Researching and Doing Professional Development Using a Shared Discursive Resource and an Analytic Tool

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Linked research and development forms the central pillar of the 5-year Wits Maths Connect Secondary Project in South Africa. Our empirical data emphasised the need for teaching that mediates towards mathematics viewed as a network of scientific concepts, and the development of the notion of 'mathematical discourse in instruction' (MDI), as an analytic tool and discursive resource for working on research and professional development. This paper describes and reflects on MDI, its emergence in a particular education context, and what this discursive resource offers more generally as it works across different discourses and practices.

Introduction

It is well known that poverty is strongly associated with poor educational outcomes, and that inequitable socio-economic conditions are the most significant factor in inequitable educational outcomes (OECD, 2013). We also know in our field of mathematics education, that despite building expertise over many years in doing and researching professional development, links between investments in such activity, the quality of teaching school mathematics, and equitable educational outcomes remain tenuous. In the light of these claims, a question must be asked as to whether, and then if so how, it is possible to impact mathematics teaching and learning in conditions of deep inequality and high levels of poverty through professional development. What might be appropriate and meaningful goals for improving learner attainment in low-income communities, or in the context of this conference, in the margins? What role, if any, is there for discursive resources in realising such goals?

The first question remains the driving force for the Wits Maths Connect Secondary Project (WMCS), a research and development project working with mathematics teachers in ten disadvantaged secondary schools in one district in South Africa. WMCS has worked in its first five-year phase (2010-2014) with a key goal of strengthening teaching and learning of mathematics through professional development of teachers in these ten schools. It is a complex project with multiple additional, and at times competing goals: goals for advancing knowledge and research on related questions and problems in mathematics education, for building research capacity through linked doctoral studies; and for developing and investigating sustainable models of professional development. These professional goals are complemented by a social justice goal in our field, where 90% of the research we do and thus much of the knowledge we build, takes place in, or in relation to, adequately resourced and functioning schools (Skovsmose, 2011). WMCS has been inspired by the challenge of investigating the research-development nexus in mathematics education in poorly resourced conditions – and so the learning and teaching of mathematics in *schools for the poor* (Shalem & Hoadley, 2009). We have learned a great deal over the past five years, and have reported results on the impact of our professional development intervention on student attainment (Pournara et al, forthcoming), and the workings of the overall project (Adler, 2014). In this paper and presentation I focus in on one key aspect of our work, and that is the discursive resource and analytic tool developed

2015. In M. Marshman, V. Geiger, & A. Bennison (Eds.). *Mathematics education in the margins* (Proceedings of the 38th annual conference of the Mathematics Education Research Group of Australasia), pp. 25–40. Sunshine Coast: MERGA.

to support our professional development work and our research, to engage the second question posed above. The more we learned with and from teachers and learners in their classrooms, the more we were able to sharpen our core research questions, and to construct a framework – called Mathematics Discourse in Instruction (MDI) – to support deliberate movement between the discourses of research, professional development and teaching, and so between the overlapping communities of practice (Wenger, 1998) in which the overall project was participating.

The MDI framework characterises the teaching of a mathematics lesson as a sequence of examples together with the tasks they are embedded in, and the accompanying explanatory talk, two commonplaces of mathematics teaching (and thus high-leverage practices (Grossman et al, 2009)), that occur within particular patterns of interaction in the classroom, and towards a particular goal or what we refer to as an 'object of learning' (Marton & Tsui, 2004). As intimated above, MDI has developed over time. In previous research work across WMCS and a similar project in primary schools, we conceptualised MDI to examine coherence within a task, and so between the stated problem or task, its exemplification or representation, and the accompanying explanations (Venkat & Adler, 2012); and more recently to examine coherence across a sequence of tasks/examples and accompanying explanatory talk within a lesson, and in relation to the intended object of learning (Adler & Venkat, 2014; Adler &Ronda, 2014). It was our empirical data that emphasised the need for coherence, and teaching that mediates towards mathematics viewed as a network of scientific concepts (Vygotsky, 1986), and towards generality (Watson & Mason, 2006). More recently we have used an expanded MDI analytic framework, illustrated in Figure 1 below, to examine shifts in exemplification and explanatory talk in classroom discourse, and have described our methodology in some detail (Adler & Ronda, forthcoming).

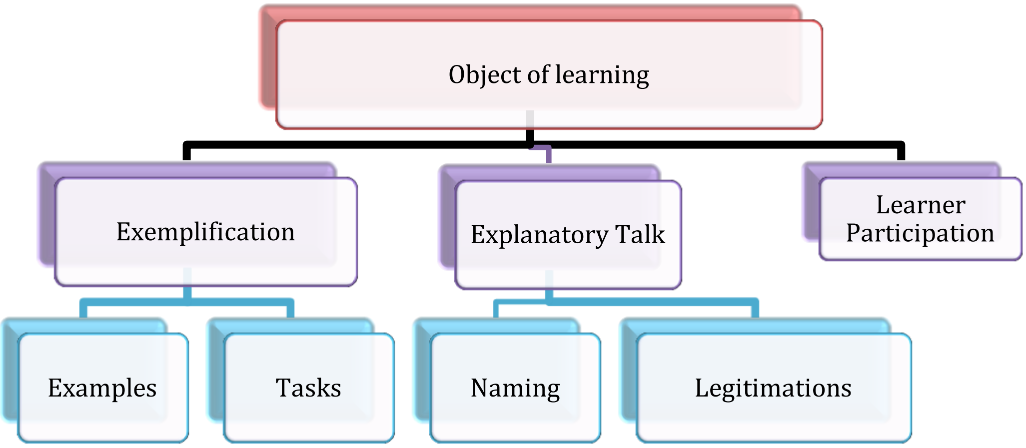


Figure 1: The MDI analytic framework (in Adler & Ronda, forthcoming)

