Young Children's Drawings in Problem Solving

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This paper explores young children's drawings (6 years old) in early number and addition activities in Malaysia. Observation, informal interviews and analysis of drawings revealed two types of drawing, and gave insight into the transitional process required for children to utilise drawings in problem solving. We argue the importance of valuing and supporting the early development of children's drawn mathematical representations to facilitate their successful use in problem solving processes.

Introduction

Representation is a key feature in teaching and learning mathematics. In many mathematics classrooms worldwide, teachers introduce various types of representations including physical and virtual manipulatives, number lines, pictures and written and spoken symbols (Ahmad, Tarmizi, & Nawawi, 2010); Elia, Gagatsis, and Demetriou (2007). Profiting from such exposure, students use various types of representation to access mathematical ideas and solve mathematical tasks. Of concern is the research finding that Malaysian primary school children rely heavily on written symbols representation (Mohamed & Johnny, 2010) and make little use of other representations. This is a worrying situation as researchers strongly emphasize the connection between children's facility in a variety of representation and their mathematical understanding (Lesh, Post, & Behr, 1987).

Research indicates that the use of representations (including physical, verbal and written symbols) can have positive impacts in the teaching and learning of mathematics, by supporting communication of mathematical ideas, understanding of concepts and the solving of problems (Elia et al., 2007). For example, teachers use representations to engage students in mathematical thinking by encouraging students to explain and justify their representations. On the other hand, students may use representation to scaffold their understanding of emerging concepts. While a number of studies have reported the positive function of various types of representations for mathematical understanding, little is known about drawing in early year's mathematics (Crespo & Kyriakides, 2007; Woleck, 2001) particularly with regards to problem solving.

Research background

Drawing is considered to be a natural aspect of children's imaginative and expressive development (Machón, 2013). Although children often begin using icons and symbols prior to starting school, the function of drawing shifts in the school context. Here, the purpose of self-created drawing is often to represent specific meanings, not only for themselves, but also for the comprehension of others. The transition in the function of drawings can be troubling for many children (Machón, 2013), and we know surprisingly little about the shift from 'natural' drawing to mathematical representation (Woleck, 2001).

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Research on mathematical drawing indicates a positive connection between drawing and problem solving. Researchers have asserted that children who produced drawings managed to solve the problems more accurately (Edens & Potter, 2007; Uesaka, Manalo, & Ichikawa, 2007). Through the processes of internalization and externalization during representation-creation and through making connections between the two (Goldin & Shteingold, 2001), problem context can clearly be 'seen' which in turn facilitates problem solutions. Drawings can function as a problem solving tool, whereby children use drawing to model the problem, and later as a counting tool (Badillo, Font, & Edo, 2014). Despite its usefulness in problem solving situations, child-generated drawing does not necessarily connect directly to problem solving performance. This is particularly true for students who lack the mathematical knowledge required for the problem solution (Essen & Hamaker, 1990). Furthermore, a correct drawing does not necessary result in a correct solution (Crespo & Kyriakides, 2007). A number of researchers particularly compared the different types of drawings that children produced and investigated their connection with children's abilities to successfully solve problems. Edens and Potter (2007) assert that children's problem solving performance was dependent on the type of drawing that they produced, with students who created schematic drawing performing better than peers who did not produce this type of drawing.

However, researchers observed that students do not always produce drawing as a problem solving strategy (Uesaka et al., 2007). One reason is associated with the status of drawing; that is, drawing is not highly valued in mathematics classrooms (Soundy & Drucker, 2009). The low usage of this particular representation seems to be associated with children's perception of drawing in the classroom context. When children perceived drawing as a *teacher-only strategy*, they thought it was too difficult for them to produce a drawing themselves (Uesaka et al., 2007), and perceived no benefits in the use of mathematical drawing (Essen & Hamaker, 1990).

