The Role of Reasoning in the Australian Curriculum: Mathematics

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The mathematical proficiencies in the *Australian Curriculum: Mathematics* of understanding, problem solving, reasoning, and fluency are intended to be entwined actions that work together to build generalised understandings of mathematical concepts. A content analysis identifying the incidence of key proficiency terms (KPTs) embedded in the content descriptions from Foundation to Year 9 revealed a much lower representation of "actions" relating to the proficiency reasoning than to the other three proficiencies. A generalised model of patterning is proposed to provide an interrelated view of the proficiencies and to further support the development of generalised understandings in mathematics education.

Mathematics is widely accepted "as a subject that consists of patterns and relationships that are understandable through mental activity that involves mathematical reasoning and logic" (Wood, 2002, p. 61). The goal of mathematics education is clearly articulated in the Australian Curriculum: Mathematics (ACM) rationale statement: "It aims to instil in students an appreciation of the elegance and power of mathematical reasoning" (Australian Curriculum and Assessment Reporting Authority [ACARA], 2015, p. 4). Reasoning is recognised as paramount in the development and growth of mathematical understanding (Ball & Bass, 2003; Mason, Stephens, & Watson, 2009). In the ACM reasoning is singled out as one of the four mathematical proficiencies: understanding, problem solving, reasoning, and fluency. These are identified as key processes that describe "the actions in which students can engage when learning and using the content" and similarly inform teachers "how the content is explored or developed" (ACARA, 2015, pp. 4, 5). The content knowledge in the ACM is structured around three strands that "describe what is to be taught and learnt" (p. 5) and the mathematical actions of the proficiencies are embedded in the content descriptions. Therefore it is the interaction within and between these content strands and the four proficiencies that builds conceptual understanding in mathematics.

Mathematical reasoning is described as the "capacity for logical thought and actions such as analysing, proving, evaluating, explaining, inferring, justifying and generalising" (ACARA, 2015, p. 5). Reasoning involves recognising similarity and differences encountered in concepts explored across multiple contexts leading to the development of abstract understandings. Explaining and justifying thinking enables knowledge to become "more general and its applicability to different situations … increased" (White & Mitchelmore, 2010, p. 2). Intentional instruction supports conceptual understanding to deepen, become more fluently recalled, and applicable in new learning contexts. Ball and Bass (2003) emphasise the role of the teacher in promoting reasoning, as "mathematical understanding is meaningless without a serious emphasis on reasoning" (p. 28). Engaging students in mathematical reasoning naturally draws students into greater levels of fluency as they connect their understandings in new problem-solving contexts.

Sullivan (2012) proposes that teacher learning should focus on "ways of identifying tasks that can facilitate student engagement with all four of these proficiencies" (p. 183) as the "intention is that the full range of mathematical actions apply to each aspect of the content"

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(Sullivan, 2011, p. 8). However, the organisational structure of the curriculum as three content strands comprising number and algebra, measurement and geometry, and statistics and probability, draws attention to content knowledge. How the proficiencies together build entwined conceptual understanding is well intended in the rationale of the ACM but not clearly articulated within the content strands. This raises key questions addressed in this paper: In what ways do the proficiencies in the ACM build generalised understandings and reasoning skills? Is this relationship between reasoning and generalised understanding of mathematics evident and transparent to teachers accessing the curriculum?

At a theoretical level, an interrelated view of the proficiencies will be discussed in light of a generalised model of patterning proposed by McCluskey, Mitchelmore, and Mulligan (2013) to highlight the importance of reasoning. An outcome of this paper is to identify how the proficiencies are articulated in the ACM through a content analysis of key language terms embedded in the content descriptions denoting the "actions" of the four proficiencies across Foundation to Year 9.

Background

In the rationale of the ACM the role of the mathematical proficiencies is highlighted: "The curriculum focuses on developing increasingly sophisticated and refined mathematical understanding, fluency, logical reasoning, analytical thought and problem solving skills" (ACARA, 2015, p. 4). They are described as capabilities that "enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently" (ACARA, 2015, p. 4). The ACM describes the field of mathematics as "composed of multiple but inter-related and interdependent concepts and systems" (ACARA, 2015, p. 4), anticipating that teachers and students will engage with the ACM in a dynamic and symbiotic way and thus implying, similarly, that the proficiencies are also interrelated.

