Game Playing to Develop Mental Computation: A Case Study

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This research investigated the use of game playing for mental computation development. As part of a larger case study of a Year 6 class, classroom observation data were used to examine the nature of students' mental computations. Findings indicated that regular playing of a number-based game that was scaffolded by teacher designed learning structures supported students' engagement in mental recall, verbalisation of computation steps, using a range of mental strategies, and experimenting with number combinations.

For a number of years teachers and others in education have advocated the use of game playing as a teaching strategy to improve children's learning in mathematics (Ainley, 1990; Booker, 2000; Hatch, 1998; Oldfield, 1991). Manufacturers of teaching aids and educational games have also suggested that children need to experience the 'fun' element of learning and that games can provide this feature (Drysdale, 1999). Further, it is argued that, since number-based games can provide opportunities for using mental computation strategies, the effectiveness of using mathematical games to support children's development of their personal mental computation needs further investigation.

It was this context — the potential of games for mental computation development — that motivated the larger study upon which this paper is based. The larger study addressed the research question: In what ways can game playing impact on students' mental computation capacities? The focus of this paper is upon the case study component of the larger study; specifically, the component of the larger study that addressed the sub-question:

> What is the nature of students' mental computation capacities within game playing activities?

The significance of the larger study lies in its potential to inform mathematics educators of curriculum practices and specific activities that support the development of number sense and mental computation capacities. More specifically, it provides insight into the type, extent, and development of computation strategies used by students when playing a number-oriented game, along with how, and if, they transferred their learning to their general mathematics performance in mental computation. The study also informs researchers and practitioners of the social aspects of mathematics learning with regard to the role of discussion, motivation, and attitudes. Finally, at a more theoretical level, this research can offer insights into frameworks or models for mental computation development.

Background Framework

Mental Computation

Many children rely on images of traditional written methods when they are attempting a calculation mentally. This has the effect of limiting their choice of strategy and in many cases the effectiveness of the calculation (Cobb, Wood & Yackel, 1990). Further, the emphasis on formal written computation does not support the development of mathematical understanding (McIntosh, Nohda, Reys & Reys, 1995) nor the everyday use of mathematics in the community outside the school (Morgan, 2000; Reys, Reys & McIntosh, 1995).

For the purposes of this study mental computation was seen as more than the rapid recall of learned facts that it is comprised of in many classrooms. Here, mental computation was used to describe calculations done mentally, in a broader sense, as Reys (1984) has suggested: "a procedure . . . performed mentally, without using external devices" (p. 548). Mental computation methods, in this sense, are personal and dependent on the problem, the numbers involved and the context. Beishuizen (1997) extended this definition of mental computation when he suggested that mental computation was "doing mental arithmetic *in your head* (number facts) and doing mental arithmetic *with your head* (mental strategies)" (p. 18). Drawing on these ideas, for this research study, mental computation capacities were viewed as consisting of two inter-related components: mental recall and mental strategies.

Mental computation development is not innate and requires experiences and practice (McIntosh, 2002). In particular, children need effective experiences and practice of mental computation to develop sophisticated strategies and to offset the tendency to adopt traditional written procedures. Game playing may be a way to provide suitable mental computation experiences. It may also help build the larger goals of number sense and personal meaningful connections as children develop their mathematical understandings and knowledge. Heirdsfield (2002) also noted that children needed time and experiences, but not necessarily in testing situations, to develop their ability with mental computations. Game playing appears to present a useful context for this to happen.

Game Playing

Games in the teaching and learning of mathematics are often ill-defined and are used sometimes as a time-filler or reward with little attempt to qualify, in terms of mathematics learning, why they are being used. In fact, the literature is not always clear in defining what constitutes a mathematical 'game'. Many activities that fall under the heading of games in many classrooms and textbooks are, in fact, not a game as defined by Gough (1999). He suggested that "a game needs to have two or more players, who take turns, each competing to achieve a winning situation" (p. 12), and he also noted that games should not be based on luck or have no interaction between players. A mathematical game, in this sense, provides players with opportunities to use their mathematical knowledge and to interact with peers, while at the same time experiencing variety and challenge.

Though there is little research supporting the effects of developing skills through the playing of games, many authors have written favourably for the inclusion of games as a mathematics teaching strategy (e.g., Ainley, 1990; Hatch, 1998). Others have suggested that playing games allows children to gain confidence, test their own thoughts, justify their ideas, interpret the ideas of others, and shape their attitudes towards new concepts (Booker, 2000). Teachers, therefore, can use mathematical games to support achievement of particular mathematics learning objectives. Usually, a number-based game does not stipulate how a calculation is done. It can therefore allow children to practise their mental computation strategies without the 'drudgery' of endless calculations on a page or in a test situation.

