Improving the efficiency of problem-solving practice for children with retrieval difficulties

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Despite the importance placed on how children come to solve single-digit addition problems, many children count on to solve these problems when they are expected to use accurate retrieval-based strategies. In this study, we assessed if a subitising intervention improved the rate at which problem-solving practice promoted retrieval, using a multiple baseline across participants design. For two of three participants, problem solving practice was initially ineffective for promoting retrieval before the intervention but after the intervention, retrieval increased significantly as a function of practice. We also examined possible reasons for why this occurred.

It is critical that children develop retrieval-based strategies for solving single-digit addition problems early in early primary school (Geary, 2011). Retrieval-based strategies encompass direct retrieval, where the answer is recalled from a store of facts in memory (Aschraft, 1995) and decomposition strategies, where an answer is derived using a retrieved fact (e.g., 3+4=3+3+1). As children develop retrieval-based strategies for solving single-digit problems, demands on working memory are reduced (Imbo & Vandierendonck, 2007) and higher-order learning is made possible (Canobi, 2009; Carr & Alexeev, 2011). Despite the importance placed on how children come to solve single-digit addition problems, many children are not solving these problems in a way that matches curricula expectations. Children's recorded use of retrieval is considerably lower than expected in second and third grade (Cowan et al., 2011; Geary, Hoard, Byrd-Craven & deSoto, 2004) and their use of counting strategies beyond third grade is much higher than expected (Hopkins & Bayliss, 2014, Cumming & Elkin, 1999).

The provision of practice is critical for the development of retrieval-based strategies but it is important to distinguish between different types of practice. Children typically become more proficient with single-digit addition as result of *problem-solving practice*; that is, practice solving single-digit addition problems using strategies of choice. Problem-solving practice with *back-up strategies* (any strategy other than retrieval) leads to the discovery and use of more efficient back-up strategies (e.g., counting-all strategies are replaced by counting-on strategies) and eventually *retrieval-based strategies* come to dominate performance (Canobi, 2009; Goldman, Pellegrino, & Mertz, 1988; Siegler & Jenkins, 1989). The distributions of associations (DOA) model and updated versions of the model (Siegler & Shrager, 1984; Shrager & Siegler, 1998) provide a comprehensive account of how practice with backup strategies leads to more efficient backup strategies and the eventual dominance of retrieval. Along with problem-solving practice, children may benefit from being explicitly taught more efficient backup strategies like the min-counting strategy, where the smaller addend is counted on from the larger addend (Fuchs et al., 2010; Tournaki, 2003) and decomposition strategies (Torbeyns, Verschaffel & Ghesquière, 2005).

While children can be explicitly taught more efficient backup strategies, it is not possible to explicitly teach retrieval. Instead, a core component of interventions designed to increase the use of retrieval for single-digit addition has been *fact-retrieval practice* (sometimes referred to as drill and practice), where children are repeatedly exposed to and

2016. In White, B., Chinnappan, M. & Trenholm, S. (Eds.). Opening up mathematics education research (Proceedings of the 39th annual conference of the Mathematics Education Research Group of Australasia), pp. 311–318. Adelaide: MERGA.

rehearse the problem with the correct answer (Fuchs et al., 2006; Powell, Fuchs, Fuchs, Cirino, and Fletcher, 2009); Poncy, Skinner, & Jaspers, 2007). Both problem solving-practice and fact-retrieval practice lead to an increased use of retrieval but each rely on a different process for establishing problem-answer associations in memory. Problem-solving practice relies on the correct use of back-up strategies, like counting and decomposition strategies, to strengthen problem-answer associations in memory (Siegler & Shrager, 1984; Shrager & Siegler, 1998); fact-retrieval practice relies on memorisation. Fact-retrieval practice was designed as an *alternative* path for children with substantial mathematics learning difficulties (MLD) to *circumvent* what is considered to be the typical route of retrieval development (Fuchs et al., 2010).

