate: No, it's not.
Nicole: It's just all the wrong way (*Nicole laughs*)
Kate: No, it can't be but …two three's are 6 times …
Pause
Kate: No, it can't be but …2 3 4
Nicole: It is. It's just round the wrong way.
Kate: (*laughing*) Oh, it is too.

Figure 2. A 2 x 3 x 4 cuboid is the same as a 2 x 4 x 3 cuboid?

As the students work out successive tasks, they connect what they are doing back to other tasks. Figure 3 gives a sequence which occurred after they had spent some time making their first two prisms of volume 24, 2 x 4 x 3 and 2 x 3 x 4. Kate announces to me, incorrectly, that only 2 shapes can be made from 24 cubes; then she gives an exposition on the commutativity and associativity of multiplication, and ventures into combinations and permutations with commendable accuracy.

Kate: We figured it out. You can only make 2 shapes from 24 cubes because ..
Nicole: They're a double; they go back the front like in factors.
Kate: Yes, like, if you're doing your times tables and you do 1 times 2 is 2 and then you turn it round and do 2 times 1 is 2… it's the same thing.
So really you can do 6 different ones but they're going to be all the same.
Just say the first length, width, height is 2, 3, 4; then the second one might be 4, 3, 2… just changing it around- but it's still all the same.

Figure 3. How many cuboids?

When the students were having trouble generating any more cuboids because they all involved at least one dimension of 1, they were forced to think very hard. There had already been a couple of statements which indicated that "1" was causing some cognitive conflict: Kate had said earlier, "If it's 1 by 2 wouldn't that make 1 cube?" and Nicole had said, "Would that be 12 by 2 by 1? Where's the 1?" In the next sequence, in Figure 4, where the girls are still trying to find other cuboids with a volume of 24, Nicole consolidates the concept, and Kate senses the breakthrough:

Kate: 2 by 3 is 6… by 4 (*Pointing at Nicole, as she says by 4*)
Nicole: 2 x 3 x 4 What about like …? …No.
Kate: What about 3 by 3 is 9 … by… (*Both girls are reflecting a great deal*)
Kate: yawns
Nicole: yawns (*the girls are distracted by another group of students. Both laugh when they remember the camera.*)
(More silence and serious reflection)
Nicole: What about 8 by 3 by (*pause*) 1?
Kate: (*excitedly*) Hey, yeah!

Figure 4. That elusive "1"

However it is evident in the next sequence, in Figure 5, where they have progressed to cuboids with a volume of 36, that Kate has finally firmly constructed the A x B x 1 cuboid concept:

Nicole: You could go 3 x 4 x 3…3 times 4 is 12 times 3 is 36. Oh yeah by 3
Kate: Hang on…3 by 4 is 12 by 3 is 36
Nicole: 3 by 4 by 3…We could do 3 by 3 by 4
Kate: Yeah …4 by 9 is 36 … 4 by 9
Nicole: 1 by 4 by 9 (*Nicole puts in that elusive 1 for Kate, sensing that she is groping for it*)
Kate: Hey that means we could write down the factors of 24 … 3 by 4 by 1 as well
(*Kate has just realized the power of this method of enumerating the factors to describe the cubes; she wants to apply it to the 24 cube model they did before, although she has made a slight slip*)
Nicole: (*too engrossed in her own thought train to acknowledge Kate's insight*) We could do 12 by 3 … by 1 (*Note the pause before that elusive 1*)

Figure 5. More consolidation of "1"

Discussion

There are many impressive features of this learning sequence: how easy and enjoyable the learning is; how important the blocks are to the learning process; and how naturally many other mathematical topics are referred to and seem to be understood in a new light. For example, there is much more incentive to master factors in order to make progress in an interesting investigation. Hershkowitz and Schwarz (1999) also refer to the web-like manner in which mathematical concepts link up during rich tasks when students are fully engaged in interactions with mathematical tools.