Method

The aim of the larger study was to determine how game playing might assist the development of students' mental computation capacities. That is, what impact could the regular playing of a number-based game have on the mental computation knowledge and skills of students? In this regard it was important that the choice of game (or games) used take into account the potential of the game(s) as environments to foster mental computation proficiencies. Thus, a first step in development of the study was selection of an appropriate game (or games). This step was followed by decisions pertaining to how the research could explore game playing in ways that could inform future curriculum planning, teaching practices, and research. These aspects of the research process are outlined next.

Selecting the Game

As an exploratory small-scale study, it was decided to focus upon a single game with a flexible and wide range of possible mental computations required for playing the game successfully. Thus, the game would have potential to be used with students of various achievement levels. The decision to focus on one game only also allowed for a comprehensive initial account of the game in use and how it might impact upon students' mental computation capacities. Further, to meet the potential of games as outlined in the literature, it was necessary to select a game that gave students opportunities to use their mathematical knowledge, to interact with peers, and to experience variety and challenge. Finally, to meet the requirement that mental computation be practised and in fact developed, it was required that the game foster: the use of number facts, the use of mental strategies, the development of meaningful number relationships, and the opportunity to experience over time a variety of mathematical computations. The game selected to meet these criteria was Numero®.

Numero® is a card game that can be played at different levels of sophistication depending on the mathematical ability of the players (Drysdale, 1995). It has four sets of coloured cards numbered from 1–15, and 'wild cards' that use the basic operations of arithmetic (namely, addition, subtraction, multiplication and division) or other mathematical representations or processes (specifically, fractions, decimals, percentages, indices, and roots). With this diversity of cards, games can be at a 'simple' level using only one or more of the basic arithmetic operations, or they can be at a more advanced level by incorporation of a selection of the wild cards.

The game begins with five cards dealt to each player, and for each player a card from the pack is turned face up to create a 'centre' collection of cards. The simplest 'take' is to match a card from the hand with a single card of the same number in the centre. For example, a 5 in the hand is played onto a 5 in the centre. A more complex 'take' is to match a single card from the hand to a combination built from cards from the centre by an arithmetic operation. For example, an 8 in the hand can be matched to a 6 and a 2 in the centre. These cards are added to give an answer of 8. An even more complex 'take', using wild cards to create an equation, would be to use a 4 and the wild card '–5' (subtract 5) from the centre along with the wild card 'x 5' (multiply by 5) from the hand to create the equation (4 x 5 – 5 = 15), and then to match this with a 15 from the hand.

During the game the players are expected to verbalise their 'takes' so that opponents can confirm that the mathematical calculation is correct. Points are assigned to a 'take' by the designation of points that are printed on each card. The object of Numero® is for players to maximise their total points for a game.

Research Sample

A case study approach was adopted to enable the study to capture a 'rich' picture of the context and happenings of the use of the game. Further, to provide opportunity for the study to examine students using a range of number knowledge and skills beyond the four basic arithmetic operations, it was decided to use upper primary students. That is, to gain a broad perspective on students' mental computation capacities it was necessary to work with students who had prior experiences with computations involving more than one operation and more than the four basic arithmetic operations (i.e. more than simple one-step addition, subtraction, multiplication, and division).

A Year 6 class at a government primary school was selected for the study. The class consisted of 28 students (21 boys, 7 girls), and was a convenience sample (Cohen, Manion & Morrison, 2000) in that the location was chosen so as to be easily accessible to the main researcher for regular classroom visits. The school was located in the metropolitan area of a large Australian city. Most students in the class had some previous experience playing Numero® (the previous school year), so they had some familiarity with the rules and structure of the game. However, the teacher had no prior experience with Numero®.

Data Collection and Analysis

As a case study, the research used a combination of qualitative and quantitative techniques to gather factual and descriptive data pertaining to the use of the game. More specifically, data collection focused on: the classroom structures for using the game (e.g. how often, when, the game level difficulties, and who plays against who); what happens during game playing (e.g. interactions of the students, what types of mental computations students display, and how they verbalise their 'takes'); students' and teachers' perceptions of the value of the game (e.g. with regard to motivation, and how the game does or does not help with learning in mental computation); and evidence of any changes in students' mental computation performance (pre- and post-tests in mental computation). Hence, two main components (one qualitative and one quantitative) of data collection were developed, consisting of: (1) classroom observations, student and teacher interviews, and student task-based interviews (qualitative component); and (2) pre- and post-tests in mental computation, and student task-based interviews (quantitative component).