Children with MLD exhibit considerable delay in learning to retrieve answers to simple addition problems (Torbeyns, Verschaffel, & Ghesquière, 2004). Circumventing how children typically learn may not be the best approach for supporting children with learning difficulties. Importantly, problem-solving practice leads to the development and use of multiple strategies (Siegler, 1995), which fosters *adaptive expertise* – the ability of an individual to choose the strategy that works best for them on a particular problem. When individuals are free to choose among strategies as they engage in problem-solving practice, they are likely to develop adaptive expertise because information about the success and efficacy of (i) a strategy, (ii) a strategy on specific problem types and (iii) a strategy on specific problems, is continually encoded as a result of experience (Siegler & Jenkins, 1989; Imbo & Vandierendonck, 2008). Children are unlikely to become flexible problem solvers if they learn only one strategy for solving problems (i.e., retrieval through fact-retrieval practice).

Problem-solving practice may be more effective for children representing a wider range of ability levels, if suitable ways of improving the efficiency of problem-solving practice in terms of promoting retrieval, could be established. One possible way this could be achieved is through strengthening children's skills with subitising. Subitising has long been recognised as important for developing an understanding of number and has been classified into two types (Clements, 1999). Perceptual subitising involves instantly recognising the number of items in a set without counting them and is limited to small quantities (around four). Conceptual subitising involves recognising a whole as the sum of composite parts to make use of perceptual subitising and can involve some counting. For example, six items can be recognised as four items and two items (using perceptual subitising) or four items (using perceptual subitising) plus two more items (counted on). An association with children's subitising skills and arithmetic skills have been noted (Gray & Reeve, 2004) but investigations into how improvements in one skill affects the other have been limited.

The aim of this study was to investigate if instruction designed to enhance children's subitising skills improved the rate at which problem-solving practice leads to an increased use of retrieval for simple addition. We were also interested in explaining why subitising instruction might increase the likelihood of retrieval. Four possibilities were derived from the literature: subitising provides a more robust backup strategy for solving simple-digit problems (Pincham, & Szúcs, 2012; Trick, 2005), subitising improves the accuracy of children's performance (Trick, 2005), subitising enhances an understanding of number to promote the use of decomposition strategies (Cheng, 2012; Starkey & McCandliess 2014), and subitising provides mental images of quantity that bolster children's confidence to use retrieval (Siegler & Shrager, 1984; Shrager & Siegler, 1998). The second aim of this study was to explore these four possibilities.

Method

Design

A microgenetic approach (Siegler & Crowley, 1991) was used to address the research aims of this study. This approach requires that participants are tested individually and involves a trial-by-trial analysis of data to document change as it happens, to be able to infer reasons for change or lack thereof. This approach was instrumental in exposing children's variability in strategy use for single-digit addition and led to the development of important theory explaining how children choose among multiple strategies (Siegler & Jenkins, 1989, Siegler, 1995). The microgenetic approach has since been used in many studies examining skill and conceptual development (see Fazio & Siegler, 2013).

A multiple baseline across participants design was used to assess the effects of the intervention. This design involves staggering the start of the intervention so that it occurs at different times for different individuals. As the intervention is started at different times, patterns of change after the intervention can be associated with the intervention rather than with chance factors. The study of change based on a multiple baseline design is typically evaluated using visual inspection, whereby a change is considered significant if it is obvious. However, visual inspection alone can be unreliable and so inferential statistics have been used to augment visual analysis (e.g., Busse, Kratochwill, & Elliot, 1995; (Crawford, Garthwaite, & Gray, 2003). Crosbie (1993) recommended using an interrupted time series analysis (ITSA) procedure for single case studies, which uses least-squares estimates of intercepts and slopes for each phase and then assesses overall change in intercepts or slopes using an *F*-test supplemented by *t*-tests. The program DMITSA (Crosbie & Sharply, 1991) was used to perform the ITSA procedure to test if the intervention produced significant change in participants' use of correct retrieval.

Participants

Participants were selected from two third grade classrooms, in one school located in an area of mid socio economic status in the Perth metropolitan area (in Western Australia). In WA, third grade children are 8 to 9 years old. The two classroom teachers of the Year 3 cohort were asked to identify children who were predominantly using counting for solving simple addition problems. Nine children were identified in this way and were observed solving a set of simple addition problems. The screening identified three children who still applied a count-all strategy. As this is a particularly immature and inefficient strategy to be using in third grade, these children were excluded for selection in the intervention study but received explicit instruction in using the min-counting strategy. The other six children predominately used a min-counting strategy. Three children were randomly chosen to participate in the intervention study. Parental and child consent was obtained. Results from the standardised testing routinely administered at the school indicated that all three children achieved a score on the Progressive Achievement Test for Maths (ACER, 2005) below the average score recorded for their age.

