Teaching Statistics in Middle School Mathematics classrooms: Making Links with Mathematics but Avoiding Statistical Reasoning

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Statistics is a domain that is taught in Mathematics in all school levels. We suggest a potential in using an interdisciplinary approach with this concept. Thus the development of the understanding of a situation might mean to use both mathematical and statistical reasoning. In this paper, we present two case studies where two middle school Mathematics teacher taught a lesson in Statistics where the students had the task create a pie graph representing the data. Results show us that their procedural vision of Statistics lead them to focus more on a graphical representation and thus led to avoid all statistical reasoning development (Garfield, 2002).

Introduction

In the 21st century, citizens must be able to solve complex problems that cannot always be done by applying one strategy or a particular algorithm. More creative approaches that require using high cognitive level thinking are needed in order to bring solutions to these problems. Do students have the necessary opportunities to develop the abilities needed to adapt themselves and be productive citizens in their society? Do they have the opportunities to develop their mathematical competencies along with citizenship competencies (Savard, Manuel & Lin, 2014)? We argue that students should be exposed to rich tasks in Mathematics and Statistics classrooms for them to be better prepared to face the realities and problems in society. Such tasks include problems that are: open-ended (can have multiple answers and can be solved using various strategies); complex (require many steps to find answers; require to investigate a particular situation or to pose a question to investigate; ask to make choices and justify them; or require to find patterns, generalise and prove results.); ill defined (missing necessary data that prompt students to search or define them to find answers); have different interpretations; and are contextualised (Manuel, 2010; Manuel, Freiman & Bourque, 2012). However, Mathematics and Statistics are still often taught in a way where students develop procedural understandings of concepts. Using these traditional teaching methods tends to have students see mathematics as a school subject that consists of rules, formulas, equations and algorithms to apply, thus enabling them to make links between mathematics and the real world or even between different mathematical concepts (Boaler, 2009). Yet, studies suggest that students who experience any form of rich inquiry-based learning tasks seem to enjoy learning mathematics, develop more conceptual understandings of mathematical ideas, achieve better in standardised testing, and develop the necessary abilities to solve unfamiliar and more complex problems (Boaler, 1998, Boaler & Humphreys, 2005).

In this paper, we examined how two middle school teachers support students in making interdisciplinary links between statistics and mathematical concepts, and between statistics and the context of the tasks they were solving. We suggest that a very large number of rich tasks could be used in order to study ideas in statistics, and interdisciplinary and intra disciplinary links could be developed while solving those tasks. The teachers selected come from another larger nationwide study on pedagogies used by middle school

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mathematics teachers in regions of Canada. Both teachers taught a lesson on pie charts. One of the teachers who teach Grade 7 mathematics in Quebec proposed a task to compare the frequency of the colours of candies found in Halloween bags (the lesson was conducted a week prior to October 31). The other teacher who teaches Grade 8 mathematics in New-Brunswick used technology to support her teaching and proposed problems for students to solve in her lesson. Kader and Perry (1994) argue that students can also develop statistical concepts and problem solving strategies using technology. The goals of our study were to examine the interdisciplinary links the teachers made between statistical ideas and mathematical concepts, and between statistics and the context of the tasks presented in their lessons. We also examined the teachers' representations of statistics to investigate if their representations guided their actions during the lesson. We defined representations as an intersection between a situation and the knowledge mobilised by a person according to this situation (Brun & Conne, 1990). The following questions guided our study:

- What links did the teachers created between statistical and mathematical ideas in their lessons?
- What links did the teachers create between statistics and the context of the tasks and/or other disciplines?
- What are the teachers' representations of statistics?

Theoretical Framework

The idea of integration curriculum orientation is not new in education (Lowe, 2002). In the context of major worldwide changes during in the last decades and new complex phenomena, it is essential to take into consideration the new social realities. Thus, instead of integrating disciplines as stand-alone school subjects, like it was suggested in the 70's (Lenoir & Sauvé, 1998), the new realities bring forth the need for interaction between them (Legendre, 1993). These interactions might be called interdisciplinarity and could be considered as a negotiation between disciplines, where the development of one discipline contributes to the development of others (Fourez & Larochelle, 2003). It can offer an extended perspective and allows disciplines to support each other. For example, collecting and interpreting statistical data might contribute to understand social or scientific phenomena.

