Young Indigenous Students en Route to Generalising Growing Patterns

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This paper presents a hypothesised learning trajectory for a Year 3 Indigenous student en route to generalising growing patterns. The trajectory emerged from data collected across a teaching experiment (students n=18; including a pre-test and three 45-minute mathematics lessons) and clinical interviews (n=3). A case study of one student is presented as a representative of high achieving students' progression and shifts in learning. Results suggest that students are capable of functional thinking, which contradicts the notion of young students can only engage with recursive pattern sequences. In addition, particular teaching actions assisted in promoting shifts in developing students' capability to generalise growing patterns.

Early algebra is considered crucial to students' mathematical success, and therefore has been at the forefront of both national and international initiatives over the past decade (e.g. Australian Curriculum, Assessment and Reporting Authority, 2012; National Council of Teachers of Mathematics, 2006). Furthermore, algebra has been labelled as a mathematics gatekeeper for all students, having the potential to provide both economic opportunity and equitable citizenship (Satz, 2007). As algebra provides a pathway for potential equality and opportunity, and a reduction in exacerbated inequalities between ethnic and socioeconomic groups (Greenes, 2008), it is important to consider the significance of early algebra for young students in marginalised communities (Gonzalez, 2009). Within the Australian context, developing an understanding as how to best support the teaching and learning of early algebra for young Indigenous students is imperative.

Studies with non-Indigenous primary students have demonstrated that engaging with early algebra assists these students to develop a deeper understanding of mathematical structures that can lead to mathematical generalisations (Radford, 2010). One particular path for developing this thinking is through students working with growing patterns (Warren, 2005). There are two ways students may generalise growing pattern structures; recursive generalisations and functional generalisations (Blanton, Brizuela, Gardiner, Sawrey & Newman-Owens, 2015). When students display recursive thinking they are considering the relationship between successive terms in the pattern. Whereas when students exhibit functional thinking they are considering the relationship between both variables (pattern and the position; Warren, 2005). Studies have indicated that young students are capable of engaging in functional thinking (Cooper & Warren, 2011; Moss, et al., 2008), however there is limited understanding of how young students develop functional thinking. Recently, Blanton and colleagues (2015) have developed a learning trajectory to demonstrate one particular path 6-year old students may take when generalising functional relationships using the context of function tables. Despite this new development, important questions remain regarding, (a) if this trajectory is reflective of how young Indigenous students reason functions from growing pattern contexts, and (b) what teaching and learning actions promote shifts in students' thinking as they move towards algebraic generalisations (Blanton, et al., 2015). The aim of this paper is to examine an in-depth case study of one student's shift from recursive thinking to functional thinking and, highlight the teaching actions that supported this student's development of more sophisticated generalisations.

Literature

The ability to generalise mathematical structures beyond the initial learning experience has been highlighted as an important components of mathematics (Cooper & Warren, 2008). Consequently, it can be understood why generalisation has been described as the heart or the heartbeat of mathematical thinking (Mason, 1996). It can be implied that the ability to generalise is intrinsic to our success in mathematics, because it enhances our capability to apply mathematical concepts across mathematical tasks (Mason, 1996). Recently, there has been a growing body of literature exploring mathematical generalisation with younger students. Results of this research have shown that young students are capable of generalising mathematical structure across a range of contexts (Cooper & Warren, 2008). These contexts include generalising relationships between numbers and pattern rules, and generalising from particular examples in real-life situations to abstract representations (Blanton & Kaput, 2011; Cooper & Warren, 2011).

Current research has used visual geometric growing patterns as a way for students to generalise functional relationships (Cooper & Warren, 2011; Radford, 2010; Rivera & Becker, 2011). However, past research indicates that common issues arise when students generalise, and these are potentially due to the way the pattern tasks are presented and taught to students (Moss & Beatty, 2006). Growing patterns are often presented that limit students' awareness and accessibility to generalise the multiplicative pattern structures (Dörfler, 2008). Thus often leading to students articulating recursive thinking to express their generalisations rather than generalising the functional relationship. Thus, often students need to be scaffolded in order to recognise the pattern term number (independent variable) in geometric patterns (Moss et al, 2008). To overcome such an issue, independent and dependent variables need to be explicitly represented (Moss et al, 2008). As it is when students begin to co-ordinate between the variables that they shift from recursive thinking to functional thinking and thus form functional generalisations (Cooper & Warren, 2008).