Procedure

Each child was individually withdrawn from their classroom to a quiet room nearby and observed as they engaged in 20 sessions of problem-solving practice. Practice sessions during the pre-intervention phase and post-intervention phase were completed on consecutive school days and each session lasted around 20 minutes. During each practice session, the child solved a set of 36 problems using strategies of choice. The problem set represented all single-digit addition problems written in the form *'$x + y =$'* (where $x \leq y$ *and* $x, y > 1$), and were presented in random order. Immediately after solving each problem, the child was asked to explain to the research assistant (RA) the strategy they had used and the RA checked that the self-report was consistent with what she had observed. This combined procedure of self-report and observation on a trial-by-trial basis is a reliable and valid way of assessing children's strategy use and has been used extensively in studies of single-digit addition skill (e.g., Canobi, 2009; Geary et al., 2000, Bailey, Littlefield, & Geary, 2012).

An analysis of reaction times on correct trials confirmed the validity of the approach used to identify strategy use in this study. Participants in this study reported using four strategies: (i) the min-counting strategy, where the smaller addend is counted on the larger addend (sometimes referred to as counting-on-from-larger), (ii) a decomposition strategy, where answers are derived using a known fact (e.g., double facts, add-to-10 facts), (iii) a subitising strategy, where children reported visualising dot patterns, and (iv) retrieval, where the answer is directly retrieved from memory and is usually accompanied by an explanation similar to "I just knew it". Reaction times decreased according to strategy efficiency: min-counting trials (n=491, mean RT=4.81s, SD=2.45), decomposition trials (n=69, RT=5.91, SD=4.07) and retrieval trials (n=702, mean RT=2.20s, SD=1.10).

Intervention

The subitising intervention was designed around the concept of a ten-frame. A ten-frame is traditionally represented as an array with five columns and two rows, and is recommended as a teaching tool for helping children visualise, compare and partition numbers within the context of each number's relationship to ten (NCTM, 2000). The tens frame was coloured in a particular way to enhance children's perceptual subitising skills as studies have shown that *perceptual* subitising capacity is limited to around four items (e.g., Pincham & Szúcs, 2012). Specifically, the first two columns were coloured green, the next two were coloured red and the last column was coloured yellow. Each number to ten was also represented as a particular pattern to enhance children's conceptual subitising skills. The intervention was developed around the theme of sleeping cats and was delivered over 10 sessions on consecutive school days, with each session taking around 20 minutes. Six sessions were focused on developing children's subitising skills and four sessions were focused on applying these skills to partition numbers (e.g., 5=2+3, 1+4, 3+2 and 4+1) and find corresponding subtraction facts. No feedback was given during the pre-intervention or post-intervention phase.

Results

Changes in strategy use for correct trials for the pre-intervention and post-intervention phases are depicted in Figure 1. Based on visual analyses, there appears to be little if any improvement in the frequency of correct retrieval as a result of problem-solving practice during the pre-intervention phase for Tanya and Mary. For Emma, an increase in the correct use of retrieval during the pre-intervention phase is evident: she correctly retrieved 6 answers at Time 1 and 11 answers at Time 8. During the post-intervention phase, a more obvious increase in the use of correct retrieval is apparent for all three children, particularly for Tanya and Mary, with a corresponding decrease in the use of the min-counting strategy. All three children reported using a subitising strategy during the post intervention phase, although use of this strategy was infrequent relative to retrieval and min-counting. Emma and Mary correctly used a decomposition strategy only in the post intervention phase.