Amidst this research work, we reported on our understanding of MDI as a *boundary object* as we were simultaneously using a form of it in our professional development work with teachers (Venkat & Adler, 2013). Drawing on Star & Griesemer's (1989) notion of *boundary objects* we viewed MDI illustrated above as "plastic enough to adapt to local needs and constraints" of our different practices and discourses, but "robust enough to maintain a common identity across sites" (p. 393). We were particularly concerned with an instrument that resonated with teachers, connecting with their practices in ways that enabled us to engage in the joint and shared enterprise of working on teaching to improve opportunities for mathematics learning in their schools and classrooms. We developed and refined MDI informally first and then through trialing across *mathematical knowledge for teaching* courses – 20 day content knowledge for teaching courses that are offered within our respective projects.

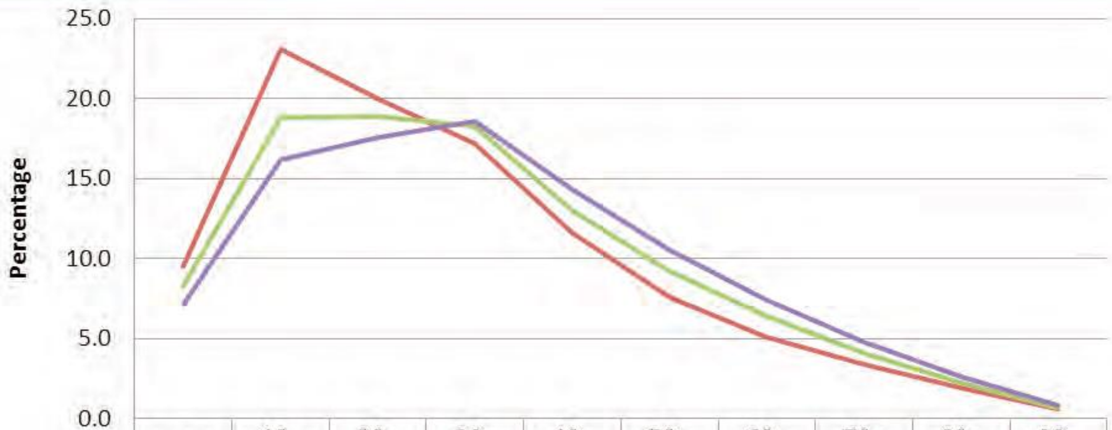
I draw from all the above work and its interconnectedness as I describe how and why MDI emerged in this form and reflect on what it does in relation to the goal of impacting the teaching and learning of mathematics. It goes without saying that the emergence of MDI is a function of its context, specifically mathematics education in post-apartheid South Africa, and the interaction of this 'ground' with discourses in the field of mathematics education and my own previous research. It is also a function of the desire early on in the project to produce a resource – an overarching frame - that could move across our overlapping communities and differing discourses. I thus begin with a brief account of the mathematics education terrain in South Africa, and the conditions of teachers' work in *schools for the poor*; followed by a brief detailing of some of the 'realities' indicated by research findings early on in our project, that further illuminate common mathematics teaching practices in South Africa, and provide the impetus for the MDI framing above. I link these with literature and research in mathematics education and so too the elaboration of MDI before moving on to illustrate how we bent MDI towards the needs and design of our professional development work, and describe how we extended and operationalised it for research. This background, I hope, will enable appreciation of the WMCS in its location, and at the same time, connect with mathematics education on the margins elsewhere. I conclude with some reflections, what MDI illuminates and obscures, and with work therefore that lies ahead.

The South African Mathematics Education Context

Broad Patterns of Performance and Conditions of Teachers' Work

We are twenty years into our new and still rather young democracy. It is deeply troubling that education in post-apartheid South Africa is described, in research and in public debate, as being in a state of 'crisis' (Spaull 2013; Taylor, Van Der Berg & Mabogoane, 20013). Research over the last decade has established that problems of low educational outcomes for a majority of learners is apparent in South Africa as early as the end of the Foundation Phase or third grade. Whilst this is the pattern across education, the problems of performance in mathematics are deeper, with Mathematics showing consistently lower levels of performance at Grade 12 level than most other subjects (South African Institute of Race Relations, 2012).

The graphs in Figure 1 below show the performance distribution curves for Mathematics (2011 - 2013), as presented in the National Senior Certificate Diagnostic report in South Africa (DBE, 2013, p. 126). While improvements in the system as a whole are visible, with failures decreasing and more obtaining better scores, the evidence is stark: the South African education system, and mathematics within this, is failing most of its learners. The performance curves in 2009 and 2010 in the WMCS schools had a similar shape, though more exaggerated, as all are relatively poorer performing schools. The challenge for the project was whether a research informed professional development project could work with teachers to shift this curve in and across schools, to reduce the large failure rate and very low performance of the majority, and increase learners obtaining scores over 60% and so with possibilities for tertiary study in the mathematical sciences.