Instructional curricula in Statistics, also known as data analysis (National Council of Teachers of Mathematics (NCTM), 2000), focus on exposing students to a statistical approach, where students from K-12 should develop the abilities to: 1) formulate questions that can be addressed with data and collect, organise and display relevant data to answer them; 2) select and use appropriate methods to collect data; and 3) evaluate inferences and predictions based on data (NCTM, 2000). Throughout this progression, it is aimed that students develop statistical reasoning where they will be able to use descriptive Statistics in order to clearly interpret data with fidelity and with rigor (Ministère de l'Éducation du Québec, 2010; Ministère de l'Éducation et du Développement de la Petite Enfance du Nouveau-Brunswick, 2012). Both the Quebec and New-Brunswick curricula align with the NCTM standards for Statistics. In this paper, we consider that Statistics and Mathematics are two different disciplines, because they have different epistemologies. The reasoning behind those disciplines is not the same. Statistics focuses on an interpretative reasoning that depends on variability, while Mathematics focuses on a deterministic reasoning (Savard, 2014). Statistical reasoning involves a conceptual understanding of important statistical ideas (delMas (2004; Garfield (2002). However, Mathematics might be used to solve statistical problems (delMas, 2004). We understand that in school systems, Statistics are considered as a branch of Mathematics, but in opposition with Carvalho and Solomon (2012), we do not consider that developing Mathematics and Statistics together as a form of intradisciplinarity, where topics or concepts in a same discipline are related to each other.

Method

Context and Participants

This study is a part of a larger nationwide study conducted with middle school (Grades 7 or 8) Mathematics teachers in four different regions of Canada. Four Anglophone teachers from Alberta, four Anglophone teachers from Ontario, four Anglophone and four Francophone teachers from Quebec, and four Francophone teachers from New Brunswick took part in this nationwide study in which the main objective is to describe regional differences in mathematics teaching and underlying pedagogies in Canada, and to relate these to differences in student achievement in mathematics.

In the nationwide study, each teacher had to video-record three lessons: one he/she considered as typical in his/her classroom; one that he/she considered as an exemplary lesson in his/her classroom; and an introductory lesson on an idea related to fractions. Members from the research team would then edit each video keeping the best 15-20 minutes of each lesson. The four teachers from each region and linguistic group would then meet with the research team for focus group meetings. During the meetings, they would watch the edited videos of each of their lessons and discuss the practices and pedagogies they observed in the videos. These discussions would follow a similar protocol. At first, the teacher would explain the lesson he/she did and answer preliminary questions (mostly clarifications about the lesson) other members of the group may have had. Second, they would watch the edited video of her lesson. Third, the teachers and the members of the research team would discuss the lesson. During that time, all the members of the group could ask questions, ask for clarifications, point out practices and strategies that they thought where great and give suggestions. We were members of the research team for three groups: the Quebec Anglophone teachers, the Quebec Francophone teachers, and the New Brunswick teachers.

One Francophone Grade 7 teacher from Quebec and two Grade 8 teachers from New Brunswick made a lesson on pie charts as their typical lesson for this project. We analysed the lessons from the Quebec teacher and one of the New Brunswick teachers. The teacher from New Brunswick is technology oriented. During the group discussion on their lessons (after watching the video), we took the opportunity to ask them about their representation of Statistics. The video of those lessons and the focus group discussions (for those lessons) were transcribed. We used pseudonyms for naming them.

The Lessons and Data Analysis

The two cases we selected permitted us to compare between a teacher who didn’t use technology in her class and one that did.

The teacher from Quebec, Ida, designed the lesson in three parts. In the first part, she presented the task by giving each group of thee students a bag of coloured candy and constructed the question they had to investigate with them (how much candy of each colour their bag contained). She then went over the steps for completing the table that helps construct the pie chart with the students. In the second part, the students solved the task. In the last part, the teacher discussed the task with the students by doing an example using the data from one group. We will mostly focus on the last part because this is where the links were made.

The teacher from New Brunswick, Danya, gave three problems on pie charts for her students to solve. The teacher focuses on three practices with her students: observe, practice and teach. On the previous day, they discussed how to create a pie chart (observing part). This part was not video-recorded. The two problems were for them to practice. They would then teach the concept to a partner. Unfortunately time ran out so the teacher had to do this part on the next day. The teacher and all the students have access to a laptop. The teacher thus used various technologies. She showed her students her blog and informed that they she placed a video that shows how to construct a pie chart in it. She informed her students that it was the best video she found on the internet and those who needed a guide should go on her blog and watch it. She also encouraged her students to get out their diagnostic test that they did at the beginning of the trimester (she gives a diagnostic test at the beginning of each trimester on the content she will cover in order to see where her students are at and guide her teaching throughout the semester) and use it as a guide. The students had to solve the first problem individually and then compare with their neighbour. The task is shown in Figure 1 (translated from French):

The artistic activities of Canadians are the following: photos, 46%, videos, 21%, drawing, 13%, and dance and piano, 10%. Draw a pie graph that represents the artistic activities of Canadians.

