Experiencing mathematics for connected understanding: using the RAMR framework for accelerating students' learning

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This paper reports on the use of the RAMR framework within a curriculum project. Description of the RAMR framework's theoretical bases is followed by two descriptions of students' learning in the classroom. Implications include the need for the teacher to connect student activities in a structured sequence, although this may be predicated on the teacher's own structural understanding of mathematics.

The *Accelerating the Mathematics Learning of Low Socio-Economic Status Junior Secondary Students* (*XLR8)* project (Cooper, Nutchey & Grant, 2013) has aimed to trial and refine a mathematics curriculum for students who enter junior secondary school with mathematical understanding that is (nominally) at a Year 4 level. The objective of the XLR8 curriculum is to accelerate these students' learning such that they are able to enter a core Year 10 mathematics classroom with age-appropriate mathematical understanding. The underlying motivation for the project is emancipatory: to develop theory and practice which may improve the life chances of such underperforming students. The XLR8 curriculum is built upon the RAMR pedagogy (standing for Reality, Abstraction, Mathematics and Reflection). The RAMR framework developed from over two decades of mathematics education research and previous projects conducted within the university's YuMi Deadly Centre (Cooper & Carter, 2016). The RAMR cycle has been adapted to the specific needs of the XLR8 project, in particular the exploration of mathematical structure in a sequence that is anticipated to promote accelerated student learning.

This paper presents the RAMR cycle and its theoretical underpinnings. The research design of the project is briefly summarised. Then, the RAMR cycle is illustrated by way of examples taken from XLR8 classrooms. A discussion is then provided in regard to how the use of the XLR8 curriculum, which embeds the RAMR cycle, has achieved the cognitive and affective aims of the project, in particular students' connected understanding.

Context

The XLR8 project has partnered with several low-SES metropolitan high schools in south-east Queensland. Typically, these schools have high enrolments of Indigenous, Torres Strait Islander, Pacific Islander and refugee students. In these schools there are significant numbers of students whose mathematical ability falls below the national minimum set by the Australian Curriculum, Assessment and Reporting Authority (ACARA). Without targeted intervention, these students will likely learn little mathematics, continue to disengage from school and enter post-compulsory years of schooling with mathematics understanding that is inadequate to access meaningful employment or tertiary education.

Building upon prior work of the YuMi Deadly Centre, the XLR8 project has developed a comprehensive suite of curriculum resources, presented as a set of units. Each unit is comprised of several RAMR cycles and is provided as a booklet for the teacher, with a set of classroom materials (activity sheets etc.) and assessment tasks (including pre/post tests). These resources were first trialled in 2013. Since then they have been progressively refined

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in response to feedback gathered from teachers and observations made by the researchers. In 2016, the XLR8 curriculum is presented as 15 units that, ideally, should be completed across three years of school (i.e., Years 7-9). The XLR8 curriculum is metaphorically described as a twin-trunked tree with lateral branches: the curriculum progressively develops students' understanding of number and operation concepts (the twin trunks) from foundational whole number and counting through fractions and their operations, to the point where linear relationships bring the two trunks together. Along that progression of the number and algebra strand of mathematics, lateral 'branch' connections to measurement, geometry, statistics and probability concepts are made. In many cases, these other strands of mathematics provide contexts for the development of number and algebra. In 2016, which is the final year of the project, the curriculum is being trialled across seven classrooms (three Year 7, three Year 8, one Year 9) at two different partner schools.

The RAMR Pedagogical Framework

The YDM pedagogy has been influenced by the work of Matthews (2009) who, as an Indigenous Australian applied mathematician, provided a description of what 'doing maths' meant for him. The pedagogy incorporates aspects of Wilson's activity type cycle (Ashlock, Johnson, Wilson & Jones, 1983) which defines six distinct types of learning activities: Initiating, Abstracting, Schematising, Consolidating and Transferring, along with Diagnosing. The resultant RAMR cycle of instruction is comprised of four distinct phases of instruction: Reality, Abstraction, Mathematics, and Reflection (YuMi Deadly Centre, 2014). These phases of instruction align with Baturo's (1998) sequence of four knowledge types: (a) entry (pre-existing knowledge); (b) representational (knowledge of materials and pictures used to develop the ideas); (c) procedural (knowledge of definitions, rules and algorithms); and (d) structural (knowledge of relationships and concepts), and are emphasised in each phase of the RAMR cycle.