	0 - 9.9	10 - 19.9	20 - 29.9	30 - 39.9	40 - 49.9	50 - 59.9	60 - 69.9	70 - 79.9	80 - 89.9	90 - 100
2011	9.5	23.1	20.0	17.2	11.6	7.6	5.1	3.4	1.9	0.6
2012	8.3	18.8	18.9	18.3	13.0	9.2	6.4	4.1	2.2	0.7
2013	7.1	16.2	17.6	18.6	14.3	10.5	7.4	4.8	2.6	0.8

Figure 1: Performance distribution curves Mathematics (DBE, 2013, p. 126)

Two additional contextual issues in South Africa are important to highlight here, as they are typically not foregrounded in the research on professional development, and both relate to the conditions of teachers' work. We learned very early on in the project, that whatever the desired intervention might be, it would interact with and thus need to be deeply cognisant of the conditions of teachers' work. We were guided in this, firstly by time spent becoming familiar with the schools in the first year, but also by an insightful analysis of the dual economy of schooling in South Arica, and the impact of this on teachers work. Shalem & Hoadley (2009) studied the relationship between inequality, teacher morale and their conditions of work, and identified four factors that impact on this work. They argue that:

> [t]eachers experience inequalities in terms of their access to:
>
> ● learners who are cognitively well-prepared for schooling, are physically healthy and whose homes function as a second site of acquisition;
> ● meaningful learning opportunities in the past and in the present and a reservoir of cognitive resources at the level of the school;
> ● a well-specified and guiding curriculum; and
> ● functional school management that mediates the bureaucratic demands on teacher time. (p.127)

The relevance of this study to our work was that it revealed resources (the authors refer to these as assets) in teaching that are less visible, but resources non-the-less. We can divide South African teachers into three analytic categories based on this understanding of assets. In one category are (roughly 20%) teachers whose experiences are mediated by the presence of all the resources listed above. They work in schools for the rich, produce good student achievement and are associated with the provision of quality education. At the other end, also roughly 20%, are teachers who work in schools for the poor and whose work and experience is shaped by the absence of all these assets. In between, and also with relatively low educational outcomes are the majority - 60–70% - of teachers in South Africa, whose work is mediated by some but certainly not all of these assets. The teachers in the schools in our project are in this last category, facing a situation where many learners in their classes are not academically prepared for the grade level they are in, and so an ongoing tension between meeting curriculum requirements for the specific grade, and at the same time meeting many learners where they are, mathematically speaking. Collectively, teachers in this schooling band, while qualified, have had poorer disciplinary and professional learning opportunities, and their schools are on lower scales of functionality. As Shalem & Hoadly argue, teachers here have to "… expend much more effort to develop their learners and the task is insurmountable given the property and organisational assets available to them" (2009, p.128). Six of the schools in the WMCS project were termed *priority* schools, which meant they were subject to significant levels of bureaucratic control. The mathematics teachers have to follow a specified term by term, week by week, teaching schedule and learners write common assessments set by officials in the district offices who also then check on the school and teachers for compliance. In broad terms, increased time pressures bear down on teachers who are subjected to high levels of bureaucratic demands that aggravate already low morale.

Linked to this, and the second area of impact on teachers' work, there is increasing curriculum prescription and an assessment regime that impacts teaching and learning - a condition shared in some countries that do not have extremes of poverty and inequality (e.g. the United Kingdom). In South Africa, we currently have Annual National Assessments in Grades 3, 6 and 9, and while these are meant to be for diagnostic and systemic purposes, they have become an additional pressure on teachers and schools. The effect of these processes in secondary schools in particular, in addition to broad low morale, is on teaching/learning time. The space for exploring and building, for example, more exploratory mathematical work and dialogic classroom interaction valued in the field is highly constrained, and markedly so in priority schools where the bureaucracy bears down heavily, expecting and monitoring teachers' compliance with official decrees.

How does or can a professional development (PD) intervention meet these conditions on the ground, where the shared goal with teachers and schools of improving opportunities to learn come up against low morale and this key tension in PD work – teachers' time? PD is premised on the availability of time; however this might be organised, for the teacher to engage in life-long learning in their work. The irony here that I have tried to make visible, is that while time constraints and pressures for improved mathematics performance affect PD everywhere, and this is well documented (see discussion of tensions in Adler (2013)), these are acute in schools with low educational outcomes – and generally then in schools *for the poor*.

Performance and Practice in our Schools and Further Rooting MDI

The first year of WMCS is best described as a time of 'getting to know' the schools, and mathematics teaching and learning in them. We piloted a diagnostic test in algebra, with Grade 8 and 10 learners. The results of these tests, and a rerun of this in Grade 9 and 11 the following year, were distressing. Not only did errors proliferate across items, but within an item errors were too diffuse to formulate clear categories to organise and enable discussion of the range of responses offered. As we shared these results with teachers, we were able to use this data to open up conversation about the absence of both skill and meaning with respect to algebraic symbolic forms for the majority of learners, even learners who had selected mathematics as a subject of study in Grade 10; and thus open discussion on the daily challenges they faced given the under-preparedness of many of their learners.

