Large-Scale Professional Development Towards Emancipatory Mathematics: The Genesis of YuMi Deadly Maths

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This paper describes the genesis of YuMi Deadly Maths, a school change process that has been used in over 200 schools to develop mathematics teaching and learning to improve students' employment and life chances. The paper discusses the YuMi Deadly Maths approach to mathematics content and pedagogy, implemented through a process of PD and school change, and looks at the strengths and weaknesses of the process and the challenges it faces.

Australian mathematics teaching is dominated by passive imitative textbook teaching where students recite definitions and rules and copy procedures (Hollingsworth, Lokan, & McCrae, 2003). These systemic weaknesses make it difficult for students to have the conceptual schema necessary to be successful in science, technology, engineering and mathematics (STEM) vocations. Within many Indigenous and low socio-economic status (SES) schools, the situation is even grimmer; large systemic mathematics performance differences exist between Indigenous and low SES schools and other schools.

Within this milieu, a group of academics and practitioners in the YuMi Deadly Centre (YDC) at QUT decided to provide an alternative teaching practice to schools. The aim was not to research for academic writing but instead to research to produce in praxis a mathematics pedagogy to directly confront existing methods. In this, we were influenced by four beliefs:

1. Powerful mathematics (in terms of emancipating the learner) cannot be told but must be constructed by each learner from activities and discussion (social constructivism).
2. Pedagogies exist that enable all students to learn, and all teachers to teach, powerful mathematics, and all communities to be advantaged by this teaching and learning.
3. The only acceptable research for Indigenous and low SES schools is the "empowering outcomes" decolonising methodology of Tuhiwai Smith (2012) where research benefits the researched.
4. The role of researchers in school change can be negative in that there is a danger that change agents can become the new oppressors (from the ideas of Gramsci, 1977).

This paper briefly outlines the components of our school change process generally known under the name YuMi Deadly Maths (YDM), where YuMi is Torres Strait Islander Creole for "you and me" and Deadly is an Australian Aboriginal term for "smart". Since the genesis of YDM in 2010 we have worked with over 200 schools. This paper describes the YDM approach to mathematics and mathematics pedagogy, explores the processes followed by YDM to bring about school change, and discusses some challenges that YDM faces as an agent for school improvement. It encapsulates much of what has been written in the overview book for participants in the YDM program (YuMi Deadly Centre, 2014).

Mathematics

Mathematics Structure

Our initial design of YDM was based on three big ideas: (a) mathematics is a structure of ideas formed into a schema; (b) mathematics is a language that concisely describes real-life situations; and (c) mathematics is a tool for problem solving. Structure tended to

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predominate, notwithstanding the importance of language and problem solving. We believe that deep learning and powerful ideas with respect to mathematics are a characteristic of structural understanding of mathematics. In this, we were influenced by the following: (a) Piaget's (1977) descriptions of schema, assimilation, and accommodation; in particular, the interpretation of Piaget's work by Ohlsson (1993) and Niss (2006); (b) Skemp's (1976) ideas of relational and instrumental understanding of mathematics; (c) Sfard's (1991) description of structural knowledge and the role of reification in determining higher levels of abstraction; (d) Leinhardt's (1988) four knowledge types as modified in Baturo (1998), namely, entry, concrete or representational, procedural, and structural (or principled/ conceptual); (e) Chi, Glaser, and Rees' (1982) description of the power and properties of rich schema in terms of defining, applying, connecting, and remembering, and the powerful effect these properties have on recall, problem solving, and future learning; and (f) our experience that structurally connected mathematics ideas can be taught in a similar manner, and that identification and use of these structures helps learning.

We designed the teaching ideas of YDM to complement mathematics structures in three ways. First, we constructed sequences that provide information on how to move in a seamless fashion from early to later ideas and vice versa, ensuring that the early teaching prepares for later ideas (called pre-empting); for example, the multiplication of whole numbers by 10 causing each digit to move one place to the left can later be extended to multiplying decimal numbers by 10. Second, we identified structurally connected topics and organised teaching so that it chunks the knowledge; for example, division and fractions are both parts of a whole and follow the inverse relation that the larger the number of parts the smaller each part. Third, we used big ideas (see Carter, Cooper, & Lowe, 2016) to integrate the teaching of topics into a coherent whole; for example, the part-part-whole big idea applies to problems in addition and subtraction, fractions and ratio, and percent. The resources and professional development (PD) for YDM are therefore designed to emphasise sequencing, connections and big ideas. In particular, one YDM resource identifies, defines, and classifies all major big ideas of mathematics (YuMi Deadly Centre, 2016).