In later work, Baturo, Cooper, Doyle and Grant (2007) discussed four general pedagogical strategies for teaching mathematics: (a) flexibility (experiencing the mathematical idea many ways); (b) reversing (teaching in both directions); (c) generalising (developing the idea into a transferrable rule); and (d) changing parameters (considering the impact of a changed parameter upon the state of a known system). The centrality of these generic strategies, along with continuous assessment, to the interpretation of the RAMR cycle that has been used in XLR8 is summarised in Figure 1. This figure is followed by guidance for implementing the four phases of the RAMR framework, including the discussion of the phases' theoretical bases. Such explanation of the RAMR cycle is similar to that given to the participating teachers during professional learning sessions, which was further discussed during group planning and post-lesson meetings.

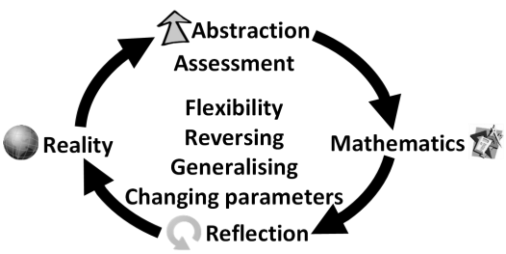

Figure 1. Central pedagogies of XLR8 RAMR

Reality. Closely aligned to the Initiate activity type in the Wilson model, the reality component of the RAMR cycle is when learners: (a) access knowledge of their environment and culture; (b) utilise existing mathematics knowledge prerequisite to the new mathematical idea; and (c) explore real-world activities related to the idea. The focus in this phase is to situate the new idea in students' everyday experiences and provide an experiential base for building connections. This aligns to Baturo's (1998) entry knowledge type. Reality phase activity should facilitate a natural transition into the abstraction phase.

Abstraction. Within the abstraction phase learners experience a variety of actions, representations and language to expresses new ideas in increasingly sophisticated or abstract ways while also developing meaning. This phase of learning, aligned to Baturo's (1998) representational knowledge, is based upon the seminal works of Payne and Rathmell (1975) and Bruner (1966). Students begin by expressing their reality in relation to the new idea using physical context-specific representations and progress to using abstract (and so multi-purpose) representations. Duval (2006) argued that mathematics comprehension results from the coordination of at least two representational forms or registers: the multifunctional registers of natural language and figures/diagrams; and the mono-functional registers of symbols. Early in the abstraction phase students are focused on a single form of representation. However, later in the abstraction phase, alternate representations may be introduced to represent the learner's reality. Features of these representations should be compared to aid students' flexible movement between them.

Mathematics. The mathematics phase of the framework is when learners: (a) appropriate and practice the formal or conventional language and symbols of mathematics; (b) reinforce the knowledge they have gained during the abstraction phase; and (c) build connections with other related mathematical ideas. This phase marks the transition from focussing on Baturo's (1998) representational knowledge to procedural knowledge, and as time progresses shifts to structural knowledge. In regard to structural knowledge, the building of connections between new and existing ideas enables better recall of mathematical ideas and improved problem solving. During this phase it may be appropriate to leverage the experiences of the preceding reality and abstraction phases to introduce additional new ideas. Rather than explicitly repeat the entire reality-abstraction-mathematics sequence, the previous experiences may be drawn upon to quickly connect to working with these new ideas.

Reflection. The reflection phase is critically important because it encompasses three different types of learning opportunities: (a) apply new knowledge back to reality to solve everyday life problems; (b) validate/justify knowledge and, when misconceptions are identified, refine knowledge; and (c) extend understanding by generalising relationships that further structure mathematical knowledge. This phase predominantly focusses upon refining and extending the students' structural knowledge. As well as considering the mathematics they have learnt in relation to the world they live in, the reflection phase should involve learners considering the journey from reality to mathematics via abstraction that they took in developing the mathematical ideas. It requires thinking about what they learnt, how they learnt it, and why they learnt it. Affectively, such reflection on learning and the validation of knowedge against their everyday life is valuable because it leads learners towards ownership of their learning and their knowledge.