Yackel, Cobb, and Wood (1998) present discussions between the teacher and younger, primary students from which sociomathematical norms are constructed. This form of discussion, akin to triadic dialogue (Zevenbergen, 2001), was not productive in my Year 8 class, even at those few times when all the groups were engaged on the same task. My study had more in common with that of Hershkowitz and Schwarz (1999) in that some of the norms were established in the absence of the teacher, and some when a single group consulted with me. For example, the norm that a 2 x 3 x 4 cuboid is the same as a 2 x 4 x 3 cuboid was arrived at, with humour, by Nicole and Kate without me. However, when they were establishing the norm that a prism can have a dimension of 1, they sought my opinion, but had in fact already arrived at this norm.

Even though evidence has been presented here from only one group, Nicole and Kate were fairly representative of the class: Some other groups established the sociomathematical norms in this task more quickly. Given a substantial problem, and mathematical tools with which to practise, interact, and experiment, students will generate their own sociomathematical norms as evidenced in this study. The latter are not received in a finished form but must be grappled with and made their own, thus minimising any dissonance between the students' habitus and the classroom field (Zevenbergen, 2001).

It is only in the freedom of open-ended learning experiences that students have the opportunity to reflect, experiment, and make choices, and it is in the understanding of the consequences of those choices that real mathematical learning occurs. The student is forced into an active mode. There can be resistance to this challenging, active role if the student has been used to a passive, receiving mode. However, compared to the higher order cognitive functioning possible in authentic learning, the memorisation or instrumental understanding of lower order cognitive skills engendered by transmission teaching represents a “pedagogy of poverty” (Ladson-Billings [1997] cited in Boaler, 2002, p. 241).

Having accepted the challenge, most students learn mathematics during authentic learning experiences in a much easier, more enjoyable manner than in a transmission classroom. The video recordings are testimony to the easy banter, strong concept construction, and exciting Ah Ha! experiences that occurred frequently during these activities. In addition, and perhaps more importantly, the students learn that learning is joyful and natural, making it more likely that they will remain lifetime learners. Students also learn about working with others; they learn that talking over problems with others can enhance understanding; and they learn that a functional group can achieve more than a single person. Mathematics becomes more human during authentic learning: Rest breaks, humour, art, and other human pursuits are encouraged along with the mathematising.

Within this milieu students learn that discussion, argument, frustration, and creativity are as much a part of mathematics as they are of any other human endeavour. They learn that the essence of mathematics is not the perfectly ordered, neat, decontextualised, deconstructed solution in a mathematics text book, but rather a way of looking at their world that enhances their understanding of that world. Given an interesting problem, mathematical resources, and plenty of time and support, most students will build strong mathematical concepts.

However, students learn a surprising amount of mathematics during authentic learning experiences as, being rich larger problems, they usually integrate many areas of mathematics, so that other concepts are continually being accessed, in a web-like manner, and therefore maintained. Connections between the different representational systems in memory are continually used and strengthened. Also, since there is an emphasis on hands-on mathematics, involving manipulatives wherever possible, the learning of mathematical concepts is multi-representational and therefore stronger (Goldin, 1998).

Early in the research there were problems with tasks where the manipulatives were more a window-dressing than integral to the problem. Other task creators and I had succumbed to the *myth of reference*, whereby everyday problems are sometimes recontextualised in order that mathematics can explain them (Dowling, 1998). As have other educators (e.g., Lubienski, 2000), I gradually realised that students are confused by such recontextualisations, and that they will enthusiastically embrace a bare problem such as How Many Cubes? which has no spurious practical reference.

Conclusion

School mathematics work programs should be re-written: Instead of the interminable, detailed listing of specific topics to be taught, it would be much more enjoyable and beneficial for students and teachers to have libraries of rich tasks from which they can make a selection. Goldin (1998) would underline the importance of such activities for the affective system of representation, for the feelings that derive from understanding and solving such tasks are very pleasant, even addictive, and students will have the urge to repeat these activities.

Currently in many high school mathematics classrooms students are practising deconstructed, decontextualised exercises with the forlorn hope that they will accrue someday into fruitful knowledge. The process is akin to an English course without essays, plays, films, poetry, and novels, but with verbs, nouns, adjectives, pronouns, simple sentences, metaphors, etc.

