Investigating the Impact of Context on Students' Performance

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This paper reports on a study that examines the effects of problem context on students' performance. The performance of 151 Year 10 students on six mathematical problems was compared with the performance on fifteen variants with more and less context familiarity (CF) and engagement (CE) across levels of context use (LCU). The latter explanatory variables (CF, CE, and LCU) are used to estimate the strength of the relationship among them and the students' performance. Results show that neither CF nor CE affect students' performance but LCU demanded in solving a problem does.

The incorporation of context in mathematical problems has been highly recommended by current reform documents and mathematics curricula around the globe (e.g., OECD, 2013) and this has led to the development of new forms of connectedness of the instructional mathematical content by focussing on solving problems in context and mathematical modelling (Stillman, 2000). Nevertheless, in this escalating emphasis on problems set in context, the role that problem context plays on students' performance seems to be an unsolved matter. The fact that the influence of context on students' performance in mathematical problems is a matter that cannot be disregarded is confirmed by ample research. For instance, this author agrees with De Lange (2007) that:

> The influence of contexts should be studied much more systematically than is presently the case, and we researchers should refrain from strong statements that we have proven to be of disputable quality until we have firmer evidence (De Lange, 2007, p. 1120).

In general, evidence is sparse. The knowledge of the findings of individual studies highlights that there is a lack of a firm body of convincing empirical evidence for the effects (in any direction) of the context of a problem on students' performance (Stacey, 2015). Thus, for this paper, the purpose is therefore: to report an empirical study on the possible effect of an alteration of problem context familiarity and engagement on the performance of Year 10 students.

Key Terminology

(Problem) Context

In this study context refers to the information that surrounds a scenario described in a problem that needs to be mathematised. The surrounding information might be necessary or unnecessary for the mathematisation of the problem, but this is independent from the problem's syntax and the stimulus.

To clarify the intended definition of context and the distinctions above, consider Problem 1 provided in Table 1.

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Table 1
Problem examples

Problem 0: Some buyers are looking at the technical information sheets of similar spin dryer machines A and B: Dryer machine A's drum spins 17 times every 10 seconds, Dryer machine B's drum spins 35 times every 21 seconds. Which dryer spins faster?
Problem 1: John works full-time, 5 days a week. He spends \$45 weekly on lunch with a drink included. He finds a very similar place to have lunch with no drinks included; If he decides to have lunch there, he will spend only \$35 weekly on lunch but he will have to spend an extra \$50 monthly on drinks. Based on this information, should John have his lunch at his regular place or try the new one?
Problem 2: For a rock concert, a rectangular field of size 100 m by 50 m was reserved for the audience. The concert was completely sold out and the field was full with all the fans standing. Which one of the following is likely to be the best estimate of the total number of people attending the concert?*
A) 2 000 B) 5 000 C) 20 000 D) 50 000 E) 100 000

* Source OECD (2006, p. 94)

The context for Problem 1 relates to having lunch when working on a full-time job. For example, context involves general aspects such as value for money and the number of working weeks of a full-time job in a calendar year to solve the problems. Here, for the mathematisation of the problem this context is needed for students to realise that a full-time job week in the state of Victoria, Australia, has 5 days and 48 weeks respectively. In this problem, the words and the gramatical structure give the syntax, whereas the stimulus is the actual physical layout written on the page, without diagrams.

Levels of Context Use (LCU)

LCU was introduced by De Lange in 1979. However, the definitions provided by De Lange (1979) need clarification; De Lange's first and second order uses of context blur because in the examination of their descriptions essential differences between them are difficult to be distinguished. In a previous work from the author of this paper (not yet published and not reported here), LCU was clarified, subjected to statistical validation, and re-defined as follows:

- *Zero order use of context*: At this level, context provides the opportunity to take direct actions or make direct inferences from the instructions given in a mathematics problem. Therefore, context of a problem is not used to interpret mathematical results or arguments.
- *First order use of context*: At this level, context is used to either identify or select relevant information, variables, or relationships for the mathematical formulation of a problem. Also, context is used to determine the adequateness of the mathematical results.
- *Second order use of context*: At this level, context is used to either define or retrieve relevant variables, relationships, or assumptions for the mathematical formulation of a problem. Also, context is used to judge the adequacy of the mathematical results and arguments in terms of the original problem.

Under this new classification of LCU, problems presented in Table 1 were classified as follows: Problem 0 in Table 1 was classified as zero order. Here, the context provides a chance to take direct actions for formulating the problem in mathematical terms. It is expected that Year 10 students know how to use the particular mathematical model needed (i.e., compare ratios) in a range of contexts. Neither assumptions nor a new mathematical model nor interpretation of the results are required, hence the emphasis is on the mathematics involved but not on the problem context. Problem 1 was classified as first order. The context of the problem is needed to mathematise the problem. Students have to identify relevant information (e.g., number of weeks of a full-time job) to formulate mathematically the problem. It is considered that results do not need to be further interpreted or evaluated against the context. Lastly, Problem 2 was classified as second order. The context of this problem is required to find the number of people that can be accommodated per square metre. Therefore, context provides a chance to define assumptions and constraints to use a model and validate the answer in relation to the context.