There is a clear intent in the introductory sections of the ACM to highlight the mathematical proficiencies as integral aspects of the curriculum. They are described in the *Key Ideas* section directly following the rationale and aims, and they are also outlined again in the next section, *Structure*, before the description of the content strands. Importantly, the proficiencies are embedded in the language of the content descriptions and achievement standards as verbs that describe the mathematical actions students engage with (Sullivan, 2012). This is demonstrated in the following content description: "Interpret and compare data displays" (ACARA, 2015, Section ACMSP070): the verbs *interpret* and *compare* identify use of the mathematical proficiencies. Throughout the ACM the proficiencies are described individually, rather than an entwined system at the beginning of each year level. However, naming and identifying individual proficiencies may not encourage teachers to focus on the potential interrelationships between the proficiencies to build and deepen conceptual understanding. It is their connectedness that is not well articulated and thus does not resonate clearly with the rationale.

In *Engaging the Curriculum-Mathematics: Perspectives from the field*, Atweh, Miller, and Thornton (2012) identified challenges that schools and educators could face in interpreting and implementing the curriculum due to this "possible lack of cohesion between the aims and rationale, the content and its articulation" (p. 2). In particular, they noted inconsistencies in emphasis between the proficiencies, such as the role of reasoning which they argued was underrepresented in the content elaborations. Therefore, in this paper an interrelated view of the proficiencies is explored to address this imbalance and support the development of generalised understandings in mathematics.

Interrelationships Between Mathematical Proficiencies

Atweh et al. (2012) highlight the interrelationships between the proficiencies, explaining that these “proficiencies are not disjointed … [and that] … some content elaborations may relate to one or more of the proficiencies” (p. 8). They refer to a model, focused on mathematical proficiency, described in the United States report to the National Research Council (Kilpatrick, Swafford, & Findell, 2001). In this model, based on five strands, the term mathematical proficiency is used to “capture what we think it means for anyone to learn mathematics successfully … the most important observation we make about these five strands is that they are interwoven and interdependent” (Kilpatrick et al., 2001, p. 5) and “represent different aspects of a complex whole” (p. 116). For Kilpatrick et al., these strands are adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency. The following descriptions explain the mathematical actions relating to these strands of proficiency from this model.

- Conceptual understanding “includes the comprehension of mathematical concepts, operations and relations”.
- Procedural fluency includes skill “in carrying out procedures flexibly, accurately, efficiently, and appropriately, and, in addition to these procedures, having factual knowledge and concepts that come to mind readily”.
- Strategic competence is “the ability to formulate, represent and solve mathematical problems”.
- Adaptive reasoning is “the capacity for logical thought, reflection, explanation and justification”.
- Productive disposition is “a habitual inclination to see mathematics as sensible, useful and worthwhile, coupled with a belief in diligence and one’s own efficacy” (Watson & Sullivan, 2008 as cited in Sullivan, 2011, pp. 6–7).

Kilpatrick et al. (2001) stressed the importance of the relationship between all strands in building resilient understandings that can be fluently applied in new situations. They refer to findings from cognitive science that indicate that “competence … depends upon knowledge that is not merely stored but represented mentally and organized (connected and structured) in ways that facilitate appropriate retrieval and application …. Organization improves retention, promotes fluency, and facilitates learning related material” (p. 118). Proficiency in mathematics involves the construction of effective neural networks that are structured in resilient and flexible ways to both connect understanding and accommodate new learning efficiently. This description proposes a view of the proficiencies in the ACM working interdependently to build conceptual understanding systematically. However, defining the proficiencies as individual strands still accentuates their separateness, not their integrated relationship in building patterns of thinking.

The ACM Proficiencies as an Opportunity for Changing Practice

Sullivan (2012) has asserted that the ACM provides an opportunity for educators to re-think and reshape mathematics learning for students by focussing on “the principles that underpin the structure of the curriculum and the use of these principles to inform teacher learning” (p. 175). These principles are that:

- the four proficiencies provide a framework for mathematical processes,
- the ACM has been designed to emphasise teacher decision making, and
- there is a focus on depth rather than breadth to address challenges of equity.

Sullivan identified the mathematical proficiencies as the first key principle that connects the other two principles, emphasising that engagement with the mathematical proficiencies encourages educators to make pedagogical decisions to explore not just the breadth but also importantly the depth of mathematical concepts. Incorporating learning experiences in relevant problem-based contexts creates opportunities for students to engage meaningfully with the mathematical proficiencies. "Mathematics … is more than following rules and procedures but can be about creating connections, developing strategies, effective communication … this view is not obvious in the content descriptions … it is part of the opportunity for those supporting teachers to communicate such views … [and is] … communicated through the proficiencies that underpin the curriculum" (Sullivan, 2012p. 179). This raises the issue of whether the language identifying the proficiencies is visible to, and used by, teachers accessing the curriculum.