Different ways to generalise have been identified in a number of research studies. Lannin (2005), for example, distinguishes between two types of generalisation: recursive and explicit. Harel and Tall (1991) theorised that there are three types of generalisation: expansive; reconstructive; and disjunctive and Radford's (2010) introduces the notion of "layers of generality": factual, contextual and symbolic generalisations. While there is agreement that students move through different stages during the generalisation process, how one generalises, and the processes that assist students to move through these stages, remains a largely unexplored realm.

Theoretical Framework

Two theoretical frameworks underpin the study, Indigenous research perspectives (Denzin & Lincoln, 2008) and learning trajectories (Clements & Sarama, 2004). In researching the cognitive interactions with young Indigenous students, it is important to acknowledge the potential for unique cultural variations with regard to how the outward displays of thought processes may be expressed. Thus, a decolonized approach has been adopted for this study with a focus on valuing, reclaiming, and having a foreground for Indigenous voices (Denzin & Lincoln, 2008). In essence, every attempt was made to ensure that the findings of this study best reflect how Indigenous students construct knowledge and engage in the learning process.

In order to understand the progression of students learning, a learning trajectory is adopted as the second theoretical framework for the study. The learning trajectory used to analyse the data from this study emerges from Blanton et al.'s work (2015) which reported the 'the levels

of sophistication in children's thinking about generalising functional relationships' (p. 244) using function tables. The following presents the learning trajectory that comprises eight levels of generalisation.

Pre-structural: "Students do not recognise that mathematical quantities could be related or notice that they were related by an underlying quantitative structure, nor did they understand how to articulate this structure" (p.525).

Recursive-Particular: "Students conceptualise a recursive pattern as a sequence of particular instances… and has not yet composed the underlying recursive pattern as a generalisation over a class of instances" (e.g. particular sequence 2, 4, 6, 8 but cannot generalise the process of adding two every time; p.527).

Recursive-General: "Students conceptualise a recursive pattern as a generalised rule between arbitrary successive values without referent to particular instances" (e.g. you add two every time; p. 529).

Functional-Particular: "Students conceptualise a functional relationship as a set of particular relationships between specific corresponding values. That is, students could describe a relationship within the specific but not generalised functional relationship over a class of instances" (e.g. 3 plus one more equals 4 (specific instance) but cannot generalise across instances; p.530).

Primitive Functional-General: "Students conceptualise a general relationship between quantities across a set of cases although their representations had primitive characteristics" (e.g. "they are all the same" – rather than the number of dogs is the same as the number of noses, or D=N; p. 532).

Emergent Functional-General: "Students reflect the emergence of key attributes of a generalised functional relationship, although their representation of the relationship was incomplete" (e.g. add the number of cares to itself (A+A) with no mention of the dependent variable; p. 533-535).

Condensed Function-General: "Students conceptualise function as a generalised relationship between two arbitrary and explicitly noted quantities" (e.g. Whatever the number, how many stops it made, if you doubled it, that's how many cars it would have - R+R=V; p. 535,6).

Function as Object: "Students perceived boundaries concerning the generality of the relationship and conceptualise the relationship structurally, as a new object in its own right on which new processes could be performed. (e.g. adding a constant value to a previously generalised structure; p. 537,8).

Research Design

This study was conducted with Year 2 and 3 students in a single multi-age classroom (7-9 year olds) from an urban Indigenous school in North Queensland. In total, 18 students, 2 Indigenous education officers, and 1 "researcher as teacher" participated in the study. The class was purposively selected for two reasons; (a) a relationship had been formed with the school prior to the study, an important aspect of Indigenous research perspectives; and, (b) students had no previous formal lessons in growing patterns.