Our observations of many lessons provided us opportunity to consider how teaching, and more specifically MDI, was implicated in the apparent incoherence in learner productions in the tests. We observed teachers explain some examples for the announced focus of a session, often with poor levels of coherence between the example and its elaboration, and/or across a sequence of examples. By way of brief example, in one lesson on the products of expressions, three different sets of rules were provided: multiplying expressions with exponents ("if the base is the same we add the exponents"); multiplying a monomial and binomial ("you multiply everything inside the bracket by the term outside the bracket"; multiplying two binomials ("we use the distributive law, and multiply first, inner, outer and last terms [FOIL]). Aside from the instructional talk being focused on the 'how to' steps of procedures, devoid of explanations that provided rationales for these steps (Adler & Venkat, 2014), there was no narrative related to operation of multiplication of different expressions that could have connected the lesson parts and reduced the inevitable result of learners having to rely on multiple visual cues and memory if they were to reproduce such products independently themselves. Compounding such practices was the ways in which teachers used words to name what they were talking about – we observed extensive use of ambiguous referents in teacher talk.

Most of the lessons we observed proceeded with examples and explanations of what was stated as the focus of the lesson, but, as illustrated above, mathematical goals or *objects of learning* were out of focus. We identified two key areas of issue within pedagogy to focus on in our professional development work: Mediating mathematical ideas (this point takes in findings related to ambiguity within teacher talk, and the lack of explanations that establish rationales for action in teachers' handling of specific examples); Progressing understandings towards ideas that build generality, effectiveness and efficiency (this point incorporates the selection and sequencing of examples and ongoing promulgation and acceptance of rule-based strategies that relied on visual cues, memory or imitation). Much of reform based mathematics teacher education engages these pedagogic issues of mediation and progression towards generality through *rich* tasks where mathematical exploration becomes possible through orchestrated dialogic teaching. These practices are viewed as providing possibilities for deepening mathematical knowledge for teaching, and advocacy of such task based or problem based teaching in teachers' classrooms.

Whether the underlying or source of the issues is in pedagogic practice or mathematical knowledge for teaching (and later I discuss the intervention and its focus on the latter), both construct the teacher and the teaching as in deficit, as wholly problematic. We believed strongly that a reform-based orientation would not be an appropriate route to take for WMCS. So we focused our attention on the object of learning being out of focus and how this might be pursued through the themes of *exemplification* (selection and sequencing of examples and related tasks) and teachers' mediation of these through *explanatory talk* taking cognizance of learners' current understandings – and so with resonances with their deeply interwoven cultural practices in their classrooms. Hence, the initial and first layer of elements of the MDI framework in Figure 1 above.

Interestingly, within mathematics education, significant bodies of literature underlie both aspects, and I turn briefly to those studies dealing with examples and talk/explanations in ways that are particularly salient to the issues we have raised above as well as to my own prior research in the field.

Linking with Mathematics Education Research

Focus on examples

The ubiquity of examples within the terrain of mathematics teaching and learning has been acknowledged (Bills, et al., 2006). This follows from a basic maxim that initial experiences of mathematical concepts and procedures, given their abstract nature, will be through some exemplification: through examples and the tasks in which they are embedded. Watson & Mason (2006), for example, have noted the importance of carefully structured example sequences that draw attention towards generality whilst working with particulars:

> the learning of particular interest to us here is conceptual development. This means to us that the learner experiences a shift between attending to relationships within and between elements of current experience (e.g., the doing of individual questions) and perceiving relationships as properties that might be applicable in other situations (p. 92)

Rowland (2008) has also emphasized the need for careful selections and sequencing of example for practice, noting that learners should also experience the range of examples that a procedure can be applied to, to have a sense of the breadth of the ‘example space’, and to build not just fluency, but also efficiency across the procedures one is practising.

Both Rowland, and Watson & Mason discuss the importance of variation amidst invariance in the teaching and learning of mathematics, referring to theoretical work on variation theory (e.g. Marton & Tsui, op cit; Runesson, 2006) that has come to figure in the literature in mathematics education and exemplification. Variation theory rests on the underlying notion that learning something depends on access to distinguishing variation in the thing to be learned. The form of example sequences ‘fits’ this model of learning well, with traditional example sets in mathematics often being set in graded forms that lend themselves to analysis through the lens of variation.

Focus on mediating talk/explanations

The ubiquity of ‘explanation’ as a form of pedagogic talk in mathematics classrooms has also been acknowledged. Andrews (2009) for example, noted the need for teacher explanations to be ‘relevant, coherent, complete and accurate’. In previous research work (e.g. Adler & Davis, 2006), we operationalised such explanatory talk through Bernstein’s (2000) key insight that pedagogy proceeds through evaluation, and through what was legitimated as knowledge in pedagogic practice. We developed tools for analysing the criteria transmitted as to what was valued in school mathematics or in teacher education, finding this productive and illuminating of what was constituted as mathematics in these pedagogic sites. We have included this in MDI as part of explanatory talk, and as a means for observing whether and how explanations in school mathematics classrooms are coherent and accurate.

In addition we also drew on previous research that foregrounded the importance of how words are used in multilingual mathematics classrooms (Adler, 2001). Mathematical objects come to life not only through activity on tasks and selected examples, but also in how they are named, and thus the importance of movement between informal or colloquial talk and more formal and literate use of mathematical words in school mathematics. In the context of WMCS work, ambiguous use of referents, and so not naming mathematical signifiers appropriately can obstruct learner participation in mathematical discourse. Hence, our specific and additional attention to naming within explanatory talk in MDI.