For this research, LCU is used as an analytical tool to analyse how the context of a problem is used to formulate a problem in mathematical terms and to interpret the answer in relation to the context of a given problem.

Context Familiarity (CF) and Context Engagement (CE)

A search for definitions of context familiarity revealed unexpectedly that the existing literature does not provide an unambiguous understanding of the term context familiarity, although its effects on the performance of students are researched and reported. Apparently, context familiarity is often implicitly meant by researchers as students' close knowledge or experience. Hence, in this study, a familiar context is drawn from students' everyday experiences. Correspondingly, non-familiar contexts in this study mean that problem contexts are not drawn from the students' everyday experiences.

Context engagement of a problem is understood as a students' desire to solve a mathematical problem within a certain context. Thus, a more engaging context for students in this study means that the context results in a desire for students to solve the problem. Correspondingly, non- engaging context in this study refer to contexts that do not result in a desire for to students to solve the problem.

The Study

In the study, the effects of context familiarity and engagement across LCU on students' performance are investigated. Subjects were from eight Year 10 classes (n = 151, 76 males and 75 females) at one volunteer co-educational selective entry secondary school in Melbourne, Australia. Data collection occurred in Term 4, 2014. Two experiments were devised to observe whether the nature of determination of CF and CE may increase, decrease or maintain the same level of students' performance across problems between experiments. In the first experiment context familiarity and engagement is determined from the researcher's opinion, whereas in the second experiment, these factors (i.e., CF and CE) were determined from the students' preferences to certain contexts using a student background survey (SBS). In both experiments, the problem context was varied whilst holding constant other features of the problem that are known to affect performance (performance is interpreted as average percent correct, score out of 2 points). There are two research questions:

RQ1: Is students' performance better on problems with more familiar and engaging contexts?

RQ2: What is the relative effect of context familiarity, engagement and levels of context use on students' performance?

Characteristics and Creation of Problems Used in the Two Experiments

Six problems (two at each LCU) were prepared and used as pivots to create fifteen extra variants on context. Pivot problems' contexts were not varied so they exhibit less context familiarity and engagement for students. Variant problems are characterised by the same mathematical core (i.e., same mathematical content, competency and model) and LCU of their corresponding pivot, but they differ in the context in which the problem is embedded. The context in which variants are embedded exhibits more context familiarity and engagement for students (either assumed or empirically determined) in both experiments.

To exemplify the latter, Table 2 shows one set of variants on context, from the first experiment, constructed from problems presented in Table 1.

Table 2
Example of one set of variants constructed from problems in Table 1

Problem A0: Some children are making pink paint by mixing together white and red. Maria uses 17 spoonfuls of red paint and 10 spoonfuls of white paint, Marc uses 35 spoonfuls of red paint and 21 spoonfuls of white paint. Whose paint will be darker pink?

Problem A1: John attends primary school 5 days a week. His mother spends $45 weekly on a lunch-box with a drink included for John. John's mother finds a very similar lunch-box at a local supermarket but with no drinks included; if she decides to buy lunch for John there, she will have to spend only $35 weekly on the lunch-box but she will have to spend an extra $50 monthly on drinks. Based on this information, should John's mother buy the lunch-box at the regular place or try the new one?

Problem A2: In a ferry, a rectangular space of size 50 m by 25 m was reserved for transporting cars. The ferry was full with cars. Which one of the following is likely to be the best estimate of the total number of cars being transported in the ferry? Show your work and explain your reasoning.

A) About 60 B) About 125 C) About 180 D) About 200

Table 3 presents the distribution in frequency of the twenty-one problems used for data collection per experiment. As can be seen from Table 3, the number of variants in the second experiment is not consistent.

Table 3
Type, frequency and distribution per experiment of problems used for data collection

Level of Context Use	First experiment		Second experiment		Total
	Pivot	Variant	Pivot	Variant	
Zero	2	2	2	2	8
First	2	2	2	0	6
Second	2	2	2	1	7
Total	6	6	6	3	21

Two reasons explain this disparity. Firstly, three contexts used in the first experiment matched the students' preferences to contexts more or less familiar and engaging to them; hence, this researcher decided to use the same problems in context in the second experiment.