Data Collection

Teaching experiments (Confrey & Lachance, 2000) and clinical interviews (Opper, 1977) were employed in the study to explore the teaching and learning process as students developed their thinking about generalising functional relationships. Each teaching experiment consisted of (a) a pre-test to ascertain what the students knew prior to the lessons; (b) three 40-minute video-recorded mathematics lesson taught by the researcher (2 video-recordings – one focusing on the students and the other on the researcher), and (c) audio-taped interviews with the Indigenous education officers. At the conclusion of the teaching experiment, one-on-one Piagetian clinical interviews were conducted with students (n=3). The interviews were also video-recorded and provided opportunity to further explore students' mathematical thinking. This paper reports data from a single case study (S1 – Aboriginal girl),

who participated in the entire teaching experiment. This student was selected as she represented a high-achiever in mathematics, as identified by the classroom teacher and Indigenous education officer.

Data Analysis

Data analysis was concurrent with the data collection and informed each stage of the research design. The pre-test analysis determined students' prior knowledge of growing patterns in terms of both their capability to answer the question, and also the ways in which students responded to tasks. Analysis of the pre-test informed the lessons that occurred in the first teaching experiment. Two phases of analysis occurred during the lessons: ongoing and in-depth analysis. At the conclusion of each lesson, analysis occurred to inform the teaching for the following day. Video-data and field notes were compiled and peer-debriefing occurred between the researcher, supervisor, teacher, and Indigenous education officer to determine conjectures for the following lesson.

After the pre-test, mathematics lessons, and clinical interviews, an in-depth analysis was conducted to capture students' verbal responses. Member checks occurred through the interviews to determine that the researcher had correctly interpreted students' responses. These responses were transcribed from the video-recordings and coded using the 8 levels of students' thinking about generalisation as described by Blanton and colleagues (2015). Once, the student data had been coded for these levels of thinking, the data were re-analysed to determine the teaching and learning interactions that occurred between each level. To ensure that all data represented students' cultural interactions the Indigenous education officers analysed the data paying attention to the cultural nuances. Audio-recorded discussions were transcribed and aligned with the student transcriptions.

Findings

The following presents findings, of an in-depth case study (S1), according to the chronological order in which the data were collected. Table 1 presents the types of functional relationships explored for each growing pattern given to students in the pre-test, lessons and interview.

Table 1.
Pattern Name and Functional Relationship Explored in Each Task

	Pattern Name	Functional relationship explored
Pre-test	Possum pattern	The number of possum tails and the number of possum eyes. $(y = 2x)$
Lesson 1	Butterfly pattern	The number of butterflies and the number of butterfly wings. $(y = 4x)$
Lesson 2	Feet pattern	The number of people and the number of feet. $(y = 2x)$
Lesson 3	Kangaroo pattern	The number of kangaroo tails and the number of kangaroo ears. $(y = 2x)$
Interview Task 1	Crocodile pattern	The number of crocodile tails and the number of crocodile legs. $(y = 4x)$
Interview Task 2	Classroom pattern	The year level (position card) and the number of student tables (square tiles; $y = 3x$)
Interview Task 3	Classroom and Teacher pattern	The year level (position card) and the number of student tables and one teacher (square tiles; $y = 3x + 1$).

During each lesson and interview task students were asked to: (a) continue the pattern with hands-on materials attending to the structure of the pattern, (b) predict and create the next/previous position of the pattern, (c) predict the quasi-variable position, (d) identify the pattern rule, and (e) generalise using alphanumeric notation. At the completion of each task the generalisation that S1 proffered was coded according to Blanton et al's 8 levels of generalisation. Figure 1 illustrates S1's progression of thinking and aligns the levels of generalisations presented in the theoretical framework (Blanton et. al., 2015).

Levels of Generalisation	Student 1						
8. Function as an object							
7. Condensed Functional-General				■			■
6. Emergent Functional-General						■	■
5. Primitive Functional-General					■	■	■
4. Functional Particular					■		
3. Recursive-General	■		■				
2. Recursive-Particular		■					
1. Pre-structural							
	PT	L1	L2	L3	IT1	IT2	IT3

NB: PT=Pre-test; L1 = Lesson 1; L2 = Lesson 2; L3= Lesson 3; IT1 = Interview task 1; IT2 = Interview Task 2; IT3 = Interview Task 3.

Figure 1. Progression of thinking demonstrated by S1 across teaching experiment 1.

As shown in Figure 1, as the lessons and interviews progressed S1's ability to generalise became more and more sophisticated. To further illustrate the shifts in S1's thinking the following data are presented with excerpts from the interview transcripts. Teaching actions that assisted S1 to shift her thinking are underlined throughout the text.