Identifying the Language of the Proficiencies

In describing this dynamic view of learning, Sullivan refers to the use of verbs identifying the actions of individual proficiencies. It is intended that teachers look within, across, and beyond the content descriptions to connect with the language that articulates the proficiencies. Taking up this point, Atweh et al. (2012) analysed the occurrence of the proficiencies stated in the ACM Year 8 content elaborations, finding that "53% relate to experiences to develop understanding … 56% relate to developing fluency … 12% relate to problem solving … and 7% refer to reasoning" (pp. 8–9). In this analysis, the proficiency of reasoning, an essential element in the development of generalised understandings, was rarely identified in the content elaborations.

However, reasoning may be represented in the use of language terms describing problem solving. Sullivan (2012) highlights the role of problem solving by engaging the proficiencies, in particular reasoning, through problem-based contexts. Investigating problem-based approaches assumes that "the teacher draws upon the various strategies used by the students" and that the learning "experience will communicate to students that there are many ways to approach mathematical tasks, they can choose their own approach, and that some approaches are more efficient than others" (p. 179). This type of thinking, authentically embedded in problem-solving contexts, builds a capacity to reason but is dependent upon teachers' awareness of "structural relationships … [and] strategies … [for]… bringing structural relationships to the fore" (Mason, Stephens, & Watson, 2009, p. 29). Structural relationships emerge from engaging in opportunities to reason. This involves generalising commonalities about concepts across contexts. Therefore the use of language terms in the ACM that relate to various proficiencies, in particular reasoning, requires further investigation.

Content Analysis: Reasoning

In the research reported here, an initial phase of a content analysis was used to identify the type of language used to describe the actions of the proficiencies. This was conducted to find evidence of terms related to reasoning that were articulated in the ACM. This content analysis extracted key proficiency terms (KPTs) that "can be thought of as verbs" (Sullivan, 2012, p. 179) from the content descriptions. (Note that some terms such as *efficiently*, *accurately,* and *appropriately* are adverbs and were included as KPTs if they modified a verb in the content description). The process occurred in the following four stages:

1. Each proficiency description in the key ideas section was analysed for KPTs.
2. A framework was constructed, identifying the KPTs that related to each proficiency. Table 1 indicates the KPTs by proficiency.
3. The KPTs embedded in the content descriptions from Foundation to Year 9 were extracted and categorised using the framework in Table 1 to compare the frequency of their use throughout the content descriptions from Foundation to Year 9. (Note, some KPTs recorded in Table 1 relate to more than one proficiency; however each KPT extracted from the content descriptions was counted to calculate the total number of occurrences relating to each proficiency.)
4. The KPTs embedded in the content descriptions from Foundation to Year 9 were counted and categorised using the framework in Table 1, to compare the frequency of their use throughout the content descriptions from Foundation to Year 9. Table 2 contains entries that summarise the total number KPTs identified across F–2, 3–6, and 7–9 content descriptions.

Table 1

Key Proficiency Terms (KPTs)

Proficiency strand	Key proficiency terms (KPTs)
Understanding	Apply, build, connect, describe, develop, identify, interpret, make, represent
Fluency	Accurately, answering, appropriately, calculate, carrying, choose, choosing, develop, efficiently, find, manipulate, flexibly, recall, recalling, readily, recognise, regularly, use
Problem solving	Apply, communicate, design, develop, effectively, formulate, interpret, investigate, make, model, plan, represent, seek, solve, use, verify
Reasoning	Adapt, analysing, compare, contrast, deduce, develop, evaluating, explain, explaining, generalising, increasingly, inferring, justify, justifying, known, mathematically, prove, proving, reached, reasoning, transfer, thinking, used

One could assume for each year level clustering (i.e., F–2, 3–6, and 7–9) that the individual proficiencies would be equally represented, with a similar proportion of KPTs relating to each of understanding, fluency, problem solving, and reasoning. However, this is not the case, with problem solving noticeably over-represented in Years 3–9: F–2: 26%, 3–6: 35%, and 7–9: 45%; and reasoning consistently under-represented across the year level clusters: F–2: 19%, 3–6: 14%, and 7–9: 13%.

Table 2

Frequencies and Percentages of Key Proficiency Terms (KPTs) Across the ACM [a]

Year level clusters	ACM proficiency strands				Total KPTs
	Under-standing	Fluency	Problem solving	Reasoning	
F–Year 2	33 (26)	36 (29)	32 (26)	24 (19)	125 (100)
Years 3–6	83 (29)	65 (22)	102 (35)	42 (14)	292 (100)
Years 7–9	33 (17)	50 (25)	89 (45)	25 (13)	197 (100)
F–Year 9	149 (24)	151 (25)	223 (36)	91 (15)	614 (100)

[a] Cell entries are frequencies (row percentages)