Figure 1. First Problem the teacher gave.

When the students finished solving the first task, the teacher would give them a second problem. She explained to her students that this one would serve as a formative evaluation so she could see who understands how to create a pie chart. The problem consisted of a table representing the number of students who used different means of transport to get to school. The students had to create the pie chart using the data. The teacher also challenged the students to create 2 questions about the data from the pie chart. However, the students did not have to answer the questions they invented. When the students finished solving the second task, they would have a longer problem to solve on their laptop. The teacher posted a problem on Google Doc. The students had to access it from their laptops and solve it. She also mentioned that it was an example of problems they could find on the final exam. The task is shown in Figure 2 (translated from French):

Julie will start university next September. She has a monthly budget of $ 1,000. Her expenses include: $ 90 recreation, 20% rent, transportation 2/10 and twenty-five hundredths for food budget. The remainder will be spent on other personal expenses. Construct a pie chart from the data above.

Figure 2. Second Problem the teacher gave.

The teacher constantly walked around the classroom during those tasks and guided the students if needed. She insisted a lot on clearly communicating the process while solving the tasks.

We used the corpuses to analyse and interpret the specific interdisciplinary and intradisciplinary actions related to knowledge building (Savoie-Zajc, 2000). In the results, we present the actions and the links made by the teachers.

Results

Intradisciplinary Links between Statistical Ideas and Mathematical Concepts

In the last part of the lesson, Ida discussed about the effectif (the number of candy in a certain colour in a bag), the frequency (a/b) for each coloured candy (data expressed as a fraction) and the relative frequency (data in percentage). She made links with numbers, fractions and percentages. She reminded the students that the total should be found using addition of the numbers representing the frequency. For the frequency (a/b), she came back to the definition of a fraction, which is a part of a whole. Ida explicitly asked the students why we write 3/9 and made it clear that 3 is the number of red candies and the 9 is the total number of candies in the bag. At the very end, she asked the students about how to fill the relative frequency column of the table but she did not give much importance to it. Ida focused her lesson on the representation and the organisation of data using mathematical notations. When it came to the idea of constructing a pie graph, Ida made links with fractions, the circle and with angles. She started off by asking the students what they knew about a circle. The students answered that it had a 360-degree angle. Following that, she asked the students how to determine the angle of the sectors for each coloured candy in the pie graph. A student answered that, "for the red candy, you can do 3/9 of 360". Ida made it explicitly clear that we can use fractions in other contexts. She claimed, "It is not for nothing that we discussed about fractions, fractions of a number and so on before this. Now you see that we can apply fractions in other contexts other than in arithmetic". She then went over different strategies of doing this calculation by saying that you could use proportional reasoning. Then she did an example with the class, focusing on the strategy of dividing 360 by 9 in order to get the measure of 1/9 and then multiplying the quotient by 3. She proceeded that way because a student suggested that strategy. She seemed very responsive to students' strategies.

In conclusion, Ida created interdisciplinary links between the representation and the organisation of data (Statistics) and Mathematics: mathematical notation, proportional reasoning, geometrical representation and measurement. But, as delMas (2004) pointed out, the lesson did not go beyond the learning of procedures and thus did not develop explicitly a statistical reasoning.

Danya supported students who struggled with changing angles into degrees, finding fractions of a number, such as 2/10 of $1,000 and transforming quantities into degrees (angles). She would question students to support them in understanding why they use particular algorithms, but she didn't make any links between the concepts involved. The entire lesson focused on making calculations to construct pie charts. It is possible that some links were made in the previous lesson, but we cannot make this conclusion since that lesson was not recorded. It is possible that Danya did put an emphasis on the interpretation of data when she challenged her students to make questions about the data on the graph. However, we did not have the students' work to make any conclusion. During the discussion on her video, we questioned her about this. However, she didn't remember what she did with the questions. She recorded the lesson 7 months prior to the meeting and did not remember what she did because she used different approaches with her students.

In conclusion, although Danya had moments where she could have made links between statistical ideas and mathematical concepts; she didn't take this opportunity to make them. The entire lesson focused on finding data by calculation and representing them on a pie chart.