This brief foray into the literature illuminates the third row of boxes in Figure 1 above, and so the expanding out of two key elements of MDI (exemplification and explanatory talk) to include examples, tasks, naming and legitimating criteria.

As suggested but not explicitly stated, our observation of teaching across classrooms in the schools in which we work is that there is a dominance of more traditional teacher-led whole class working rather than the more dialogic interactional forms described in the international literature. Thus, the focus on teacher's selections and use of examples and explanations 'fits' with the prevalence of more traditional pedagogies. A critique of this twin focus relates to the relative *absence* of the learner in this frame. Linked therefore to the earlier mention that the goal of pedagogy is to improve mathematical learning, we added in a focus on *participation* alongside the other two categories in MDI, guided by the need to explore mediation and progression of mathematical ideas across these features. We have used this discursive resource, underlain by the lenses gained from the more local, and broader research findings, as a tool for analysing videotaped lessons, and as a boundary object for developing pedagogy for mathematics learning. In the remaining sections of the paper, I turn now to discuss our PD work, and related research. There are constraints on space here, and so I only provide illustration of our work in relation to MDI.

MDI in WMCS Professional Development Work

Earlier, I mentioned the "20-day" mathematics for teaching courses in our PD intervention. These are the major components of our work. We have two courses: Transition Maths 1 (TM1), which is aimed at the transition from Grades 9 to 10 (in our system between what are referred to as General and Further Education); and Transition Maths 2 (TM2) aimed at the transition from Grades 11 and 12 into tertiary study. As we got to know and appreciate the diverse knowledge and experience of the range of mathematics teachers across the ten schools, so it became necessary to organise our mathematics focused PD at different levels. The TM courses were not part of our original plan, but became the form in which we could meet teachers mathematically, so to speak, as well as practically. Teachers come to the University for 16 full days in eight 2-day sessions spread over the academic year. We negotiated with the district and schools for teachers to be released from school on 10 of those days, with 6 days then committed from teachers' own time (on Saturdays or in school vacation time). The additional 4 days of the course were allocated for in-school work. This arrangement dealt with the practicality of time for PD work for teachers. Mathematically, we realised that it would be of most value if teachers had adequate opportunity to engage with mathematics in their PD time, hence the two-day sessions; but also that they would have time in between sessions for working on their own mathematics with their colleagues, independently from course tutors. Between each of the two-day sessions, teachers had mathematics assignments that included work on strengthening their fluency and conceptual understanding, as well as a teaching assignment to try out in their classroom or with some learners.

The bulk of each course, 75%, was on mathematics, a function of our developing understanding that an underlying difficulty for many teachers was articulating what it was, mathematically, they wanted learners to know and be able to do. Our starting point then for strengthening this was to provide opportunity for teachers to strengthen their own relationship with mathematics in the first instance.

The remaining 25% of time in the courses focused on MDI and its elements (exemplification, explanation, and learner participation, all in relation to an object of learning) and we called this a Mathematics Teaching Framework in the PD. We worked on each element separately and then together in various sessions in the courses, structured by the discursive resources in Figures 3 and 4 below. For example, in the first day of a course we would have a two-hour session where we worked on selecting and sequencing examples, typically for a lesson related to content being dealt with in the mathematics sessions earlier that day. Teachers examined textbooks, and other teaching materials for what were exemplified, and how, in a particular topic; whether these were *good* examples, and well sequenced. This opened space for discussion of what made examples, and sets of examples, *good*, or *coherent with the object of learning*, and we shared with teachers, key tenets of variation theory, of seeing similarity and difference, as a means for doing this work. At some point following, we would introduce the framework, and so our boundary object recast for work in the PD. In following sessions we then dealt with each of the columns in Figure 3, elaborating these, as illustrated in Figure 4 for explanation.

In the latter half of the course we have a *lab* lesson during one of the course days, where a class of learners from one of the schools comes to the University (this was typically arranged for a Saturday session). The course leader and teachers planned the lesson together in a session on the previous day. They used the framework to bring attention to the mathematical goal, and how the selected examples and tasks, their sequencing and their mediation in talk through naming and justifying, supported the intended learning object. Attention was then also turned to learner participation – to what learners would be asked to do, say, write and how this would enable their learning. The course leader then taught the lesson, teachers observed, and made notes, using the framework, on an empty version of the table in Figure 3. After the lesson, the course session would be a reflection on the lesson, again using the framework to guide discussion. This adapted version, drawing from both Lesson Study (with resonances with the Japanese model) and Learning Study (the Swedish model), is also then carried out in schools. Teachers from neighbouring schools come to one school once a week in the afternoon for three consecutive weeks once a term, to work in a similar fashion as described above. Planning takes place in week 1, the lesson is taught by one of the teachers in week 2 with one class of learners, and revised, and the revised version is taught by a different teacher with another class in week 3. While the project assists with co-ordination and planning, teachers themselves teach the lessons, and collaborate on its design, reflection, redesign and so on. One WMCS team member works with each group of teachers. Here too, the framework is used as a discursive resource to guide planning and reflection.

This in-school work, while occurring after school hours, provided an opportunity for teachers to collaborate on and study their teaching with their own learners (or those of a colleague), and on an agreed and shared problem. Questions like: “What do we want learners to know and be able to do?” “How do the examples and tasks selected support this?” “Where the examples well sequenced?” “What of the talk? How did it move between every day or informal and then mathematical talk”. “How full were explanations that evolved?” “Were learners participating and how?” “Ultimately, did learners learn what we intended? How can we know?”