Across the early years of school (F–2), a total of 125 terms were extracted from the F–2 content descriptions. From these, 19% related to reasoning, 29% related to fluency, and 26% each for KPTs relating to understanding and problem solving. This reflects the emphasis in the early years of developing conceptual understanding and fluency of procedural knowledge and processes through problem-solving contexts. However, reasoning is critical in the development of mathematical concepts. Further analysis will reveal if KPTs identifying reasoning are represented more in the later years of school. Throughout the primary years (3–6) there is an increasing incidence of KPTs embedded overall in the content descriptions. KPTs identifying understanding and problem solving were noted more frequently than were those identifying fluency and reasoning. KPTs relating to reasoning were identified 42 times from an overall count of 292 KPTs, resulting in only 14% of the total terms extracted. Similarly, in the middle years (7–9) an increasing focus on exploring content through problem-solving contexts is recognised, as 45% of the total KPTs identified across Years 7–9 related specifically to the proficiency problem solving. Fluency received 25% of the KPTs, understanding 17%, and reasoning 13%.

Overall, problem solving is predominantly represented in this content analysis, with 36% of total terms relating to developing this proficiency across years F–9. Understanding and fluency are similarly weighted, with 24% and 25% of the KPTs respectively. However, only 15% of KPTs from Foundation to Year 9 describe actions that relate specifically to students engaging in reasoning in their learning in mathematics. A higher representation of KPTs identifying problem solving could be attributed to the intent described in the ACM rationale "these proficiencies enable students to respond to familiar and unfamiliar situations by employing mathematical strategies to make informed decisions and solve problems efficiently" (ACARA, 2015, p. 4). It could be inferred in the ACM that reasoning would be built into this process of problem solving. However, this is not evident in the KPTs extracted. This is a limitation of the analytic process used here and the problem that differentiating the proficiencies individually presents. If reasoning is embedded in problem-solving contexts, this could be made explicit in the description of the proficiencies as an integrated system.

Integrating the Proficiencies

A generalised model of patterning (McCluskey, Mitchelmore, & Mulligan, 2013) has been proposed as a means of describing the abstraction of patterning across differing domains of knowledge. It was noted that patterning moves through a progressive cycle in building generalised understandings within and beyond mathematics in that:

- a sense of familiarity is experienced with known situations,
- similarity experienced across contexts is encoded in the conceptual structure of the pattern,
- patterns are activated when similarity is recognised, and
- familiar patterns are accessed more fluently and applied in new contexts.

Thus, we propose that, all four proficiency strands of *understanding, fluency, problem solving*, and *reasoning* in the ACM can naturally work together as an integrated whole, in a cyclic structure, building and deepening generalised patterns of mathematical understanding with a focus "on depth of learning rather than breadth" (Sullivan, 2012, p. 185). For example, as *understanding* is connected across *problem-solving* contexts, similarities about mathematical concepts are recognised, and students develop *reasoning* as they construct generalisations. Over time, *fluency* in recognising and engaging with similar problems is strengthened with an increasing capacity to transfer *understanding* to new contexts. The four proficiencies have a combined role in systematically building patterns of generalised understandings through this pedagogical cycle.

Summary and Recommendations

The ACM heralds in an opportunity for educators to focus on the interrelated development of the mathematical proficiencies, a key principle that underpins the curriculum (Sullivan, 2012). The importance of reasoning is clearly articulated in the rationale in the ACM. However, the KPTs that articulate reasoning appear to be noticeably under-represented in the content descriptions from Foundation–Year 9. In contrast, a clear emphasis on students engaging their thinking through problem-solving contexts was identified throughout the F–9 curriculum content descriptions. Sullivan (2011, 2012) has emphasised pedagogical use of relevant problem-solving contexts and approaches as a means of engaging a greater breadth and depth of proficiencies through teachers' choice of task design and consequent learning experiences for students. Similarly, the heavier weighting of KPTs relating to problem solving, identified through the content analysis, could encourage teachers to adopt practices and design learning experiences that will realise the intention of an integrated view of the proficiencies.

We propose a pedagogical cycle that could support teachers in engaging students' sense of reasoning systematically through problem-solving contexts. This structure acknowledges the mathematical proficiencies as being interrelated aspects that together build conceptual understanding through opportunities for students to:

- engage their current understandings through familiar experiences,
- identify and describe similarities in concepts,
- question and engage in mathematical discourse to communicate their thinking,
- generalise their conceptual understanding about concepts across contexts,
- develop fluent patterns of knowing how to engage with similar type problems,
- apply these patterns of understanding in new and unfamiliar contexts, and

- explain and justify their reasoning, which in turn re-shapes and strengthens conceptual understanding.

Adopting such an integrated view of the role of the mathematical proficiencies has implications for professional learning to ensure teachers' pedagogical content knowledge and promotion of reasoning enables their students' to develop generalised understandings of mathematical concepts.

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