Culture and Mathematics

Since our initial work was in Indigenous and low SES schools, we explored the connection between culture and mathematics for two reasons: (a) to value the cultural capital these students bring to the classroom; and (b) to challenge the Eurocentric nature of Australian school mathematics. We worked with Aboriginal mathematician and mathematics educator Dr Chris Matthews to identify what mathematics is. The result of this collaboration is encapsulated in Figure 1 (adapted from Matthews, 2009). It illustrates how mathematics: (a) starts from an observer in reality who *chooses* a real-life problem (the grey circle); (b) creates an *abstract* representation of that situation using a range of mathematical symbols; (c) uses mathematics to *explore and communicate* particular attributes and behaviours; and (d) *reflects and validates* the mathematics back to the reality to see if it is worthwhile (and if it is, applies, extends and transfers the mathematics to other situations).

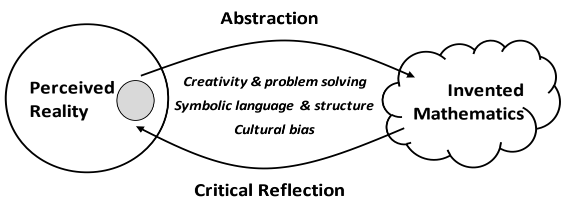

Figure 1. Ontology of mathematics (Matthews, 2009).

As shown in the figure, three other features of the model are a consequence of the reality-mathematics-reality cycle: Both abstraction and reflection are *creative and problem-solving* acts; mathematics as a language and structure is built around *symbols* that carry concepts, strategies and relationships from reality to abstraction and back to reality; and the mathematics and how it is used in reality is framed by the *cultural bias* of the person creating the abstraction and reflection.

Mathematics Pedagogy

Past Pedagogies

Our initial focus was on improving mathematics teaching in Indigenous and low SES schools. In these situations, the model in Figure 1 was useful and the decision was taken to see if it could be turned into a pedagogical framework, or teaching cycle, following the sequence: reality, abstraction, mathematics, reflection (RAMR). To do this, we looked at the pedagogies we were already following. The major ones are summarised below.

Wilson's Activity Type cycle (Ashlock, Johnson, Wilson, & Jones, 1983; Wilson, 1976). This five-step cycle includes: (a) initiating by teaching the idea informally in real-world situations; (b) abstracting to formal mathematical language and symbols; (c) schematising by connecting the new knowledge to prior knowledge; (d) consolidating through practice; and (e) transferring by solving problems and extending knowledge to new ideas. The cycle advocates continuous checking and diagnosis of student understandings.

Payne and Rathmell's triangle (Payne & Rathmell, 1975). This framework connects models (physical, virtual, and pictorial), language, and symbols and advocates an initial pedagogical sequence of *story → models → language → symbol*, then relates all the parts in all directions. As Duval (1999) argued, mathematics comprehension results from the coordination of at least two representational registers: the multi-functional registers of natural language and figures/diagrams, and the mono-functional registers of symbols and graphs. Learning is deepest when students can integrate these registers.

Levels of instruction and generic strategies (Baturo, Cooper, Doyle, & Grant, 2007). This framework identifies: (a) three levels of instruction, namely, technical (proficiency with the use of materials), domain (materials and activities that provide effective experiences for learning a topic), and generic (instructional strategies that hold for all topics); and (b) four generic strategies, namely, flexibility (experiencing the idea in many ways), reversing (teaching in the opposite direction), generalising, and changing parameters.

Learner-centred principles (Alexander & Murphy, 1998). The five principles are: (a) prior knowledge serves as the foundation of all future learning; (b) learning is as much a socially shared knowledge as it is an individually constructed enterprise; (c) learning, while ultimately a unique adventure for all, progresses through various common stages of development; (d) metacognition is central; and (e) affective factors play a significant role in the learning process. In this, we were strongly influenced by the importance of context in mathematics learning.