The tables in Figures 3 and 4 below are examples of the resources that structure this working on practice together, using MDI in its practice-based version.

<table>
<tr><th colspan="3">Object of learning : teaching x to y</th></tr>
<tr><th>Examples and tasks</th><th>Explanations and talk</th><th>Learner activity</th></tr>
<tr>
<td>What examples are used?
• To start off the lesson
• To develop the lesson
(these may be “examples of”)
• To introduce a concept
• To ask questions
• To explain further
• For learners to practise/ consolidate
(these are “examples for”)

What are the associated tasks?
• What are learners required to do with the example/s?

➢ How do these combine to build key concepts and skills?</td>
<td>What kinds of explanations are offered?
• What (and why)
• How (and why)

• What representations are used?

➢ How do these help to build the key concepts and skills?</td>
<td>What work do learners do?

e.g. listening, answering questions, copying from the board, solving a problem, discussing their thinking with others, explaining their thinking to the class

➢ How does their activity help to build key concepts and skills?</td>
</tr>
<tr><td colspan="3">Coherence: Are there coherent connections between the object of learning, examples, tasks and explanations?</td></tr>
</table>

Figure 3: The Mathematics Teaching Framework

<table>
<tr><th colspan="5">Object of learning</th></tr>
<tr><td rowspan="6">Examples and tasks</td><th colspan="3">Explanation
What does the teacher say and do to help learners make sense of the mathematics beyond the current lesson?</th><td rowspan="6">Learner activity</td></tr>
<tr><th>What is written?</th><th>What is said?</th><th>How is the maths justified?</th></tr>
<tr><td>What does the teacher write (publicly) regarding the mathematical object?</td><td>How does the teacher talk about the mathematical object?</td><td>How does the teacher justify the mathematics?</td></tr>
<tr><td rowspan="3">Words, phrases, sentences

Terminology and expressions

Graphs, illustrations, figures

Definitions

Procedures

Solutions

Proofs</td><td>Colloquial language
Everyday language
e.g. “taking x to the other side”
Ambiguous referents for objects
e.g. this, that, thing</td><td>Non-mathematical cues
Visual cues, mnemonics
e.g. smiley parabola
Metaphor related to features of real objects
e.g. This is how it “looks”, “sounds” , “how you remember”</td></tr>
<tr><td>Some mathematical language
to name object, component
e.g. factor, parabola, derivative
Reading a string of symbols
e.g. “x into x plus 2”,</td><td>Local mathematical
Specific/single cases
e.g. triangles in standard position, expressions with only positive terms
Established short-cuts and conventions
e.g. FOIL, SOHCAHTOA</td></tr>
<tr><td>Extended and appropriate mathematical language to name mathematical objects and procedures
e.g. “the product of two binomials”, “subtracting the additive inverse”</td><td>General mathematical
equivalent representations, definitions, properties, principles, structures, previously established generalizations

Note: A general mathematical justification could be partial/incomplete/full.</td></tr>
</table>

Figure 4: The Mathematics Teaching *Framework*, with elaboration of explanation.

As anyone working with Lesson Study would know, building and sustaining such communities is not trivial work, nor is the functioning of the study group. It is beyond the scope here to elaborate our trials and tribulations in this work in detail – I will talk to this in the presentation. I focus some discussion, however, on the salience of the framework and the discursive resources that support it and teachers working with it.

We know from our research study (see below) of videos of lessons of teachers prior to taking the TM1 course and then some time after completing it, that the selection and sequencing of examples improved – with respect to criteria we established - across many of the teachers in the research sample (Adler, 2014; Adler & Ronda, 2014; forthcoming). This research result confirms our experience with the lesson/learning studies we have done in 2014 that planning, reflecting on and critiquing the example sets in the lesson is the part of the framework and tool that teachers engage with most easily. There were also shifts in our research data on attention to word use, and working between informal and formal mathematical talk across teachers over time. Here too, and this is not a surprise in a multilingual setting, teachers noticed learners' use of words, and the particular words or phrases that they found difficult – and were aware of their own challenges in navigating and revoicing these. Teachers who taught the lessons in our after school work often raised these language issues as the first things to discuss in the lesson reflection.

At the same time, these shifts in parts of exemplification and explanatory talk were not supported by tasks that required more than simple known procedures by learners, and where learners had more opportunity to enter the discourse both through what they did and what they were able to communicate. Alongside this, the kinds of explanations for both procedures and concepts did not seem to move from justifications asserted by the teacher, stated rather than derived, or single case examples and so without connections and moves to greater generality and mathematical power. These difficulties were visible in the research data and remain a challenge in our lesson/learning study work. The discursive resources do not function at this point, as a means for thinking about and talking about this key aspect of teaching and so MDI in the classroom. Task demands and how justifications are built are linked with learner participation, and how teachers connect learners with mathematics. The entrenched cultural forms of pedagogy in these classrooms remain difficult to shift. Where these shifts are visible, and across our data are numerous attempts by teachers to invite learners into more complex tasks, and to agree, disagree with what is being offered, their mediation has tended to reduce the task demands. In managing discussion where disagreements were voiced, the mathematical substance of these remained largely hidden or implicit.