References

Barone, T. & Eisner, (1997). Arts-based educational research. In R. M. Jaeger (Ed.). *Complementary methods for research in education* (2nd ed.). (pp. 73-99). Washington DC: American Educational Research Association.

Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behaviour, 18(4),* 379-397.

Boaler, J. (2002). Learning from teaching: Exploring the relationship between reform curriculum and equity. *Journal for Research in Mathematics Education, 33(4),* 239-258.

Dowling, P. (1998). *The Sociology of Mathematics Education: Mathematical Myths/ Pedagogic Texts.* London: The Falmer Press.

Driver, R. & Scott, P. H. (1996). Curriculum development as research: A constructivist approach to science curriculum development and teaching. In D. F. Treagust, R. Duit, & B. J. Fraser (Eds.). (1996). *Improving teaching and learning in science and mathematics.* (pp. 94-108). New York: Teachers College Press.

Goldin, G.A. (1998). Representational systems, learning and problem solving in mathematics. *Journal of Mathematical Behaviour, 17(2),* 137-165.

Greeno, J.G. (1997). Theories and practices of thinking and learning to think. (Reforming the third R: Changing the school mathematics curriculum). *American Journal of Education, 106(1),* 85-127.

Hershkowitz, R. & Schwartz, B. (1999). The emergent perspective in rich learning environments. *Educational Studies in Mathematics. 39: 149-166.*

Hiebert, J. (1999). Relationships between research and the NCTM standards. *Journal for Research in Mathematics Education, 30(1),* 3-19.

Ladwig, J.G., Lindgard, R., Mills, M., & Land, R. (2001). *Queensland School Reform Longitudinal Study (QSRLS)*. Brisbane: Department of Education.

Lerman, S. & Tsatsaroni, A. (1999). Why children fail and what the field of mathematics education can do about it: The role of sociology. Retrieved October 22, 2000, from http://www.nott.ac.uk/csme/meas/plenaries/lerman.html

Lubienski, S.T. (2000). Problem solving as a means towards mathematics for all: An exploratory look through a class lens. *Journal for Research in Mathematics Education, 31(4),* 454-482.

McKernan, J. (1991). Curriculum action research. A handbook of methods and resources for the reflective practitioner. London: Kogan Page.

Mukhopadhyay, S. & Greer, B. (2001). Modeling with purpose: Mathematics as a critical tool. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural Research on Mathematics Education* (pp. 295-311). Mahwah, N.J., USA: Lawrence Erlbaum Associates.

Pesek, D.D. & Kirshner, D. (2000). Interference of instrumental instruction in subsequent relational learning. *Journal for Research in Mathematics Education, 31(5),* 524-540.

Peshkin, A. (1993). The goodness of qualitative research. *Educational Researcher, 22(2)*, 23-29.

Skemp, R. R. (1976). Relational understanding and instrumental understanding. *Mathematics Teaching 77,* 20-26.

Uhl, J. & Davis, W. (1999). Is the mathematics we do the mathematics we teach? In E.A. Gavosto, S.G. Krantz, , & W. McCallum (Eds.). *Contemporary Issues in Mathematics Education* (pp.67-74). Cambridge, UK: Cambridge University Press.

Wiggins, G. (1993). Assessment: authenticity, context, and validity. *Phi Delta Kappan, 75,* 200-214.

Wolcott, H. F. (1997). Adequate schools and inadequate education: The life history of a sneaky kid. In R. M. Jaeger (Ed.), *Complementary methods for research in education* (2nd ed.). (pp. 370-398). Washington DC: American Educational Research Association.

Yackel, E., Cobb, P., Wood, T. (1998). The interactive constitution of mathematical meaning in one second grade classroom: an illustrative example. *Journal of Mathematical Behaviour, 17(4), 469-488.*

Zevenbergen, R. (2001) Issues in Mathematics Education. In B. Atweh, H. Forgasz, and B. Nebres (Eds.), *Sociocultural Research on Mathematics Education* (pp. 295-311). Mahwah, N.J., USA: Lawrence Erlbaum Associates.