Knowledge levels (Bruner, 1966). Bruner argued that three levels of knowledge—enactive, iconic, and symbolic (which we renamed body, hand, and mind)—are required and the mind moves forward and back through them, in learning and problem solving.

RAMR Framework

The RAMR pedagogical framework (YuMi Deadly Centre, 2014) summarised in Figure 2 was designed to be a cycle of activities that, along with the resource books and PD sessions, would scaffold and provide teachers with the confidence to write their own lesson and unit plans. The RAMR framework begins and ends with the reality of the students' lives. It starts with something that interests the students, and then acts this out with kinaesthetic or whole-body activities to build visual images or pictures in the mind of the mathematics idea(s). It then moves to consolidation, which involves making connections as well as practice, and finally reflects back to the students' reality.

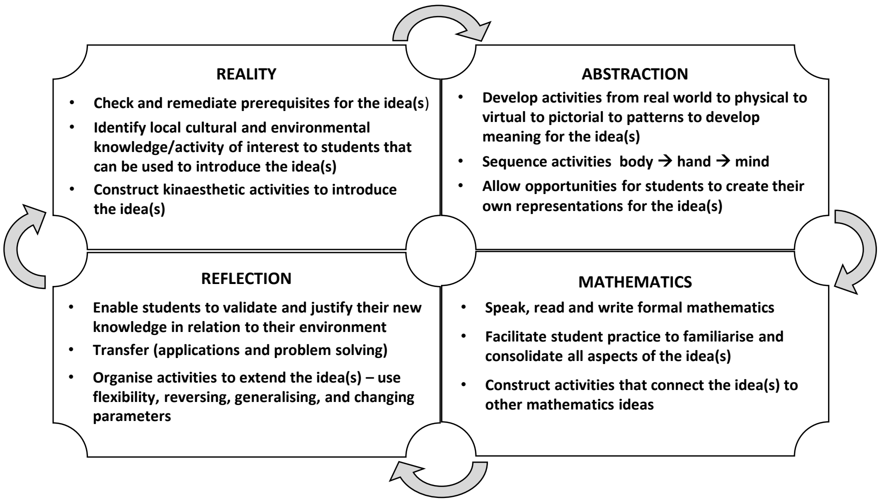

Figure 2. RAMR pedagogical framework.

The two core sections are abstraction and reflection, with reflection ensuring that the idea(s) are extended as far as they can using the four generic actions. The framework is not fixed, either in theory, or in most schools' practices. Teachers use it for unit plans and lesson plans. They move back and forth between the parts of the cycle, and use the extension strategies in reflection (flexibility, reversing, generalising and changing parameters) across the whole framework.

PD and School Change

Our first trial of whole school change using YDM occurred in 2010. We worked with 140 Years P to 3 teachers from 35 primary schools for one year. The schools were placed in three large clusters, with up to 48 teachers attending PD sessions that covered all topics in mathematics. In the second year, we focused on Years 4 to 7 mathematics teachers and in the third year on Years 7 to 9. After this project, we decided to no longer focus on particular year levels but to train Years P to 9 mathematics teachers across two years.

Prior to this, our collaborations had been with small numbers of schools and teachers, and focused on particular topics. However, our experiences in smaller scale projects influenced what we did in these larger projects and determined the future development of YDM. Our particular influences were: (a) the stronger-smarter work of Sarra (2011) that emphasised the need for whole school change, and involved challenging poor behaviour, ensuring cultural safety, building pride in heritage, collaborating with community, introducing local leadership, and having high expectations; (b) the powerful effect of metacognitive frameworks, such as Polya's (1957) four stages in enabling teachers to construct their own lessons, which led us to develop such frameworks for teachers (e.g., RAMR); (c) our experience in seeing how differences in student situation, background and culture affect mathematics teaching, leading us to focus our PD and resources on supporting teachers, not just preparing textual material; (d) our experiences with PD in observing the positive effects on teachers of motivating, effective, and innovative ways to plan and run lessons that they felt they could immediately use, mixed with theory on effective pedagogies which they saw would enable them to construct their own lessons; (e) the ideas of Hord (2004) that show the efficacy of professional learning communities and knowledge building communities that point to the importance of group knowledge building for students and teachers; and (f) the theories of Clarke and Peter (1993) and Baturo, Warren, and Cooper (2004), shown in Figure 3, arguing that implementation should be a cycle of affective readiness for change, pertinent external input, effective classroom trials, positive student responses, and supportive reflective sharing.