In summary, the RAMR cycle has been presented to teachers as a framework for organising their students' learning based upon their students' reality. This interpretation of the RAMR cycle has been used as the pedagogical basis for the XLR8 curriculum.

Research Design

The project has integrated decolonising approaches with predominantly qualitative methodology. The decolonising approach (Smith, 1999) focuses upon research which benefits the researched (the students and their communities) and will lead to empowering outcomes. The qualitative methodology was teaching experiment, a form of design research whose primary purpose is to provide opportunities for "researchers to experience, firsthand, students' mathematical learning and reasoning" (Steffe & Thompson, 2000 p.267). This research design has allowed the researchers to gather a variety of evidence during the project, including classroom observations, post-lesson discussions with teachers and more formal semi-structured interviews with teachers. In this paper, classroom observations provide examples of the RAMR cycle in use and comments offered by teachers during semi-structured interviews have been analysed to identify themes regarding the impact of the XLR8 curriculum in relation to the cognitive and affective aims of the project. This, in turn, is shaping to ongoing work of the project.

Two Examples of RAMR-based Classroom Practice

The following sub-sections present two examples of the use of the RAMR framework in two classrooms, taught respectively by Rod and Stan (pseudonyms). A brief introduction to each teacher is provided along with summary of observed lesson(s) in each classroom. Accompanying the examples are comments made by each teacher regarding how the XLR8 curriculum and RAMR pedagogy have influenced their students' learning.

Rod's Classroom

Rod had an undergraduate qualification in Tourism Management and a graduate diploma teaching qualification (Business and Geography); he had no formal mathematics education training. Prior to 2014, Rod taught for four years in a variety of locations, including rural, and had taught mathematics to a range of age groups. In 2014, the leadership at Rod's school decided to implement the XLR8 curriculum beginning in the second term of the school year. Rather than provide the XLR8 curriculum as a replacement for the normal mathematics curriculum, the XLR8 curriculum was used as a targeted intervention to address low levels of numeracy for a group of Year 8 students who had been identified during Term 1 as requiring additional help. These students attended two XLR8 lessons each week (nominally one hour each) in addition to their regular mathematics lessons. This class followed the XLR8 curriculum as written with little modification and was not aligned to the content of the regular mathematics lessons. This account of Rod's classroom focusses upon the use of the RAMR during a single lesson.

This single lesson was taught at the beginning of Term 2, 2014 when Rod and his class were very new to the XLR8 project. Upon entering the class, Rod announced the title of the lesson 'Gathering and Representing Data' and wrote the learning goal for the lesson on the whiteboard: 'Understanding different ways of representing data'. Rod's reality activity used a digital presentation: the first slide simply showed the numbers 5 and 15. Successive slides revealed more and more detail until the students recognised the column graph describing the gender of students in their class. As the details of the graph were revealed, Rod questioned the students to ascertain their pre-existing understanding of the content.

The class spent some time creating 'human graphs': Rod placed labels on the wall for each category (i.e., 0, 1, 2, 3, 4+) and asked students to stand in front of the label that represented how many pets the students had. This approach was repeated for several other topics, including eye colour and favourite takeaway food. The last example, takeaway food, was translated to the whiteboard. Each child wrote on a sticky-note their favourite food and placed the note on the whiteboard against each category. Rod queried the students as to how best arrange the notes, which led to each category of notes being arranged in a straight line and the annotation of the graph with features such as a title and axes labels. Finally, in this sequence of activities that allowed students to translate the mathematical ideas from physical to abstract mathematical representation, each student made their own copy of the graph in their notebook, and was reminded to include all the features of a well formed graph. As students finished drawing their graphs, Rod distributed a simple survey that he intended to use in future activities. Rod continued into the mathematics phase of the cycle by introducing different ways to represent the data, including stacked columns (to highlight the total class as the sum of its parts) and tally tables. These were to be used in subsequent lessons as the students explored the survey data, as part of the reflection phase, although this lesson was not observed by the researcher.