The Study

The study reported in this paper was part of a larger project investigating the variety and use of mathematical representation by children (aged 6 years), in their first year of school in Malaysia. The purpose of this particular study was to explore the children's disposition towards drawing in mathematics, the types of drawings they produced, and the ways in which they used drawing in problem solving. Specifically this study will address the following research questions:

1. What were the children's dispositions towards drawing?
2. What types of drawings did the children produce?
3. When and how did children use their drawings?

Based on the constructivist theory of learning, children were provided with the experience of creating and using drawings to solve addition problems. Children were not taught specifically what and how to draw; rather, they were prompted to create drawings that made sense to them. By prompting the children to represent their own meaning of addition and to later solve problems, children were actively exploring and building their own understanding rather than passively receiving knowledge from the researcher. As Goldin and Shteingold (2001) state, internal and external representations are linked to each other, and internal representations may be inferred through their externalization - in this study, in the form of drawing. As the teacher reported that the children had no experience in making drawings in mathematics, the opportunity existed to explore the children's earliest attempts at self-created mathematical drawings.

The study took place in a 'pre-school' in Malacca, Malaysia. Six children in the same class were purposefully selected to represent differing levels of early mathematical achievement. The initial selection was based on the teacher's recommendation and then confirmed by the pre-test. The researcher worked only with this group of children throughout the study, while the teacher continued to work with the remainder of the class. No data were collected from children outside the focus group. Since the study was conducted during the first term of school, their mathematical achievement was solely based on a pre-test that focused on counting abilities and the application of counting skills to basic addition and subtraction tasks – this being the scheduled topic for study in the curriculum. Although not included in the data presented in this paper, it should be noted that a post-test showed that all children substantially improved their basic number skills during the study period.

During their first two months of school, the children had been working on counting skills, but had not yet received any formal instruction on the concept of addition and thus had not been specifically taught to solve addition problems. For this study, the researcher acted as the teacher with the group of six children over a period of five weeks, introducing them to the addition process through modelling with concrete materials. Using manipulatives (e.g. pens and cubes) as a starting point, the researcher guided the children to also explore a variety of representational forms, including drawings and with symbols. Then the children were asked to work independently on similar addition problems. In the final tasks, they were invited to create any representations (including drawing) that would help them find the solution to the problems verbalized by the researcher. The data for this paper is predominantly drawn from the final task, with a particular focus on the drawings produced by the children, although other types of representations may also have been used to help solve the same problem. The problems were all about finding the total number of legs for a small number of animals, but the challenge level was varied according to each child's performance in the pre-test. (Problem A: 2 tigers and a chicken; Problem B: An elephant and a tiger; Problem C: 2 tigers).

Four of the children (Qaisya, Rozy, Ali and Amy) were assessed as being *perceptual counters*, meaning they required contact with actual items whilst counting. However, Qaisya and Rozy demonstrated greater confidence and stability in applying counting. Norman and Nadia obtained the highest scores in the pre-test and were identified as *figurative counters*, meaning they could count effectively without actual items being present. Norman also showed some recall of number combinations to ten, though the depth of his understanding of the addition process was unclear. Nadia was capable of doing mental counting. The more able counters (Norman and Nadia) were given addition problems involving three addends. Less able counters (Qaisya, Rozy, Amy and Ali) were requested to solve addition with only two addends. However, the problems given to Qaisya and Rozy involved two different types of animals, while the problems given to Amy and Ali involved the same type of animal.

Data Sources and Analysis

Data collection included observations, conversations with children, field notes, artefacts (students' drawing), audio and video recording. Initially, children's drawing were analysed and categorized into different types of drawing. Video analysis of children working exhibited various ways of using drawings at different phases during the problem solving. Children's talk helped to explain the drawings they created and to further clarify the thinking involved for working out the tasks. The data were summarised and organised by compiling a table annotating each child's pre-test score, drawings and associated talk, as well as events and behaviours that informed their mathematical thinking. By using such tables, a child's drawing could be linked across various data sources and thus provide a rich picture about his/her drawings. Also, the table allowed for comparison of drawing and thinking among different children. The drawing products, in combination with the processes involved (identified through observations and conversations with children) provided further details about the role of drawing, and revealed the challenges that students faced in relation to drawing in problem solving.