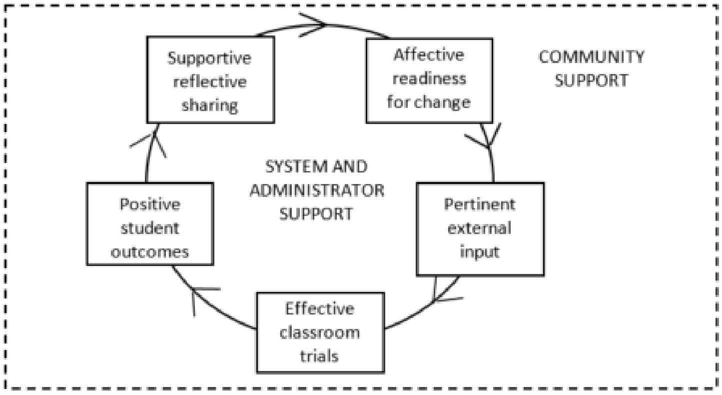

Figure 3. The YDM effective professional learning cycle.

Figure 3 recognises that positive student outcomes along with initial readiness are crucial to successful change. These are facilitated by: (a) inputs of pertinent, relevant, and innovative ideas and materials (YDM resources, PD activities, and website); (b) just-in-time support before and during in-school training and classroom trials (in planning and by modelling training and instruction); (c) support of community, system, principal and other administration staff in achieving positive student outcomes; and (d) responding to feedback in data gathered through an action-research process during in-school training and classroom trials.

As a result of the above, YDM resources and PD workshops have been designed so that: (a) their focus encompasses school change and leadership (principals, community, system, and administration support) as well as mathematics and its learning and teaching (teachers); (b) they provide a pedagogical framework (RAMR), supported by examples of classroom activities designed to maximise learning outcomes by valuing local culture and knowledge, engaging student interest, building high teacher expectations and enabling positive student identity; (c) they provide a framework for principals and the trained teachers (called *trainers*) to work together to set up a supportive in-school training and trialling process; (d) they set up contact between school (principal and trainers) and YDC staff to provide online support for in-school training and trialling; and (e) they provide information so that each school can use action research to provide feedback to both teachers and YDC to improve resources and processes. Despite this comprehensive approach, however, there are some parts of effective PD and school change that YDM has not addressed; these are discussed in the last section of this paper.

Practical considerations (including funding limitations) have prevented us from directly working with every teacher of mathematics in individual schools. Consequently, we have used a *train-the-trainer approach*, working with geographical clusters of 4 to 12 schools. Each school selects four staff members (we recommend one of these is an administrator) to be trained. We provide four three-day PD workshops across two years for these trainers, as well as a Sharing Summit at the end of each year. We ask the principal or an administrator to attend the first day of the first and third PD workshops. The four workshops cover YDM philosophy and pedagogy; in-school processes for implementation and planning, school change, community involvement, and sustainability; and mathematical content for Years P to 9 in all strands of the *Australian Curriculum: Mathematics*. We ask schools to prepare a plan to enable the trainers to trial the YDM pedagogy, train other teachers in YDM using an action-research approach, and report back to us on how the school is implementing the pedagogy. In other words, schools are asked to enable four staff to be trainers, change agents and researchers, and to provide time and space for all other mathematics teachers to be involved. Besides the PD workshops, we provide trainers with books, resources, and online access to training modules, videos, discussion groups, and lesson plans, as well as access to YDC staff by phone and online. Overall, sequencing, connections, and big ideas are central in PD sessions, as are examples of highly effective classroom activities. However, the most powerful idea is the RAMR framework; thus significant time is set aside in PD sessions for teachers to plan using it.