Classroom observations. The component of the study reported here was based upon data collected from the classroom observations. The main researcher visited the class four times a week for a 10-week period starting in mid Term 1 and ending in mid Term 2. The researcher adopted an 'observer-as-participant' role (Cohen, Manion & Morrison, 2000) for classroom observations that focused on: (i) the human setting — characteristics of how the game was structured and played; and (ii) the interactional setting — students' interactions, types of 'builds and takes' made, verbal and non-verbal actions. Observation notes were made during each visit, and some small-group game sessions were audio-recorded and later transcribed.

Data analysis. Three key aspects of the game playing context and activities were initially identified as a framework for data analysis that addressed the sub-question focused upon in this paper (i.e. the nature of students' mental computations within the game playing activities):

- Structure of the use of the game — how often, which students played together, and the difficulty levels of games played;
- Participation and communication — the nature of student interactions, and the nature and role of their verbalisations; and
- Mental computation methods — the combinations of cards used, and what mental recall and mental strategies these reflected.

This framework was initially used to categorise data. At the same time an inductive analysis approach (Powney & Watts, 1987) was used to identify key aspects of data within these components.

Findings and Discussion

Structure of the Use of the Game

The teacher chose to use the game at the end of each mathematics lesson for about 20 minutes, with the students allocated to groups of 4 students of similar 'ability'. Students played with a partner within their group, and were required to rotate between different partners on different days so that they had experiences in working with different students. Different groups were assigned different sets of 'wild cards' so that the games varied in their levels of difficulty. For example, initially Group 1 played with all the wild cards while Groups 6 and 7 played only with the subtraction, multiplication and division cards. Records of students' scores when playing the game were kept, and students who consistently won against their fellow group members were moved to a higher group. Thus, through the overall group structure the teacher designed the game activities to provide students with exposure to the ideas and strategies of a variety of other students, along with the challenges of having to play more difficult games or games against 'better' players. These facets of the context of use of the game are noted here because they were specifically planned for by the teacher to support the students' mental computation. They are in contrast to a situation where the game might not be played regularly, or in which there are not requirements for students to play more difficult games, with different people, or with better players.

Participation

The general classroom observations indicated the students enjoyed playing the game and were keen to participate each day. When the teacher announced it was time to play the students quickly organised themselves and enthusiastically entered into playing the game. Their enjoyment of the game and the friendly rivalry that ensued were reflected in the laughing and smiling that accompanied comments such as: "Hey, look, I am flogging them"; "It's payback time"; and "You could have thrown out something decent". Perhaps of more relevance is that the students participated in ways that indicated they were willing to 'have a go' and try new ideas without fear of making a mistake. Partners, as well as rivals, would ask for help or provide assistance related to how a particular wild card might be used. For example, here M asked K for clarification on how to multiply fractions:

M: With these cards do you multiply by the top and divide by the bottom?
K: Yes, so 3/5 of 15 would be 15 divided by 5, is 3, then 3 times 3 equals 9. (Week 2, Day 2)

Students also often made suggestions to one another for builds and takes, for example: “You could do that, make it minus 10, then use these and make minus 6 or minus 2”. In checking on one another’s builds and takes, students provided, when necessary, explanations of mathematical operations:

S: No, that’s not how you do it. It’s 4 times 2 is 8, and half of 4 is 2, so 8 add 2 is 10.
T: How is that 10?
C: Cos it’s 2 and a half of the first number 4. 2 lots of 4 are 8, and a half of 4 is 2, so 2 and 8 are 10.
T: Oh, so that’s 10, and then… [continues with the build] (Week 5, Day 3)

These examples of how peer assistance was an integral component of student participation in the game playing indicate that the game playing environment provided opportunities to learn about as well as practice and consolidate mental computations. Through verbalisation of a mental computation, to describe the steps of a computation, students’ thinking was made explicit to themselves and others. This suggests that verbalisation during playing of the game allowed for correct as well as incorrect mathematical knowledge to be revealed and tested, and thereby learning to be enhanced.

However, it must also be noted here that it was observed that as students became more experienced and confident with the game they often ceased stating every step of a computation, instead stating only the answer. This then inhibited opportunities for mathematical discussion, and it then was observed that some errors were missed by students. However, it was also observed that some students would return to full verbalisation for particular types of calculations. For example, S verbally stated her computations up until week 7 and then ceased to do so, except for fractions, for which she continued to verbally state every step. Thus, this study’s findings indicate a need for further investigation into the role of verbalisation in students’ mental computation development.