At the end of 2014, Rod was interviewed regarding his perceptions of the XLR8 program and its impact upon his students' learning. Rod indicated that most of his class time was spent on the Reality and Abstraction phases of the cycle and that little time was spent on consolidating the students' procedural knowledge in the mathematics phase or building connections in the reflection phase. In regard to the first two phases, Rod commented that "[the students] realised that they can do things and they want to do parts of the [activities]". Rod commented regarding the benefits of using materials when teaching concepts related to fractions and operations: the students were drawing diagrams to represent their understanding and so they had to "do it rather than just write down some numbers". Rod described this use of materials as "strategies to help them answer questions". Rod indicated that he believed the students' confidence had increased as a result of the XLR8 curriculum, citing examples of students' attitude towards completing post-test questions without commenting on them being too hard. Overall, Rod indicated that the XLR8 program had been beneficial for his students.

Stan's Classroom

Stan had a Bachelor of Education (Secondary Science and Physical Education) teaching qualification. At the start of 2016, Stan had five years of teaching experience, one term of which was at a secondary school participating in the XLR8 project. Stan had transferred to this secondary school to the role of Pedagogy Coach focussing upon numeracy and classroom teacher of junior secondary mathematics. Although Stan was new to the XLR8 program, he had experienced YuMi Deadly Mathematics and RAMR at his previous school. Stan's Year 7 students were new to the school and YuMi Deadly Mathematics. This account of Stan's classroom spans two RAMR cycles observed across several consecutive one-hour lessons.

In the first RAMR cycle students participated in discussions about quantities of discrete objects that could be counted as singles, groups or groups-of-groups. This led into an abstraction sequence that moved from concrete representations of quantity to symbolic representations using place-value charts for 2- and 3-digit numbers. The physical activity of creating singles, groups and groups-of-groups, and the concrete experience of grouping and ungrouping on the place-value chart were not completed as the teacher believed that the students' understanding of place-value was sound. Students demonstrated recognition of the symbolic value of digits in 5-digit numbers within the Mathematics phase as they engaged with the Wipeout game, using calculators to 'wipe out' a digit (i.e., make zero) in a number using a single subtraction calculation. The Reflection activity of relating singles, groups and groups-of-groups back to the real world was not completed with these students.

At the beginning of the second RAMR cycle, Reality phase conversations to connect students' thinking to singles, groups and groups-of-groups activities in preparation for exploring the multiplicative structure of place value and flexible renaming of numbers in readiness for later operations work did not occur. In the Abstraction phase, students did experience the kinaesthetic activity of forming into a group of nine students as a baseball team, a group of three for a basketball team, and solo players. Students explored deconstructing the baseball team into basketball teams and from there into solo players. The solo players were also grouped to create basketball teams. This activity was then represented symbolically on a place-value chart without students experiencing grouping/regrouping using materials. As a class, the multiplying factor between the places when changing teams was discussed as a comparison between the digits, but not connected to as a consequence of the grouping/regrouping action. The students then completed activities to explore base-10 place-value multiplicative relationships.

Students then explored a number of different ways to set up the float for a hypothetical shop using \$100 notes, \$10 notes and \$1 coins (written symbolically). They determined how many different combinations of the denominations they could use to represent the same amount. Students also experienced using number expanders to connect the language for reading and saying number names whilst manipulating the numbers. A human place value chart was used to act out movement of digits when the number is multiplied by ten. This was not explicitly connected to the previous activity of forming singles, groups and groups-of-groups, nor was the reason for multiplying by ten discussed. Students then proceeded to practice multiplying and dividing given numbers by ten without the use of place-value charts for assistance. This concluded the second RAMR cycle.

Stan commented that students in his class "have gained a sense that maths is attainable and relevant to them." He also stated that he has seen "students' confidence in the maths classroom improve so much that they truly believe that they can achieve", demonstrated by students "answering more questions, visibly enjoying activities and engaging more with activities in the classroom". Stan also believes that "several students have significantly improved their mathematical reasoning to raise conjectures and evaluate them, make generalisations, and come to conclusions based on reflecting what has been learned".

Discussion and Conclusion

Within Rod's implementation of the RAMR framework, the activities followed a connected sequence through physical actions to generate a graph, concrete representation using sticky notes, to students' own drawn representation in their books. The use of clear and consistent language and repetition of the same construction (albeit more abstract in each instance) in each of the abstraction activities meant that students were able to form the connections required between real-world representations of data, their abstraction phase activities, and the mathematical skill of representing their own collated data.