Findings

Due to space limitations, only the data from Norman, Nadia, Rozy and Amy are reported in this paper, with their responses considered to be representative of the range of findings for the whole group.

What were the children's dispositions towards drawing?

At the beginning of the study, Norman, in his response to the 'drawing in problem solving task', claimed that he did not know how to draw. Similarly, the other children showed reluctance. They stayed silent for some time without doing anything with the paper and pencils in hands. They only began after assurance from the researcher that the 'quality' did not matter and it was more important that the drawing made sense to them. However, the children repeatedly expressed their concern about their constructions and sought reassurance from the researcher. For example Norman showed his drawings to the researcher and asked, "Is this alright?" Only after receiving positive responses from the researcher was he confident to continue drawing another picture.

In an early task, Norman ignored the instruction to produce a drawing as one means for reaching the answer. Since he had mastered the required number facts, he used his number knowledge instead of making drawings as part of the problem solving. Norman resisted the use of drawing throughout the study. He did not voluntarily use drawing to assist with the task he found most challenging – Problem A. He was not able to successfully process the three addends using numerals and operation signs only, which was always his preferred approach.

What types of drawings did the children produce?

The prompt to produce drawings for solving the problem resulted in the creation of two types of drawing – *pictographic* and *iconic*. Drawing is categorized as pictographic if it has realistic depictions of the objects stated in the problem. On the other hand, iconic drawing contains only simple lines and shapes to embody the intended objects. See Figures 1 and 2 for examples.

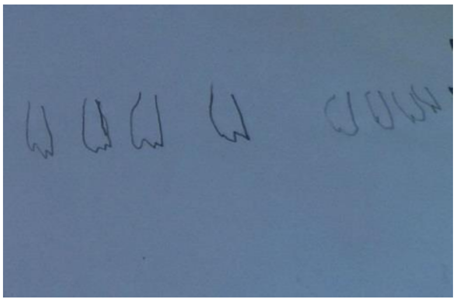

Figure 1: Rozy's drawing: Pictographic

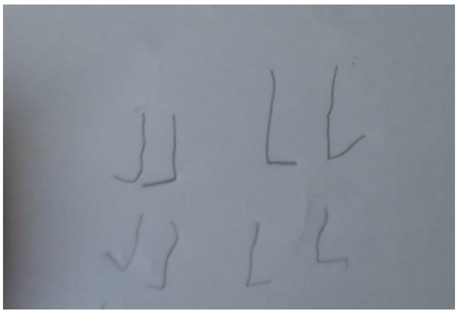

Figure 2: Amy's drawing: Iconic

Given Problem B, Rozy created a *pictographic* drawing as in Figure 1. To illustrate the tiger and elephant legs, she drew the animals' feet with claws. Although elephants have toenails rather than claws, she drew sharp claws to depict wild animals' feet. In contrast, Amy produced an *iconic* drawing to solve problem Problem C. As in Figure 2, Amy drew simple lines in an L-like shape and mirrored L-like shape. She first drew a longer vertical line to represent the animals' leg and then attached a shorter horizontal line to portray the animals' foot.

Given Problem A, Nadia and Norman also produced pictographic and iconic drawings, but they both added symbols to their drawing thus producing an additional two types of drawings- *Pictographic with symbols* and *Iconic with symbols*.