The implementation of YDM is a combination of centrally organised PD inputs, school organised in-school activities, informal ad hoc contact, and training-support and research activities. Of these activities, we control what is central, formal, and planned, but not what is school organised, informal, and ad hoc. Yet, it is these latter activities that are the most powerful and effective. The informal in-school processes provide a unity to the PD inputs and school staff's actions and, together with the formal inputs, enable opportunities for change, in spite of their apparent separateness (see Carter et al., 2016).

Present Situation and Future Challenges

At present, the YDM process is running in over 30 schools across Queensland. Over 200 schools have experienced the two-year program in the past six years. In nearly all these situations, funding has limited YDM to the provision of resources and PD with no in-school follow up. Yet, approximately 70% of the schools were still using YDM pedagogy three years after the training ended. Many of these schools have experienced dramatic and positive changes in their mathematics teaching and/or learning outcomes. We have found that schools become active in YDM after 12 days of PD spread over two years provided that there is: (a) a school plan for implementing YDM, supported by the principal; (b) reasonable staff continuity; and (c) at least one curriculum administrator and two or three teachers with enthusiasm for the implementation.

The clustering of schools and the use of action-research approaches with trainers and in-school activities appear to overcome many of the problems with limited school and teacher contact. The RAMR model seems particularly important as it enables teachers to become autonomously active in the pedagogy after one three-day PD even when teaching topics have not yet been covered in the PD sessions (see Carter et al., 2016). The strength of the YDM process is that it has worked in schools without having to provide external in-school resources. It seems to work because: (a) the PD focuses on what the school has to do to implement YDM as well as what is being implemented (the YDM); and (b) the first PD provides enough of what teachers have to do (in theory and practice terms) for them to trial the ideas with some success.

Since its initial large project funding ended in 2012–13, YDC has maintained school programs through packaging and marketing the YDM program for self-funding by individual schools or clusters. In 2015, YDC was selected by CSIRO to train Years P–9 teachers of mathematics in over 60 schools across Australia with relatively high Indigenous enrolments as part of a wider Indigenous STEM education project running from 2015 to 2019.

Despite these successes, we are confronting several challenges for the YDM approach/process. First, in some government schools there are mandated school-wide pedagogical approaches to teaching that can be summarised as "I do, we do, you do". This imitative teaching approach does not fit with the YDM approach and makes it more difficult to implement pedagogical changes in these schools. However, we have developed a six-step model that accommodates these pedagogies if the school is willing to take a flexible approach. Second, YDC's model for PD and teacher change (Figure 3) is based on teachers accepting the need for change and support from the school administration. To encourage support, we design our PD to engage teachers, build in regular involvement of administrators, and base intervention on action research which has built-in feedback. There can be significant decline if support ends. Third, Figure 3 also highlights the need for first trials to be successful. However, the YDM process comprises a sequence of researcher to trainer, trainer to teacher, and teacher to students, with the important classroom trials two links removed from the researchers. We control only the first link in this chain. Yet it is the outcome of the third link (student improvement) that determines whether schools and teachers persevere with YDM. We attempt to alleviate this challenge by focusing trainer PD and school plans on supporting in-school trials. In our experience this mostly works (see Carter et al., 2016). Finally, the YDM approach is to train teachers. If there is constant short-term changeover of staff, schools have difficulty sustaining YDM. We work with schools to develop special programs in these cases, but staff instability is difficult to overcome.

Some of the challenges are at the core of YDM and are presently the focus of discussion regarding changes to YDM. First, we are debating whether RAMR is best seen as a framework for teacher planning or for student learning. It appears that it can be applied successfully to either purpose, but the purpose affects the placement of components. For example, the *reversing* extension strategy in the Reflection stage is correctly placed if we look at RAMR in terms of student learning. However, if we look at RAMR as a way to plan teaching, reversing is a powerful method at any stage. Second, our focus on mathematics structure, in sequencing, connections and big ideas, may not be appropriate to students' development. Sometimes the logical development of mathematics runs contrary to the psychology of children's mathematical development (e.g., using set theory to introduce number). Finally, YDC has an internal challenge: to ensure its viability by generating sufficient revenue to continue operating. However, attempts to secure external funding for disadvantaged schools to access YDM programs (for example from government projects, school systems or benevolent endowments) often result in demands from the provider of the funding for the acquisition of the intellectual property developed by YDC over years at great cost and effort.

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