Recursive General (Pre-test): S1 predicted that for 10 possum tails there would be 20 eyes. When asked, "how would you work it out for any number of possum eyes?" S1's response was "I can count in twos".

Recursive Particular (Lesson 1): S1 expressed that on day five there were five butterflies in her garden with 20 butterfly wings. However, S1 needed to count the butterfly wings each time and only discussed this pattern in particular instances. S1 had difficulty describing the multiplicative structure ("fourness") of the pattern.

Recursive General (Lesson 2): S1 predicted how many people there were if 20 feet were on the ladder (10 people; Inverse function). When asked to explain how she arrived at her answer, S1 responded, "Counted in two's till I got to ten people." S1 was then asked, "What would you do for 60 feet?", S1 responded, "Count forwards two, four, six, eight, ten.... 22."). This response by S1 suggests that she was attending to only one variable, and therefore only the recursive structure of the pattern (plus two each time).

Condensed Functional-General (Lesson 3): S1 attended to both variables when working with the kangaroo pattern. She was able to express further predictions of the pattern using both the tail and the ears to communicate her understanding. This task was designed so that both variables were explicit and embedded in the kangaroo pattern. S1 explained to the class that if she had 1 million kangaroo tails she needed to "double the number of tails to work out how many ears" there were.

Recursive General to Functional Particular (Interview Task 1): S1 was using an additive process to determine how many crocodile feet were in each term of the pattern ('growing "in

fours"). S1's attention was <u>drawn to the structure of the pattern, this included explicitly introducing the mathematical language in conjunction with gestures from the researcher (pointing between the tails and the feet).</u> This was intentional and framed the structure to assist S1 to move beyond the additive rule. Once the language was grasped by S1, and the connection was made between the mathematical language and the structure, she was asked to determine what the rule would be if she had any number of crocodiles. S1 responded, "Times four".

Functional Particular to Emergent Functional-General (Interview Task 2): S1 was asked what the classroom pattern would look like at Grade 20 (response - 20 rows of three) and Grade 1 million (response - 1 million rows of three). S1 applied the structure of the pattern to <u>predict quasi-variables</u> (Cooper & Warren 2011). At this point in the interview the researcher <u>unpacked the mathematical language of multiplication</u>, S1 selected "times" and trialled it for each of the pattern positions presented (e.g. Four rows of three is twelve, Four times three is twelve). S1 was then asked to generalise the classroom pattern, "What if I had a class called grade n? What would I have to do?". S1, replied "Times it by three".

Primitive Functional-General to Emergent Functional-General to Condensed Functional-General (Interview Task 3): S1 was then presented with the classroom and teacher pattern that incorporated a variable with a constant (+1), modelled as "the teacher's desk" (orange tile). S1's trialled new rules (e.g. multiply by four), and was unsure how to explain the new teacher desk. The researcher <u>prompted her to think about the terms used in mathematics when we join two things together</u>. S1 was then able to articulate that she was adding the teacher on each time. She quickly moved to being able to provide a quasi generalisation and the pattern rule for any number. S1 was asked "What do you think our new rule might be? S1 respond, "Times three plus one". S1 rapidly went on to providing a quasi generalisation ("100 times three plus one"), the pattern rule for any class ("any number [*gesturing to the position cards*] times three plus one"), and the pattern rule for n classes ("times three plus one").

In summary, particular teaching actions that appeared to make a shift in student thinking included making variables explicit, attending to the underlying multiplicative structure and mathematical language, and moving students towards quasi-variables.

Discussion and Conclusion

Analysis from the data suggests that young Indigenous students are capable of identifying and articulating the functional relationship of growing patterns. It appears that the learning trajectory presented is similar to findings from studies conducted using function tables (Blanton et al., 2015). The case presented demonstrates that S1 was able to identify and articulate the general structure of a growing pattern in a number of ways and during the teaching experiment she moved in and out of different levels of generalisations. Similar to the case presented in Blanton et al.'s study, S1 moves through a comparable learning trajectory.

