A highly capable Year 6 student's response to a challenging mathematical task

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Highly capable mathematics students are not usually considered strugglers. This paper reports on a case study of a Year 6 student, Debbie, her response to a lesson, and her learning involving a challenging mathematical task. Debbie, usually a highly capable student, struggled to complete a challenging mathematical task by herself, but as the lesson unfolded she was able to solve the problem. Rather than relying on teacher explanations during the lesson introduction, Debbie benefitted from the support of her teacher during the lesson and sharing of other students' thinking when applying her knowledge to an unfamiliar task. [*Debbie is a pseudonym.]

This paper is a research report on a lesson that promoted meaningful learning for one Year 6 student and her response to a challenging mathematical task involving angles and parallel lines. Sullivan, Aulert, Lehmann, Hislop et al., (2013) suggest that, mathematics should be robust and students should be capable of adapting knowledge to different context. This paper includes discussion of the potential of using a challenging mathematical task to teach a geometry lesson. Traditionally geometry lessons were often taught in a mechanical way (Crowley, 1987). Today's classroom culture has changed as we consider the impact of how research and best practice have developed. In particular, since the introduction of the Australian Curriculum (ACARA, 2015) five years ago, teachers have had to consider how they might incorporate the proficiency strands of understanding, fluency, problem solving and reasoning into their mathematics lessons. The resource materials used in this study rely on students' making connections, choice of problem solving strategies, understanding and reasoning. The expectation is that these resources or challenging mathematical tasks have been designed so that they can be introduced with minimal explanation, have multiple pathways to the solution and more than one possible solution (Sullivan & Mornane, 2014).

Affective factors play an important role in mathematics learning. Dimarakis, Bobis, Way and Anderson (2014), in their review of literature, highlighted that student's lack of mathematical confidence or self-belief could be potentially detrimental to their learning outcomes. More specifically, Pajares (1996) described beliefs as important in determining how much effort one might apply to an activity such as how long one might persevere when having difficulties. Similarly, Sullivan et al. (2013) stated that it is also important for students to have the confidence to attempt and persist in solving unfamiliar mathematical problems.

The focus of The Encouraging Persistence Maintaining Challenge Collaboration during 2015 examined what happens when teachers pose challenging tasks to students and ways that teachers can encourage students to persist on those tasks (Sullivan, Holmes, Ingram,

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Linsell, Livy, & McCormack, this issue). This iteration was informed by the notion of *participatory appropriation* proposing that, guided by the teacher, individuals engage in an activity and are responsible for their own learning (Rogoff, 1990). This paper extends our understanding of a highly capable student Debbie, (Livy, in press), and her struggle and persistence when responding to a challenging task. This study sought to answer the following research question: How did a highly capable student Year 6 student respond to a difficult challenging task? Debbie was described as highly capable by her classroom teacher and not considered someone who would have difficulty learning mathematics. Parish (2015) suggests that students who are mathematically highly capable may learn differently and still require teacher support when learning.

Method

The approach chosen for this small study was a case study design including qualitative methods and analysis. Participants were Year 6 students (aged 11), eight second-year pre-service teachers and the classroom teacher. Data collection included students' pre-test and post-test responses, learning task responses and reflections to a task related to angles within parallel lines, and snippets of video footage taken by the classroom teacher as well as her reflection of the lesson and discussion with Debbie during the lesson. One pre-service teacher sat next to Debbie, observing her and taking field notes.

The first author taught a fifty-minute lesson to nine students in an Australian primary school under the observation of the classroom teacher. This was the second lesson using a challenging task completed with these students and was considered the most difficult within eight lesson suggestions (Sullivan, et al., this issue). At the beginning of the lesson students completed a pre-test item and recorded the missing angle on a straight line.

The lesson included three phases. First the learning task was launched and the students were asked to read and complete the task by themselves (see Figure 2). Students were given no further instructions or revisions of terms. The explore phase followed where the students attempted the task without input from the teachers. After at least five minutes, students who experienced difficulties were provided with an enabling prompt and were asked: 'Which angles are the same as each other in this diagram?' The third phase was the summary phase where students were chosen to share their thinking. For the conclusion of the lesson the students were asked to complete the test item again (see Figure 1) and write a reflection about their learning experience.

Results and Discussion

Debbie correctly responded to the pre-test and post-test identifying the missing angle m as equal to 150. When recording her thinking she wrote, "I subtracted 30 from 180 and got the answer 150 [degrees]."