Interdisciplinary Links between Statistical Ideas and the Context of the Problems or Other Disciplines

Ida did not make explicit links between statistical ideas and the context of the problem or other disciplines. She simply focused on the procedure of representing the data on a pie graph. After she finished constructing the pie graph on the board, she stopped her lesson by asking the students if they had any questions. When there were none, she ended the lesson by giving the student problems to solve in the textbook. It seemed that Ida's focus was on creating a pie chart correctly because during the discussion, she put a lot of emphasis on how she corrected problems on assessments and what she needed to see to be able to give marks. It is possible that she chose this action because there was less than 10 minutes remaining. During the discussion group, the other members noticed the fact that she didn't spend time on the interpretation of data (NCTM, 2000) and they made that suggestion to her. The other members mentioned that for example, they could have compared the data between groups, they could have made a set of data of the whole class and them see how the data of each group is similar and different from the whole class data, and to make links with probability, business, economy and other fields that would be interesting to discuss. In conclusion, Ida focused her lesson on the idea of representing data (NCTM, 2000). No inference or predictions were made based on the data.

Danya did not make explicit links between statistical ideas and the context of the problems or other disciplines. When the students worked on the problem on Google Docs, she mentioned that this is where she can see if you can do French along with Mathematics at the same time. However, that comment was focused on students being able to understand the problem since its text was longer than the others and it was more complex. During the discussion group, Danya did realise that she did not put enough emphasis on interpreting data. In conclusion, Danya, just like Ida, focused her lesson on the idea of representing data (NCTM, 2000). No inference or predictions were made based on the data.

Teachers' Representation of Statistics

Ida had a clear representation in mind: "My representation of Statistics is to be able to represent and compare the data. In this activity, I wanted them to be able to use the data and create a graph with it". However, when related to the three ideas of Statistics grounded by the NCTM (2000), we noticed that the ideas of evaluating inferences and making predictions were not in her representations. She saw Statistics as a way to collect, organise, represent and analyse data although the question was built with the class at the beginning of the lesson and that she didn't spend time on analysing the data at the end of her lesson. She didn't mention the ideas about formulating questions in her representation of Statistics. However, in the first part of her lesson, we observed that she spent a good quantity of time stressing the importance of creating clear and rigorous questions. The classroom had a small debate on this aspect. They struggled a bit with creating the question. Ida would often ask if the idea proposed was a good question or not. For example, one mentioned that just saying the representation of the coloured candy is not enough so that it is important to say where the candy came from. The students added the ideas together to create the question. In conclusion, Ida showed a procedural vision of Statistics and her lesson focused on a graphical representation and thus led her to avoid all statistical reasoning development on the stochastics process (delMas, 2004; Garfield, 2002).

Danya also had a clear representation in mind. To her, Statistics was a branch of Mathematics. “It is collect and an organisation of data, interpretation and display them. In this activity, I wanted them to be able to interpret, analyse and display data”. However, in her lesson, she only focused on displaying data. In all the tasks, the data was given to the students. We could argue that she focused on interpreting data when she challenged her students to invent two questions about the data. We did not have access to the necessary data to make this specific claim. In conclusion, although Danya’s goals for her lesson were aligned with the NCTM (2000) standards, the ideas of making predictions and inference were missing from her representations. She saw Statistics as data given to students in order to display them and compare some of them. She also showed a procedural vision of Statistics.

Conclusion

This study compared two lessons on pie charts by two Francophone teachers from two different provinces and using two different types of lessons. Ida used an experimental approach where students collected data (the colour of the candies in their bag) and represented the results, while Danya used problem solving tasks for her students to practice creating pie graphs. The results revealed that both teachers had a similar representation of Statistics. However, their representations did not seem to influence their attempt to make interdisciplinary links with Statistics. Ida made intradisciplinary links between statistical ideas and mathematical concepts, but Danya seemed to approach Statistics as a stand-alone concept. Both lessons were oriented on procedural understandings instead of the interpretation of a phenomenon. Some factors may have influenced our results. We only had access to one recorded lesson. We had no knowledge of what happened prior and after both lessons were video-recorded. It is possible that intra- and interdisciplinary links were made during those times. Also, it is possible that the milieu influenced the choices the teachers made. For instance, in Danya’s case, all Grade 8 students from New Brunswick have to write a provincial exam at the end of the school year. Questions related to statistics on this exam are only on representing data and in some cases interpreting data. It is thus possible that Danya focused her attention on the procedural aspects of creating pie charts to prepare her students for that assessment. These results highlighted some important questions to consider. First, how do teachers’ actions and representations affect students’ learning of Statistics? Second, how does technology support students in making intra- and interdisciplinary links with Statistics? More research is needed in order to bring insight to these questions.

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