Secondly, creating variants on context was difficult, and reveals important aspects about how context is needed for solving the problem, especially for the second order use of context. Although the basic underlying mathematical core and LCU may be the same, a change in context introduces new assumptions and information and this, sometimes, changed the problems, hence it was not possible to find a balance between pivot and variant problems at each LCU. In creating the matched pair, context was changed, but keeping the same mathematical core of the problem and, to the best extent possible, the problems' syntax and stimulus were held constant (these are not discussed here).

Participants and the Rotated Test Design

The performance of one hundred and fifty one students (n=151) from eight Year 10 classes at one volunteer school in Melbourne, Australia, on fifteen problems was compared using paper-and-pencil tests in two experiments. Sixty three students (n_1=63) were tested in the first experiment, whereas eighty eight students (n_2=88) in the second experiment. All participants were tested in a class period. Because of the limited time for testing in a class period and potential inter-problem effects, a rotated design of eight booklets was used, so that students did not solve both a pivot and its variant. This means that direct comparison of performance at the level of the student was not possible. Each booklet contained three problems, either pivot or variant problems, at each LCU. Booklets were distributed at random to each student.

The Preliminary Rating Study (SBS)

Ratings on CF and CE, obtained from students, were used to guide the construction of problems for the second experiment. The same students who answered the SBS participated in the second experiment. Thirty contexts were presented in a questionnaire to students (n = 88) in the classes assigned to the second experiment. The questionnaire consisted of two sections (each containing the same thirty contexts), which explored separately students' preferences to context familiarity and engagement. Contexts that received the lower and higher frequencies on the two sections of the questionnaire, containing a 5-point Likert scale (scale covering from Not familiar at all to Extremely familiar, and Not engaging at all to Extremely engaging, correspondingly), were selected as less and more familiar and engaging contexts for students respectively. To observe whether the students' ratings of the thirty contexts in the questionnaire all reliably measure the same latent variable (i.e., CF and CE, and therefore a 5-point Likert scale could be constructed); a Cronbach's alpha was run on the sample size (n=88). A Cronbach's alpha coefficient of the ratings' scale on CF and CE was found to be 0.793 and 0.761 respectively. Therefore, the scales can be considered reliable with the sample (Pallant, 2001) on determining CF and CE.

Results

Table 4 below presents summaries of the distribution of the students' performance, in frequencies, as well as means and standard deviations of all the explanatory variables. The students' performance was scored as No Credit, Partial Credit and Full Credit. Also, it offers a summary of the odds ratios of students obtaining Full Credit versus No Credit performance at one level of the ordinal variable. In general, overall performance across contexts more familiar and engaging for students (either assumed or determined) does not show large changes from problems set in contexts less familiar and engaging for students (either assumed or determined). Also, it can be seen that students' performance decreased as LCU increased.

Table 4

Frequency distribution of students' performance by LCU, CF and CE, and a summary of the odds of students obtaining Full Credit versus No Credit performance at one level of the ordinal variable

		Student Performance			Total	Mean	S.d	Odds Ratio (ORs)	P value
		No Credit	Partial Credit	Full Credit					
LCU	Second	65	52	34	151	0.795	0.786	6.639	0.000
	First	28	63	60	151	1.212	0.736	2.627	0.000
	Zero (Reference)	30	14	107	151	1.510	0.807	1	
Total		123	129	201	453				
Levels of CF	Extremely Familiar	21	24	24	69	1.043	0.812	1.358	0.366
	Very Familiar	21	25	28	74	1.095	0.814	1.332	0.360
	Moderately Familiar	35	25	64	124	1.234	0.866	1.05	0.860
	Slightly Familiar	19	24	42	85	1.271	0.808	0.994	0.983
	Not at all Familiar (Reference)	27	31	43	101	1.158	0.821	1	
Total		123	129	201	453				
Levels of CE	Extremely Engaging	10	9	12	31	1.065	0.854	0.652	0.335
	Very Engaging	7	14	11	32	1.125	0.751	0.833	0.642
	Moderately Engaging	40	37	54	131	1.107	0.844	0.888	0.648
	Slightly Engaging	27	39	75	141	1.340	0.782	0.536	0.014
	Not at all Engaging (Reference)	39	30	49	118	1.085	0.863	1	
Total		123	129	201	453				

To test Research Question 1, a Kruskal-Wallis test showed that there is not a statistically significant difference in the mean in students' performance between pivot and variant problems (first experiment: $\chi^2(1)=0.010$, $p=0.922$, with a mean rank of pivot scores of 117.14 for pivots and 117.94 for variants; second experiment: $\chi^2(1)=0.353$, $p=0.553$, with a mean rank of 107.53 for pivots and 112.31 for variants). Therefore, the null hypothesis that performance was equivalent on pivot and variant problems, in both experiments, is accepted. Further Kruskal-Wallis tests showed that there was not a statistically significant difference in students' performance between pivot and variant problems ($\chi^2(1)=0.094$, $p=0.759$), and levels of context familiarity ($\chi^2(4)=4.750$, $p=0.314$) and engagement ($\chi^2(4)=8.420$, $p=0.077$) from the first and second experiment. Considering these results and as pivot and variant problems have, to the best extent possible, the same mathematical core, syntax and stimulus, and were categorised into LCU using the same construct, but varied on the context in which they are set, so it was decided that pivot and variant problems from the two experiments could be blended for the next analyses.