Mental Computation Methods

Mental recall was observed as a key component of some children’s builds and takes, although the degree to which they could ‘re-create’ particular facts was not always clear. For example, for computations such as 4 + 11 = 15, 10 + 13 = 23, or 13 + 7 = 20, there were instances of students saying they “just know” the answer. They did not relate the answers to strategies such as ‘complements of ten’. Hence, K’s explanation of “you just get used to them” reflects another component of game playing and mental computation in need of further investigation — that of the role of repetition in transforming computation outcomes into facts for recall. The nature of some of the facts students displayed via mental recall was surprising in that the facts were not always involving only whole numbers and/or the four basic arithmetic operations. For example, when K played the 1/5 wild card on 15 and stated “3”, and the researcher asked him how he did that, he replied, “You know some off by heart because you play with them so long you get to know them, like you know 3/5s of 15 is 9” (Week 3, Day 3).

With respect to strategies, sometimes the mental computation strategies the students used were obvious. This was the case, for example, when students used fingers to ‘count on’, or counted out loud using ‘skip counting’. When it was not so obvious and the researcher had opportunity, she would ask them how they had worked through a computation. In this way it was found that ‘sequencing’ (e.g., 13 + 6 + 1 = 13 + 1 + 6 = 14

+ 6 = 20), ‘rounding’ a number (up or down to make a calculation easier; e.g. 32 – 5 = 30 – 3) and ‘partitioning’ (splitting a number into components; e.g. 13 + 7 = 3 + 7 + 10) were commonly used by students. The students also were often observed using ‘chaining’, which is when a calculation includes more than two numbers (e.g. (9 + 3) ÷ 3 ÷ 2 = 2).

Further insights into students’ number sense, specifically their understandings of relative sizes and properties of prime and composite numbers, were revealed when the researcher asked if they had favourite cards. For example, the ‘x $2\frac{1}{2}$’ wild card was disliked because it “makes the numbers too large [for easy calculations]” and does not “usually go with the numbers you want to go with”. Students also noted they would try to quickly use or discard the 11 and 13 number cards because “there is little you can do with them, 13s can’t be divided by anything”. In comparison, 12 was cited as a “favourite”.

Students appeared to be continually thinking about what they could do with the cards on display so that they could make builds with the cards in their hands. Many of them would try different mathematical operations, and different choices or sequences of numbers or wild cards, to manipulate numbers and explore possibilities for builds and takes. For example, one of P’s sets of trial computations was:

> P: [Plays 9 + 3] 12. [Plays ÷ 3 card on 12] 12 divided by 3 is 4. [Plays ÷ 2 card on 4] 4 divided by 2 is 2. [Removes this and tries again. Plays 9 + 3] 12. [Plays – 5 card] 12 take 5 is 7. [Removes this and tries again. Plays ÷ 3 card on 9] Is 3. [Plays + 3] 3 add 3 is 6. [Takes with 6] And I’ll take with the 6. (Week 8, Day 2)

The students did not appear to be hesitant or cautious about making trials when developing their builds. This is in agreement with Heirdsfield’s (2002) findings that children need time and experiences to develop mental computation understandings and skills. The students in the class, when unsuccessful with a trial build, simply tried again. Thus, it appeared that the different attempts at builds provided students with opportunities to practice mental computations, and thereby develop understandings and skills with numbers.

Conclusions

In this exploratory small-scale study the teacher designed the use of game playing for mental computation so as to scaffold for learning. More specifically, students were exposed to new ideas by playing with different people, and they were challenged to practice and possibly learn new skills because they were required to progress through playing higher levels of the game. The structure of game playing used in the class allowed the students not to be daunted by the number and variety of wild cards. That these structures supported students’ *engagement* in mental computation was evidenced by the enthusiastic nature of students’ participation in playing the game, as well as the range of mental recall, experimentation with number combinations, and mental strategies they displayed.

This small-scale exploration, utilising only the classroom observation data, cannot provide findings concerning the degree to which mental computation *development* (i.e. learning of new things) was fostered. The larger study, using the pre- and post-test data and student task-based interviews, began to address this broader question. However, within this more focused component, there were several aspects of students’ use of verbalisation, mental recall, and mental strategies that warrant further study in relation to mental computation capacities because they appeared as prominent during game playing. The related questions concerning mathematics learning in general, mental computation development in general, and the use game playing for mental computation are offered here as avenues for future research:

- What impact does verbalisation and sharing of ideas have on development of understandings and strategies related to mental computation capacities?
- How might game playing be used to develop specific, efficient, and sophisticated mental computation strategies?
- What are effective teaching strategies to complement the use game playing as a tool for mental computation development?
- How might game playing affect children's mental computation performance if introduced in the early primary years and played regularly, in a scaffolded way, as children progress through the year levels?

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