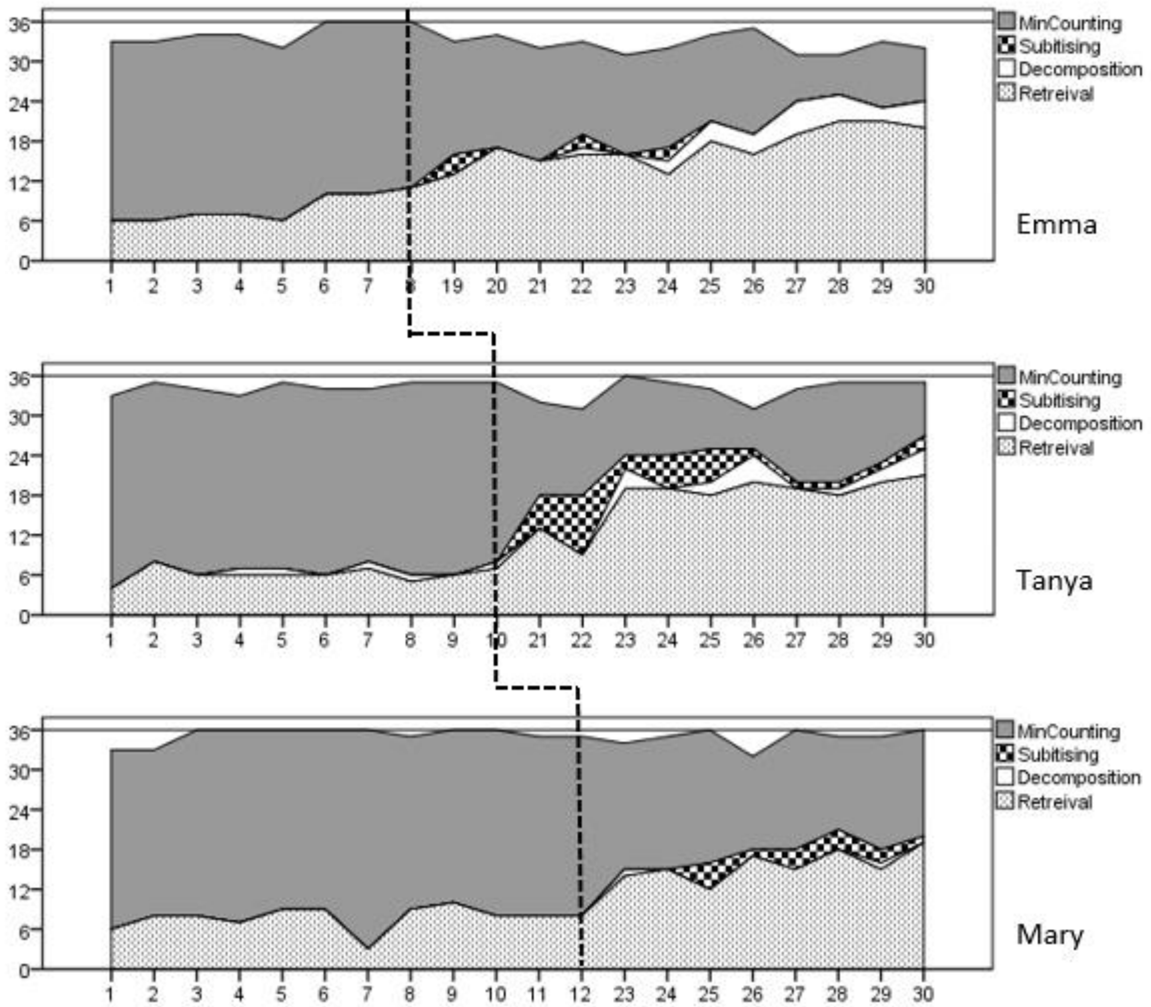

Figure 1. Area graphs displaying the strategy mix used to correctly solve single-digit problems across practice sessions. Numbers along the y-axis indicate the number of problems in the 36-problem set. Nunbers along the x-axis indicate consecutive school days. Dotted line separates pre-intervention phase and post-intervention phase

To assess whether the intervention significantly improved the rate at which problem-solving practice lead to correct retrieval, a test of significance was individually applied to the data (the number of correct retrieval trials was used as the dependent variable). This involved fitting a regression line to the pre-intervention data and another to the post-intervention data, and testing for significant differences in intercept and slope. The results indicated that the intervention did produce significant improvement in correct retrieval for Tanya, $F(2, 14) = 6.294$, $p = 0.011$, and for Mary, $F(2, 14) = 9.879$, $p = 0.002$, but not for Emma, $F(2, 14) = 2.046$, $p = 0.166$. While Emma's use of retrieval increased throughout the study and her reliance on min-counting decreased, this finding suggests that she might had achieved the same result had she just been exposed to problem-solving practice. (The analysis was repeated using the number of trials correctly solved using retrieval-based strategies and the finding was not significant for Emma.)