Stan's students had limited physical or iconic experiences of grouping/renaming quantity in place-value; they mostly experienced these ideas using symbols. During the regrouping/renaming activity that entailed flexible representations of numbers (e.g., 3 hundreds, 4 tens and 5 ones could be 34 tens and 5 ones or 3 hundreds and 45 ones), several students identified correctly that they could empty higher places by renaming down the place-value chart, but then also included examples where they grouped lower places up the place-value chart (e.g., 3 hundreds and 45 tens). Flexibility of number representation in this instance requires a clear understanding of the fact that smaller places may only be grouped up if there are sufficient smaller units to do so. Clear connections between grouping/reunitising activities on the place-value chart (using bundling sticks and/or MAB blocks) and symbolic representations may have prevented this misconception.

Student misconceptions were also evident when students were tasked with multiplying/dividing numbers by ten to explore the multiplicative relationship between place-values. One student had not realised that she could use the place-value chart as a tool to assist with renaming a number when multiplied/divided by ten. She became confused with how to read larger numbers that had been changed by multiplication as she was trying to use extended tens facts accompanied by a rote idea of increasing or decreasing the number of places. When a place-value chart was drawn, the initial number placed on the chart and the resulting number then placed accordingly, the student was able to use this visual prompt to correctly identify the changed value of the number. The student stated, "I did not know I could use the place-value chart to do that" and proceeded to draw a place-value chart up to one hundred million in the back cover of her book for later use as a tool to help with place value and larger numbers. For students to recognise the multiplicative relationship between place-values, students needed to experience the grouping/ungrouping activities with materials on the place-value chart before engaging with the symbolic activity of multiplying/dividing by ten. Without these connections, students were engaged in resizing a quantity rather than a regrouping/renaming activity. One student described the multiplier of ten as "magic". They did not seem to see it as a direct consequence of the regrouping/renaming of digits within a number. In this instance, good pedagogy and teaching still resulted in student misconceptions due to an incomplete focus on the structure of number and place-value stemming from a lack of experience (either physical or iconic) to which the symbols of mathematics could be applied.

Both teachers described in this paper interpreted and applied RAMR pedagogy and XLR8 curriculum to engage their students mathematically. With student engagement and participation, teachers were also able to observe improvements in student confidence and capacity to reason mathematically. While Rod's students appeared to develop a connected understanding of content, Stan's students demonstrated misconceptions. Stan's decision to skip aspects of the learning experiences within the curriculum resulted in the students experiencing a reduced range of representations for them to use in their construction of place-value understanding (Duval, 2006). Stan's false beliefs of students' understanding of place-value, formed as a result of not clearly ascertaining student knowledge in the reality phase and the absence of explicit links between reality-abstraction-mathematics further contributed to the disconnected nature of students' learning experience; subsequently reducing their capacity to accelerate (Cooper & Warren, 2011) towards deeper understandings of multiplicative relationships in place-value.

Considering both teachers' practice along the continuum of interpretation (Nutchey, Grant, Cooper & English, 2015), Stan appears to be an Improver of pedagogy, while Rod may be described as a Follower of pedagogy. Similarly, Stan appears to be a weak Follower of structure and sequence while Rod may be described as a questioning Follower of structure and sequence. Within the XLR8 program, the connections between mathematical structure and sequenced learning are carefully designed with a view to promoting connected understanding and accelerated learning of mathematics in underperforming students. However, an ongoing challenge within the XLR8 program has been the successful engagement of teachers with professional conversations around the value of structure and sequence. Teacher decision-making about activities to include within a teaching sequence is predicated on their recognition of student understanding and own knowledge of mathematical structure and sequenced learning.

In conclusion, student learning in both these classrooms demonstrated student improvement in confidence and ability to reason mathematically. For connections to mathematical structure to be made more overtly within the teaching sequence, teachers need further assistance with developing their understanding of sequencing and structure of mathematics. The misconceptions held by students in Stan's class may have been addressed had the students experienced the place-value concepts using materials. This supports our claim that the use of materials to model and experience mathematical concepts remains a critical component of effective learning sequences for junior secondary students, despite the challenges that working with materials may present.

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