For Research Question 2 the relative effect of all the explanatory variables (i.e., CF, CE and LCU) on the students' performance is investigated. This quantification is needed, mainly, to have a clear picture of the impact of the research design on the students' performance. An ordinal logistic regression analysis is used to test Research Question 2. Data was collected from students using a 5-point-Likert scale for context familiarity (scale ranging from Not familiar at all=1 to Extremely familiar=5*)* and a 5-point-Likert scale for context engagement (scale ranging from Not engaging at all=1 to Extremely engaging=5). The other explanatory variable included levels of context use (coded as: zero level=0, first level=1, and second level= 2). The dependent variable, students' performance, was coded as No Credit=0, Partial Credit=1, and Full Credit=2. As every variable in the model to be reported below is a categorical variable (later coded as ordinal), the odds ratios refer to the odds of students obtaining Full Credit versus No Credit performance at one level of the ordinal variable relative to a baseline level of that variable (i.e., to the lowest ordinal code at each variable).

Table 4 presents concisely the distribution of students' scores across all the explanatory variables, as well as a summary of the ordinal regression containing the odds ratios (ORs), and the statistically significant p-values. In this sample, a test of the full model against a constant only model indicates that there is a statistically significant relationship between the predictors and the dependent variable ($\chi^2(10)=76.621$, $p=0.000$). The Nagelkerke's R^2, an indicator of the goodness-of-fit of the model was 17.6%. Variables whose corresponding odds ratios are significantly greater than 1.0 have significant effects on the dependent variable (i.e., students' performance) in the model. Among the explanatory variables, presented in Table 4, the following were significantly more likely to affect students' performance in this sample: second order use of context (OR, 6.639, 95% CI [2.685, 16.411]) and first order use of context (OR, 2.627, 95% CI [1.673, 4.125]). The model shows that the odds ratio of a student obtaining Full Credit versus No Credit performance was 6.639 times higher on zero level of context use problems than on second level of context use problems, when the effects of other predictors (i.e., levels of context familiarity and engagement) are held constant (i.e., controlled). Similarly, the model shows that the odds ratio of a student obtaining Full Credit versus No Credit performance was 2.627 times higher on zero level of context use problems than on first level of context use problems, after controlling for the effects of other predictors.

Discussion and Conclusion

The present study was designed to investigate the effects of familiarity and engagement of problem context across levels of context use on the performance of Year 10 students. In particular, the context in mathematical problems was varied in CF and CE across LCU whilst holding constant other features of the problem that are known to affect performance. To elucidate how the latter mentioned factors influence students' performance, two experiments were designed. The cohort of participants was divided into two groups according to how CF and CE were determined. The first research question in this study sought to determine the effect of familiar and engaging contexts on students' performance. This study found that students did not perform better on contexts that were more familiar and engaging for them, suggesting that embedding mathematical problems in contexts that were more familiar and engaging for students does not necessarily enhance performance. This is a surprising result given that mathematical problems in context are promoted especially for their supposed contribution to the use of students' knowledge (interpreted here as context familiarity), students' motivation, confidence, engagement and interest in and within a problem. The second research question was related to the relative effect of context familiarity, context engagement and levels of context use on students' performance. Results showed that neither higher levels of CF nor higher levels of CE affected students' performance but rather higher LCU. In the current study, a student obtaining Full Credit versus No Credit performance on a zero order use of context problem was approximately seven times higher than a second order use of context problem and three times higher than a first order use of context problem. It is not clear from the literature whether problem contexts, classified in terms of LCU, have such effect on students' performance. Hence, this result must be interpreted with caution because of the type and number of second order use of problems used in this research. Only modelling and reasoning and argumentation were considered as the mathematical competencies used in the seven second order use of context problems. Creating variants on context was extremely difficult, especially for the second order use of context. Therefore, it was not possible to find a balance between pivot and variant problems at each level of context use. In addition, the latter influenced the smaller number of contexts and mathematical competencies included for this research. Because of this, it was not possible to have as a big picture as desired of the impact of context on students' performance. As a result, with the small sample of problems and competencies used in this research, caution must be applied with the previous results, as these might not be transferrable to other contexts involving different mathematical competencies. Lastly, in order to add robustness to the methodology and result of the present study, a further large-scale quantitative study is needed to investigate the research questions posed for this study, but taking into account more mathematical competencies, problem formats, numbers of problems, and secondary students at different levels of instruction.

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