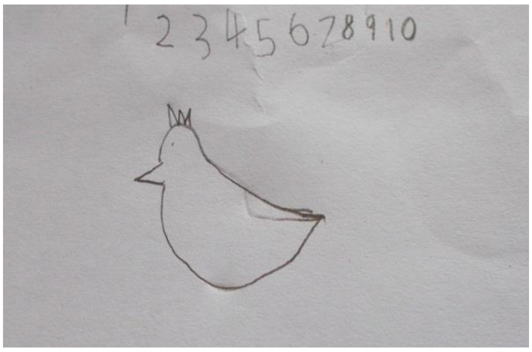

Figure 3: Nadia: Pictographic with symbols

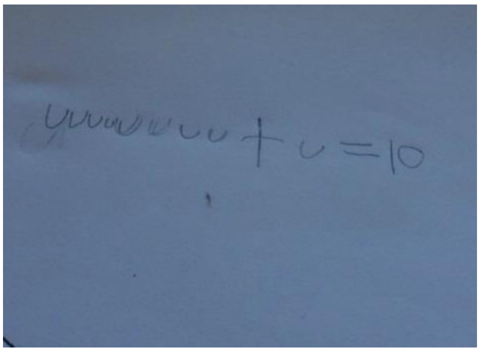

Figure 4: Norman: Iconic with symbols

Nadia produced a *pictographic* drawing (Figure 3) - a picture of a chicken that depicts the animal in everyday life. She chose not to attempt an illustration of a tiger, but rather included the numerals 1 to 10 to her drawing, as a count of all the imagined legs. In contrast, Norman produced an *Iconic* drawing by creating a simple U-like shape to illustrate each foot. He did not add any details that did not directly relate to the mathematics of the problem question. He inserted the '+' symbol and '=' symbol, and finally wrote the numeral 10.

When and how did children use their drawings?

Amy created drawings immediately when starting work on the problem. She drew the legs then pointed to them as she counted one-by-one. As a perceptual-counter, the external representation (drawing of legs) was needed for her to apply her counting strategy. Therefore Amy's drawing was integral to her problem-solving strategy. However the relationship between Amy's external representation (drawing of legs) and her internal representation (e.g. mental pictures of tigers) was not apparent.

In contrast, Rozy, Norman and Nadia created their drawings *after* solving the problems using other means. These children created drawings to demonstrate how the answers had been obtained. Initially, both Rozy and Norman used manipulatives to solve the problem. They then recreated the solution in the form of a drawing (Figure 1 and Figure 4). Both children made a '*translation'* between two forms of external representation - from manipulatives to drawings. Nadia also produced a drawing (Figure 3) *after* performing the counting in her head. She intentionally created the drawing to show her thinking processes. When asked to explain how she performed the mental counting, she pointed to her drawing and said that she first imagined the animals and then performed the counting in her head, starting from 1. In Nadia's case, the relationship between internal and external representations was apparent. She made a 'translation' from images in the mind to pictures on the paper.

In their drawings, Rozy and Amy represented the quantities for the addends accurately. However, the children did not explicitly show in their drawing that addition involves combining two groups of objects. Rather the children exhibited their early understanding of addition, whereby addition contains two groups of objects (evidenced through separation of the groups). Their understanding of addition was implicit through their actions on the artefacts, in which they counted all the marks to get the total.

Discussion

Previous studies suggested drawing as a popular, comfortable and familiar activity among children, particularly when it was a child's choice to make a drawing (Papandreou, 2014). Yet, the children in this study did not spontaneously produce drawing as part of mathematical activity. Although they expressed a 'lack of ability' in drawing, it was apparent that the difficulty actually arose from the lack of experience in *drawing for the purpose of mathematical representation*. The children were daunted by the change in the function of drawing from a personal expression to a communication of precise meanings (Machón, 2013). The children's lack of confidence tended to be worsened by their attempts to draw realistic pictures of the animals (Pictographic drawing). In Norman's case, his ability to use known number facts for most of the tasks reinforced his preference for writing numerals, even when he encountered a problem that he could not solve with this approach.