Evident in this discussion of using MDI in our professional development work with teachers, the particular form it has taken and what has taken place is deeply interwoven with our research work and insights from analysis of video lessons. As these happen coincidently, each has informed and shaped the other. I now move on to discuss how we have used MDI in research.

An Analytic Framework for MDI

Table 1 in Figure 5 below presents the framework. I briefly elaborate each of the analytic resources, and our analytic categories, derived from the research literature mentioned earlier and in interaction with our empirical data. Visible in the categories is our interest in scientific concepts and increasing generality in examples; increasing complexity in tasks; colloquial and formal mathematical talk and mathematical justifications for what counts in the discourse. With respect to participation, we are interested increasing the opportunity for learners talk mathematically, and teachers' increasing the use of learners' ideas.

Our unit of study is a lesson, and units of analysis within this, an event. The first analytic task is to divide a lesson into events, distinguished by a shift in content focus, and within an event then to record the sequences of examples presented. Each new example becomes a sub-event. Our interest here is whether and how this presentation of examples within and across events brings the object of learning into focus, and for this we recruit constructs from variation theory (Marton & Pang, 2006). The underlying phenomenology here is that we can discern a feature of an object if we can see similarities and differences through what varies and what is kept invariant. Variation through similarity is when a feature to be discerned is varied (or kept invariant), while others are kept invariant (or made to vary), with possibilities then for seeing generality; contrast is when there is opportunity to see what is not the object, e.g. when an example is contrasted with a non-example; when there is simultaneous discernment of aspects of the object is possible, further generalisation is possible. These three forms of variation (similarity, contrast and simultaneity) can operate separately or together, with consequences for what is possible to discern – and so, in more general terms, what is made available to learn. In WMCS we are interested in analysing the teacher's selection and sequencing of examples within an event and then across events in a lesson, and then whether and how, over time, teachers expand the set of examples and the sequencing constructed in a lesson.

Object of learning				
Exemplification		**Explanatory talk**		**Learner Participation**
Examples	**Tasks**	**Naming**	**Legitimating criteria**	
Examples provide opportunities within an event or across events in a lesson for learners to experience variation in terms of *similarity* **(S),** *contrast* **(C),** *simultaneity* **(U)**	Across the lesson, learners are required to: *Carry out known operations and procedures* **(K)** e.g. multiply, factorise, solve; *Apply known skills, and/or decide on operation and /or procedure to use* **(A)** e.g. Compare/ classify/ match representations; *Use multiple concepts and make multiple connections*. **(C/PS)** e.g. Solve problems in different ways; use multiple representations; pose problems; prove; reason.etc	Within and across events word use is: *Colloquial* **(NM)** *e.g.* everyday language and/or ambiguous referents such as this, that, thing, to refer to signifiers *Math words used as name only* **(Ms)** e.g. to read string of symbols *Mathematical language used appropriately* ***(Ma)*** to refer to signifiers and procedures	Legitimating criteria: *Non mathematical* ***(NM)*** *Visual* **(V)** – e.g. cues are iconic or mnemonic *Positional* ***(P)*** – e.g. *a* statement or assertion, typically by the teacher, as if 'fact'. *Everyday* ***(E)*** *Mathematical criteria:* *Local* **(L)** e.g. a specific or single case (real-life or math), established shortcut, or convention *General* **(G)** equivalent representation, definition, previously established generalization; principles, structures, properties; and these can be partial **(GP)** or 'full' **(GF)**	Learners answer: *yes/no questions or offer single words* to the teacher's unfinished sentence **Y/N** Learners answer (what/ how) questions in phrases/ sentences **(P/S)** Learners answer why questions; present ideas in discussion; teacher revoices / confirms/ asks questions (D)

Figure 5: Analytic framework for mathematical discourse in instruction.

Of course, examples do not speak for themselves. There is always a task associated with an example, and accompanying talk. With respect to tasks, we are interested in cognitive demand in terms of the extent of connections between concepts and procedures. Hence, in column two we examine whether tasks within and across events require learners to carry out a known operation or procedure, and/or whether they are required to decide on steps to carry out, and some application, and/or whether the demand is for multiple connections and problem solving. These categories bear some resemblance to Stein et al's (2000) distinctions between lower and higher demand tasks.

With respect to how explanation unfolds through talk, and again the levels and distinctions have been empirically derived through examination of video data, we distinguish firstly between naming and legitimating, between how the teachers refer to mathematical objects and processes on the one hand, and how they legitimate what counts as mathematics on the other. For the latter, we have drawn from and built on the earlier research discussed above. Specifically, we are interested in whether the criteria teachers transmit as explanation for what counts is or is not mathematical, is particular or localised, or more general, and then if the explanation is grounded in rules, conventions, procedures, definitions, theorems, and their level of generality. With regard to naming, we have paid attention to teacher's discourse shifts between colloquial and mathematical word use.

Finally, all of the above mediational means (examples, tasks, word use, legitimating criteria) occur in a context of interaction between the teacher and learners, with learning a function of their participation. Thus, in addition to task demand, we are concerned with what learners are invited to say i.e. whether and how learners have opportunity to use mathematical language, and engage in mathematical reasoning, and the teacher's engagement with learner productions.