Prior research suggests that an understanding of multiplicative thinking is fundamental for older students when generalising growing patterns (e.g. Rivera & Becker, 2011), however, this study suggests prior understanding of multiplicative thinking is not a prerequisite for generalising growing patterns for young students. Though past research suggests that addition and multiplication of whole numbers are pre-requisites when generalising linear patterns (Rivera & Becker, 2011), this was not necessarily the case for this study with young Indigenous students. It is argued that in the early years context, linear growing patterns provide a platform for developing an understanding of mathematical operations and arithmetic. In the case of the present study, S1 initially used the pattern structure to explore additive relationships, but quickly moved to the exploration of multiplicative thinking.

Consequently, while having a strong understanding of addition and multiplication would assist students to generalise the pattern, and definitely deduce the pattern, it was found not to be a necessity for these young Indigenous students. What did appear to assist S1 to engage with the pattern and develop generalisations, stemmed from: (a) the choice of pattern type; (b) supporting S1 to access the underlying structure and the mathematical language; and, (c) asking S1 to generalise patterns using quasi-variables.

Past research has highlighted an issue that arises from functional situations is the need to coordinate two data sets, and identify the relationship between these sets (Blanton et al., 2015). Thus, in this present study the growing patterns selected for the tasks were deliberately chosen to ensure that this relationship was transparent and explicitly represented. This was achieved by using hands-on materials where the variables were explicit (pattern term cards, and coloured tiles) or could not be physically separated (embedded; e.g. plastic toy kangaroos and crocodiles). It is conjectured that representing growing patterns in this manner assisted in S1 attending to both variables in the pattern, potentially pushing her towards functional thinking rather than recursive thinking.

This study adds research that suggests that in conjunction with how the pattern was presented; gesture, language, and hands-on materials contribute to making the underlying structure apparent for non-Indigenous students (Radford, 2011). To highlight particular structures of the pattern, specific and purposeful gestures were used as the researcher deconstructed the pattern. It was essential to gesture between the variables (pattern term, pattern quantity, and constant) as the pattern was deconstructed. During this process, there was a deliberate coordination between these gestures with the hands-on materials and the use of mathematical language (Radford, 2011). An example of the coordination of gesture and language is identified in Interview task 1 (cf. Miller & Warren, 2015).

Finally, it appears that the use of quasi-variables potentially assists young Indigenous students to generalise growing pattern structures. By using quasi-variables, S1 was able to observe the general structure of the pattern regardless of the fact she had limited prior formal teaching of multiplicative thinking. The quasi-variable (e.g. 371th position) pushed S1 to see the structure of the pattern. This is potentially because students find it challenging and unproductive to apply an additive rule to a quasi-variable to determine the pattern quantity. This notion has also been supported in past studies with young students (e.g. Cooper & Warren, 2008). The findings of this present study also highlight that challenging young students to extend beyond their computational knowledge, results in a shift from an arithmetic approach when describing the pattern, to engaging in algebraic thinking (Radford, 2011).

This paper begins to illuminate the learning trajectory in which young Indigenous students may take en route to developing functional generalisations while engaging with growing pattern tasks. Additionally, it adds to the literature by providing potential teaching actions that may assist students to move through these levels of sophistication. As one case is presented, it is acknowledged that there are limitations to the generalisablity of the data. Thus, further analysis of the larger cohort is to be conducted to determine if (a) both Indigenous and non-Indigenous students have similar trajectories, and (b) if the teaching actions employed with S1 contribute to changes in the larger cohorts thinking about growing pattern generalisations.

References

Australian Curriculum, Assessment and Reporting Authority (2012). The *Australian curriculum: Mathematics*. Retrieved from http://www.australiancurriculum.edu.au/Mathematics/Curriculum/F-10

Blanton, M., & Kaput, J. (2011). Functional thinking as a route into algebra in the elementary grades. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives.* (pp. 5–23). Berlin, Germany: Springer.

Blanton, M., Brizuela, B. M., Gardiner, A. M., Sawrey, K., & Newman-Owens, A. (2015). A learning trajectory in 6-year-olds' thinking about generalizing functional relationships. *Journal for Research in Mathematics Education*, *46*(5), 511-558.

Clements, D., & Sarama, J. (2004). Learning trajectories in mathematics education. *Mathematical Thinking and Learning*, *6*, 81–89.