This finding is consistent with the claim made by Uesaka and colleagues (2007) that both cognitive and behavioural factors, including children's perceived efficacy and confidence, influenced children's decision to use, or not use, drawings spontaneously. As the activities in this study were apparently the children's first encounter with mathematical drawing, it seems likely that their disposition towards drawing as a problem-solving tool would become more positive with further successful experiences.

The influence of the problem context – the animal 'story' - should also be considered. The animals were familiar to the children and they made various efforts to overcome the difficulty of making pictures of the animals. The majority of them drew only the most relevant part of the animal (the legs), instead of the whole body. By creating the animal legs in various forms, including drawing 'real' legs or simple shapes, the children demonstrated the ability to represent the *quantities* necessary for solving the problems.

Thus the children were able to extract the implied quantities from the word problems even though no actual numbers were specified. On other occasions, they substituted drawings with symbols to overcome the difficulty of making realistic depictions of the animals. Nadia created the chicken as a whole, as she felt capable of doing so (Figure 3), but did not illustrate the tigers. Instead, she visualised the tigers and wrote a series of numerals to represent the quantity of tiger legs she had mentally counted. Norman too replaced the drawing of multiple legs with the numeral 10, as he found it easier and quicker to do so. Additionally, the inclusion of symbols ensured meanings were communicated clearly. As in Figure 4, Norman used signs ('+' and '=') as well as numerals to exhibit how he obtained the answer. The drawing of claws, the invention of shapes (L and U-like shapes) to signify animals' legs and feet, and the inclusion of symbols demonstrated children's different strategies for making sense of the problem context.

Regardless of the type of drawing (pictographic or iconic) created, the children's drawings contributed to the problem solving process. Unlike the study reported by Edens and Potter (2007), problem-solving performance in this study was not dependent on the amount of detail included in the drawing. This may be due to the lesser complexity of the problems in this study compared to the problems in Edens and Potter (2007) study.

The children's choices of when and how to use drawing revealed the different roles drawing can have in problem solving. All students used drawing to communicate their thinking. However the sharing of the mathematical thinking was done in different phases during the problem solving and for different purposes. Less able counters, like Amy, relied heavily on the pictures she created to process the mathematics. As observed by other researchers, the drawing permitted the children to initially model the problem situation, then use the pictures as a counting tool to reach the answer (Badillo et al., 2014). These children benefitted from the drawing at all phases during problem solving including the beginning of tasks. However, more able counters tended to produce drawings after obtaining the answers using other approaches, such as modelling with concrete materials, mental counting or recalling known facts. They drew to share the solution. Thus their drawing was a *translation* from other types of representations, used to confirm and explain their answers. According to Lesh et. al. (1987), facility in making such translations, provides evidence for children's mathematical understanding.

The findings from this study generally support the advantages of using drawing in problem solving reported by other studies (Uesaka et al., 2007), but highlight the individual differences in children's early attempts to utilise mathematical drawing.

Conclusion and Implications

Despite the reluctance to draw exhibited by children throughout this study, with prompting they produced drawings with various degrees of sophistication at various phases during the problem solving process. The different functions that drawing served in this study indicate the potential of drawing as a problem-solving tool for children with different skill levels. The findings from this study support the idea that the transition to drawing as a mathematical representation in the first year of school is not a natural one, and appears to require deliberate pedagogical activity from the teacher. For example, providing children with rich opportunities to reflect on a variety of examples of drawn representation, and to explain and justify their own creations, may assist children to further develop their representational strategies. Further research, including longitudinal study, is needed to determine effective teaching strategies and learning experiences that will build young students' confidence and skill in using drawing as a problem-solving tool. Such research should attend to the problem-solving process in its entirety, rather than viewing 'drawing' only as a completed artefact. Consideration of the relationships between drawing and other forms of representation also appears to be important. The range of responses from just a small group of children suggests that learning experiences that allow for individual student difference may be a more appropriate frame for pedagogy than skills-based training or direct instruction approaches alone. Above all else, drawing as a mathematical representation, must be meaningful to the drawer.

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