Graphs shown in Figure 1 are also useful for illustrating participants' accuracy during different phases of the study. A horizontal line is drawn at 36 on each graph (the number of problems in the set). The distance from the top of the area graph to this line represents the number of incorrect answers produced in each practice session, by each participant. Visual inspection of these graphs suggests that the rate at which accuracy increased as a result of problem-solving practice did not differ markedly during the pre-intervention and post-intervention stage. This was confirmed using a single case study test of significance: $F(2, 14) = 3.619$, $p = .054$; $F(2, 14) = 2.214$, $p = 0.146$, and $F(2, 14) = 1.044$, $p = 0.378$.

It is not surprising to find that the rate at which problem-solving practice improved accuracy was similarly low across both phases. Firstly, the number of errors recorded for each participant were low to begin with. For example, Mary correctly solved all 36 problems on seven occasions during the pre-intervention phase. Secondly, participants received no feedback to indicate if their answers were correct or not. The important finding here is that the rate of correct retrieval increased during the post-intervention for Tanya and Mary, without concomitant increases in the accuracy of back-up strategies. The findings are now discussed.

Discussion

The design of this study is unusual as the strategies used by participants were identified using self-reports on a trial-by-trial basis. This method has been used extensively in studies describing the characteristics of children who have difficulties with simple addition (e.g. Geary et al., 2000) but is rarely used in intervention studies. This method allowed us to evaluate the intervention in terms of changes to strategy use rather than rely on measures of accuracy and/or speed. The design of this study is also unusual as it encompassed the time consuming collection of data on a trial-by-trial basis over many occasions to depict growth as it happened, rather than analyse static measures of effect after the event. This microgenetic approach allowed us to assess the subitising intervention in terms of its effect on the rate at which problem-solving practice led to the correct use of retrieval and also to explore reasons why the intervention worked or not.

Two clear findings were evident from this study. The first finding is that the subitising intervention was effective at increasing the rate at with which problem-solving practice increased correct retrieval for two of the three participants. These two participants showed little benefit from problem-solving practice in terms of improved correct retrieval during the pre-intervention phase, but both showed significant benefit during the post-intervention phase. The increase in retrieval could *not* be explained by an increase in accuracy. Although the increase in retrieval appeared too rapid for problem-answer associations to have been strengthened using a subitising and/or decomposition strategy, these possibilities could not be ruled out. A likely reason why subitising improved the rate at which problem-solving practice led to retrieval for these children is because subitising improved their confidence to use retrieval. The link between subitising and retrieval needs to be further investigated.

The second finding was that the subitising intervention did not benefit the third participant, as she displayed a similar rate of improved correct retrieval during both phases. This finding exposed a weakness in the study design rather than a weakness in the intervention, as this child displayed more typical improvement in retrieval as a result of problem-solving practice rather than a retrieval difficulty. Future studies need to adopt stricter criteria for selecting participants thought to have a retrieval difficulty. Another limitation of this study was that the intervention focused on subitising but also encompassed other learning (e.g., partitioning numbers and the complementary nature of addition and subtraction). On reflection, the intervention should have been focused solely on subitising, to make the results clearer.

The importance of developing retrieval-based strategies for solving single-digit addition problems and the strong association between proficiency with single-digit addition and mathematical achievement is well documented in the literature (Cowan et al., 2011; Geary 2011). Interventions aimed at improving simple addition skills have involved explicit instruction using the min-counting strategy (Fuchs et al., 2010; Tournaki, 2003) and conceptual instruction in understanding how numbers can be partitioned (Cheng, 2012), but these interventions were not evaluated in terms of the effectiveness at promoting retrieval. Intervention studies aimed at improving retrieval have largely relied on the process of memorisation (Fuchs et al., 2006; Powell, Fuchs, Fuchs, Cirino, and Fletcher, 2009); Poncy, Skinner, & Jaspers, 2007) and appear to be inconsistent with what is considered to be good teaching practice. Findings from this study suggest that enhancing children's subitising skills may well be an effective alternative to memorisation, for helping children with retrieval difficulties.

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