Confrey, J., & Lachance, A. (2000). Transformative teaching experiments through conjecture-driven research design. In A. Kelly & R. A. Lesh (Eds.), *Handbook of research design in mathematics and science education* (pp. 231–265). Mahwah, NJ: Lawrence Erlbaum Associates.

Cooper, T., & Warren, E. (2008). The effect of different representations on Years 3 to 5 students' ability to generalise. *ZDM*, *40*(1), 23–37.

Cooper, T. J., & Warren, E. (2011). Years 2 to 6 students' ability to generalise: Models, representations and theory for teaching and learning. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives.* (pp. 187–214). Berlin, Germany: Springer.

Denzin, N. K. & Lincoln, Y. S. (2008). Critical methodologies and Indigenous inquiry. In N. K. Denzin, Y. S. Lincoln, & L. T. Smith (Eds.), *Handbook of critical and Indigenous methodologies* (pp. 1–30). London, UK: SAGE Publications.

Dörfler, W. (2008). En route from patterns to algebra: Comments and reflections. *ZDM*, *40*(1), 143–160.

Gonzalez, L. (2009). Teaching mathematics for social justice: Reflections on a community of practice for urban high school mathematics teachers. *Journal of Urban Mathematics Education, 2*(1), 22–51.

Greenes, C. (2008). Preface. In C. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp. ix–xii). Reston, VA: National Council of Teachers of Mathematics.

Harel, G., & Tall, D. (1991). The general, the abstract, and the generic in advanced mathematics. *For the Learning of Mathematics*, *11*(1), 38–42.

Lannin, J. (2005). Generalization and justification: The challenge of introducing algebraic reasoning through patterning activities. *Mathematical Thinking and Learning, 7*(3), 231–258.

Mason, J. (1996). Expressing generality and roots of algebra. In N. Bednarz, C. Kieran, & L. Lee (Eds.), *Approaches to algebra: Perspectives for research and teaching* (pp. 65–86). Dordrecht, The Netherlands: Kluwer Academic Publishers.

Miller, J. & Warren, E. (2015). Young Australian Indigenous students generalising growing patterns: A case study of teacher/student semiotic interactions. In Beswick, K., Muir, T., & Wells, J. (Eds.) *Proceedings of the 39th Conference of the International Group for the PME* (Vol. 3). Hobart, Australia: PME.

Moss, J., & Beatty, R. (2006). Knowledge building and knowledge forum: Grade 4 students collaborate to solve linear generalizing problems. In J. Novotná, H. Moraová, M. Krátká, & N. Stehlíková (Eds.), *Proceedings of the 30th Annual Conference of the International Group for PME* (Vol. 4, pp. 193–199). Prague, Czech Republic: Charles University.

Moss, J., Beatty, R., Barkin, S., & Shillolo, G. (2008). What is your theory? What is your rule? Fourth graders build an understanding of functions through patterns and generalizing problems. In C. E. Greenes & R. Rubenstein (Eds.), *Algebra and algebraic thinking in school mathematics* (pp. 155– 68). Reston, VA: National Council of Teachers of Mathematics.

National Council of Teachers of Mathematics (2006). *Curriculum focal points for prekindergarten through grade 8 mathematics: A quest for coherence*. Reston, VA: Author.

Opper, S. (1977). Piaget's clinical method. *Journal of Children's Mathematical Behavior*, *1*(4), 90–107.

Radford, L. (2010). Layers of generality and types of generalization in pattern activities. *PNA*, *4*(2), 37–62.

Radford, L. (2011). Grade 2 students' non-symbolic algebraic thinking. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 303–322). Berlin, Germany: Springer.

Rivera, F. D., & Becker, J. R. (2011). Formation of pattern generalization involving linear figural patterns among middle school students: Results of a three-year study. In J. Cai & E. Knuth (Eds.), *Early algebraization: A global dialogue from multiple perspectives* (pp. 323 – 366). Berlin, Germany: Springer.

Satz, D. (2007). Equality, adequacy, and education for citizenship. *Ethics*, *117*(4), 623–648.

Warren, E. (2005). Young children's ability to generalize the pattern rule for growing patterns. In H. Chick & J. Vincent (Eds.), *Proceedings of the 29th Annual Conference of the International Group for the PME* (Vol. 4, pp. 305– 312). Melbourne, Vic.: University of Melbourne.