> The pre-service teacher noted: Debbie silently read the learning task, quickly wrote two different answers on the bottom of her sheet for A and B … after writing her answers she erased them, wrote the same answers again, shook her head, erased them again … She wrote 90 degrees next to A and 105 alongside B … Debbie became confused and was not able to progress any further and raised her hand to the teacher.

The author was observing and providing enabling prompts to other students. The classroom teacher noticed that Debbie was having difficulty, and asked Debbie to think about how she responded to the pre-test item and if she could apply this understanding to the parallel line task. This may have assisted Debbie's progress and ability to persist with the

task. Similarly, the New Zealand teachers reported careful consideration of the types of questions they posed when their students responded to the angle tasks (Ingram, Holmes, Linsell, Livy and Sullivan, 2016). Following the lesson, the classroom teacher mentioned that she was surprised with how Debbie struggled as she was usually capable of completing mathematical problems independently during mathematics lessons.

Then the author stopped the lesson and commenced the summary phase focusing on student reasoning, thinking and strategies. The author chose three students who were at different stages of the task. They were encouraged to explain their responses using an example on the white board at the front of the class. Each student took turns clarifying how they considered the value of the different angles, for example pointing to opposite angles that were equal; highlighting the two triangles in the diagram; identifying angles in a triangle adding to 180 degrees; finding a quadrilateral that adds to 360 degrees; and stating that a full revolution is 360 degrees. Whilst the students shared their strategies the pre-service teacher heard Debbie say, 'I knew that, I just didn't read the question properly'.

After the summary phase students were given extra time to continue solving the task and/or record two different solutions, listing all the angles in the diagram. Figure 1 is Debbie's logical solution to the task after the summary phase and evidence of incorrect response at the start of the lesson listing A as 90 degrees and B as 150 degrees. The authors recognise that it would have been better to use lower case letters for the angles A and B.

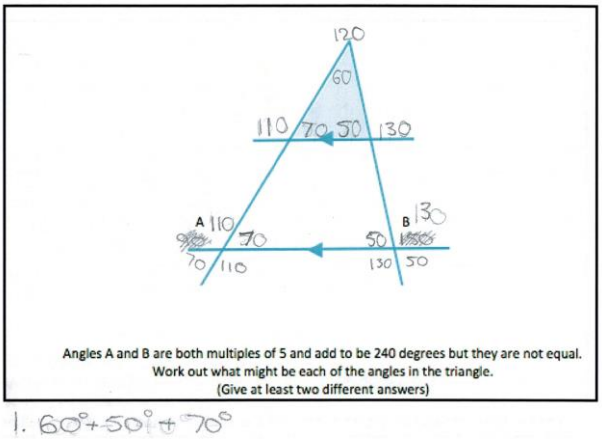

Figure 1. Debbie's response to the challenging task angles and parallel lines.

At the conclusion of the lesson Debbie was asked to share her experience and said,

> Today was very challenging. I struggled with understanding the question. My understanding of angles and the explanations on the board helped me. [I need to] try to think outside my comfort zone.

Debbie might usually rely on the teacher's explanation, such as a taught procedure to guide her thinking and strategies for solving mathematical problems. Whereas, the lesson sequence of having to attempt a difficult task without guidance from the teacher provided an opportunity for Debbie to struggle then extend her mathematical understanding. The support of the teacher encouraged Debbie to persist, and she was able to make connections possibly during the summary phase when other students shared their strategies and solutions.

Conclusion

Although this paper only reports on one student never-the-less it was interesting and surprising that a student who would not usually struggle during mathematics had difficulty

when attempting a challenging task. The teacher did not need to revise key concepts and terms at the beginning of the lesson. The student was able to engage in challenging thinking, relying on her own strategies after considering how other students explained and justified their understanding. Similarly, Sullivan and Morane (2014) considered that students can benefit when they are encouraged to persist with a challenging task. Furthermore, highly capable students need to understand that hard work, effort and perseverance are a normal and expected part of learning (Parish, 2015). The results of this study highlight that teachers should carefully consider how to plan their approach to teaching mathematics so that students are encouraged to experience struggle and success with the aim of enhancing how they learn mathematics. Limitations of this study were that only one student's response was reported, but the implications of this lesson approach are significant when considering how Year 6 students might engage and extend their knowledge of mathematics.

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