Illustration of the use of this framework first on one selected lesson appeared in Adler & Ronda (2014). Further extension of the framework and its use in comparing lessons and so shifts in MDE over time can be found in Adler & Ronda (forthcoming), where categorising events over time accumulate into levels based on our privileging of development towards scientific concepts and generality in the discourse. I do not reproduce these here and refer to the full research papers. Nevertheless, in Figures 6 and 7 below, taken from Adler & Ronda (forthcoming), is the coding of events and how these accumulate into levels for one teacher's lessons in 2012 and then 2013. I present these here, despite the analysis on which they are based not appearing here, so as to enable the discussion following.

Events	Exs	Tasks	Naming	Leg Criteria	Lr Partic
1 – Review Exponent laws	NA	K	Ms, Ma	NA	P/S
2 – Application numerical bases	U, S	A, K	Ms, Ma	L, GP	P/S
3 – Application – literal bases	U, S	A, K	Ms, Ma	L, GP	Y/N
Cumulative level	L1	L2-L1	$L2^{-}$	L2	L2

Figure 6. Summary codes and analysis of Lesson of Teacher X in 2012, in Adler & Ronda (forthcoming)

Events	Exs	Tasks	Naming	Leg Criteria	Lr Partic
1 – Meaning of a Term	S, C, U	K	NM, Ms, Ma	G	Y/N
2 – Meaning of common factor	NA	K	Ms, Ma	G	Y/N, P/S
3 – Simplify algebraic fraction	S, C, U	A - K	NM, Ms	NM, L	Y/N
4 -Divide algebraic fractions (+)	S, U	A - K	NM, Ms	NM, L, G	Y/N
5 – Extension to (-) coefficients	S, U	A - K	Nm, Ms	L	Y/N
Cumulative Code	L3	L2- L1	L2	L2	L1

Figure 7. Summary codes and analysis of Lesson of Teacher X in 2013, in Adler & Ronda (forthcoming)

Discussion

Our MDI framework allows for an attenuated description of practice, prising apart parts of a lesson that in practice are inextricably interconnected, and how each of these contribute overall to what is made available to learn. It co-ordinates *various variables, situations and circumstances* of teacher activity (Ponte & Chapman, op cit). There is much room for this teacher to work on learner participation patterns, as well as task demand (and these are inevitably inter-related); at the same time her example space evidenced awareness of and skill in producing a sequence of examples that bring the object of learning into focus, hence the value of this specific aspect of MDI. Contrasting levels in earlier observation of this teacher indicates an expanded example space and more movement in her talk between colloquial and mathematical discourse. The MDI framework is thus helpful in directing work with the teacher (practice), and in illuminating take up of aspects of MDI within and across teachers (research). As noted, our analysis across teachers suggests that take-up with respect to developing generality of explanations is more difficult.

The MDI framework provides for responsive and responsible description. It does not produce a description of the teacher uniformly as in deficit, as is the case in most literature that works with a reform ideology, so positioning the teacher in relation to researchers' desires (Ponte & Chapman, op cit). We are nevertheless aware that we have illustrated MDI on what many would refer to as a *traditional* pedagogy. We have *tested* it out on lessons structured by more open tasks, but this requires more systematic study on varying classroom practices.

Conclusion

I have written this paper to capture the work of the WMCS project and the development and use of MDI. It is a descriptive paper, as the more directed research is reported elsewhere. Through this I hope to have shared some of the *in betweenness* in our work, as many of us are simultaneously practitioners in mathematics teacher education and in research – and thus boundary crossers in our work. As a keynote paper and without the boundaries imposed by research practices on the one hand, or development descriptions on the other, I have been able to share how we worked within and between these. I hope too that by setting WMCS explicitly in its location, and linking with research literature, I have enabled connections between this work with mathematics education on the margins elsewhere.

As we reflect back and look ahead we are aware and it is important to explicitly acknowledge this here, that there is the learning progression in professional development implicit in the WMCS model as it has developed. The courses and their greater focus on content knowledge of teaching, and teachers' own relationship with mathematics in contrast to attention to pedagogy indicate that we view this as primary. We hold strongly to this view but understand at the same time, that the lesser focus on pedagogy, and further how we have done this with MDI is implicated in that it is teachers have taken it up and what are clearly more challenging aspects of teaching and related opportunities for learning, specifically setting up and maintaining more demanding tasks, and orchestrating greater learner participation in classroom discourse. At this point we do not see this as necessarily as a weakness in the programme, but more an indicator of how learning progresses over time. As we move into Phase 2 of the project, our plan in the first instance is to develop MDI further, where we bring learner participation and the nature of tasks into focus with the teachers, and research what is entailed in this work. An additional focus for our future work is that while we have evidence of the impact of the courses on teachers' knowledge and their learners' performance, we are aware of the time invested as we developed the courses, of our own learning and developing expertise as these were implemented and refined. What then is entailed in making the materials and rationales for the course available for others to use and so more teachers to have such opportunities? What happens as these are taken up and expanded out – to the mathematical experiences offered in the courses on the one hand, and to the interweaving of MDI within and across the sessions? There will be recontextualisation! But of what, how and with what consequences? There is much work to do going forward.

Acknowledgements

This work is based on research of the Wits Mathematics Connect Project, University of the Witwatersrand, supported by Department of Science and Technology, and FirstRand Foundation, and administered by the National Research Foundation. Any opinion, findings, conclusions, recommendations are those of the authors. The authors also acknowledge the WMCS project team all of whom have contributed to MDI.

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